Should Do Ch 4 Growth Accounting Report

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1 Explaining the GDP Prediction Gap in the Textbook Model

In doing an analysis of the textbook model, we find that it breaks down the equation for finding the total GDP by this equation:

$$Y = AK^{\alpha}L^{1-\alpha} \tag{1}$$

where:

- Y = Total output (GDP)
- A = Total Factor Productivity (TFP)
- K = Capital
- L = Labor
- α = Capital share of income

At first glance, this seems like an accurate way to describe GDP, and it is. However, the issues that arise with this model arise when we try to predict GDP per capita. GDP per capita is defined as:

$$\frac{Y}{L} = A\left(\frac{K}{L}\right)^{\alpha} \tag{2}$$

In essence, the variable A, or Total Factor Productivity (TFP), represents all other factors that affect productivity and are endogenous to the model. TFP can account for technology, institutions, human capital, or any other elements that enhance a country's productivity. When predicting GDP across countries, most models, including those in textbooks, assume that each country has the same level of TFP, typically set at 1. This assumption implies that all countries utilize their capital with equal efficiency. As a result, differences in GDP between countries are attributed solely to variations in capital and labor, disregarding disparities in productivity. However, empirical evidence demonstrates that countries differ significantly in how they utilize their resources due to various factors.

A clear example of this discrepancy is seen in Burundi. Burundi has a very low level of capital per person, amounting to only 1 percent of the U.S. level per capita. However, the model predicts that Burundi's GDP per capita should be approximately 20 percent of the U.S. level. This suggests that Burundi, in addition to having less capital, is also not using its capital as efficiently as countries like the United States. This inefficiency is explained by Burundi's lower TFP. To better understand why Burundi experiences lower GDP per capita, let's examine the factors that contribute to its TFP.

The first major factor affecting a country's productivity is human capital. Human capital refers to the stock of skills and knowledge that individuals accumulate, making them more productive in society. One of the most significant contributors to human capital is education—an

area where the U.S. excels due to the higher returns to schooling. The average number of years spent in school in the U.S. is 13, compared to just 3 years in Burundi. Countries that invest in human capital development enable their citizens to contribute more effectively to the economy, resulting in higher incomes and growth. Burundi's lack of investment in human capital is a major reason for its lower productivity, yet this factor is overlooked in the textbook model.

The second factor that affects a country's ability to produce is technology. Technology is defined by state-of-the-art computer chips, software, new pharmaceuticals, production techniques, and other advanced innovations. In short, any capital that allows firms to produce at a higher level than others falls within this category. Technology influences TFP because most poorer countries do not have access to the same technological advancements that wealthier countries do. This creates a larger disparity in their production capabilities, particularly in the case of Burundi. A good example can be found in the way farmers in Burundi operate. Generally, they use hand tools and farm for the sustenance of their families or local communities. In contrast, farmers in wealthier countries often use pesticides and larger agricultural machinery, which make the farming process much more efficient. This technological gap contributes to the lag in Burundi's productivity compared to more developed nations.

The last two factors that affect the model's total factor productivity are institutions and misallocation. Institutions are the formal and informal rules, norms, and structures that shape economic, political, and social interactions in an economy. Misallocation refers to the inefficient distribution of resources and firms within a country's economy. These factors often go hand in hand when influencing TFP, especially in poorer countries. Often, this manifests in corruption, bribes, or monopolistic firms that control most of the resources, all of which lower the TFP of the country in question. All of these issues are present in Burundi, which leads to lower productivity than what could be achieved.

All of these factors break down Total Factor Productivity and help give us a more holistic view of a how a country's GDP per capita is affected. However, we can take it further and see how efficiently each country uses their capital and labor by analyzing the marginal product of capital and labor for each term respectively.

The Marginal Product of Capital (MPK) is given by the derivative of the production function with respect to capital (K):

$$MPK = \frac{\partial Y}{\partial K} = A\alpha K^{\alpha-1} L^{1-\alpha} \tag{3}$$

Similarly, the Marginal Product of Labor (MPL) is the derivative of the production function with respect to labor (L):

$$MPL = \frac{\partial Y}{\partial L} = A(1-\alpha) K^{\alpha} L^{-\alpha} \tag{4}$$

Although TFP provides a broader perspective on productivity, we can further refine our understanding by examining how efficiently each country uses its capital and labor through marginal products. MPL and MPK gives us a specific value in how the economy is affected with each unit of labor and capital is added. For example, a country could have a higher efficiency in its use of capital but a lower efficiency in how it utilizes labor. Generally, in countries where capital is scarce, like Burundi, the MPK is higher because capital is less abundant. However, because there is less capital in total the total productivity stays low. Also, investment in countries like Burundi remains low due to political instability, weak institutions, and credit constraints, as referenced in the paragraph before. In contrast, capital rich countries like the U.S., the MPK is lower because there is a lot more capital in respect to labor. However, these countries compensate for this by having highly productive labor and better technological infrastructure. These values allow us to analyze how each country utilizes its resources, providing greater context to the models presented in the textbook.

The textbook model's prediction gap stems from its failure to incorporate key determinants of productivity, particularly TFP. While TFP itself is not a perfect measure, it offers a more comprehensive view of economic output by accounting for human capital, technology, institutions, and misallocation. Additionally, incorporating MPK and MPL helps us understand how efficiently a country utilizes its resources. By expanding our analysis beyond the textbook model,

we can develop a more accurate understanding of the factors shaping economic development across countries.

2 Evaluating the Hall-Jones Model's Accuracy Across Countries

Previously we analyzed the Cobb-Douglas production function that the textbook utilizes, represented by:

$$Y = AK^{\alpha}L^{1-\alpha} \tag{5}$$

We discussed what each variable signifies and its strengths and weakness as a measurement of GDP per capita. In this section we will introduce The Hall and Jones (HJ) model, which modifies the standard Cobb-Douglas function by incorporating human capital as part of the function h explicitly:

$$Y = AK^{\alpha}(hL)^{1-\alpha} \tag{6}$$

where:

- h = Human capital per worker

The intention of the model is to improve upon the standard Cobb-Douglas function by accounting for differences in human capital, which affects labor productivity across countries. The standard production model makes the assumption that all workers are the same and contribute the same amount of labor, while we know for a fact that this is not the case. A clear example is the comparison between South Korea and Nigeria. Both countries have similar levels of capital investments, yet South Korea's GDP per capita is far higher. This gap cannot be explained by the Cobb-Douglas model, which assumes all labor contributes equally to output. The Hall and Jones model improves this by incorporating human capital h, which accounts for South Korea's highly educated workforce and explains why it outperforms Nigeria economically. The HJ model attempts to correct this by incorporating human capital. Our analysis will assess its effectiveness and how this adjustment impacts cross-country GDP comparisons.

The first step in evaluating the accuracy of each equation is to construct a model fit statistic and compare their performance. To do this, we calculate the sum of the squared differences between each model's predicted GDP and the actual GDP, using the equation below:

$$\sqrt{\sum (GDP_i - y^*)^2} \tag{7}$$

The average deviation for each model is as follows:

- Cobb-Douglas Function: 3.140
- Hall and Jones Function: 3.062

Since the Hall and Jones model has a lower average deviation, it provides a better fit to the data. A smaller deviation means that the predicted GDP values are closer to actual GDP values, reducing overall error. The sum of squared differences captures how well each model explains variation in GDP, and since the Hall and Jones function has a lower value, it is a more precise predictor than the Cobb-Douglas model

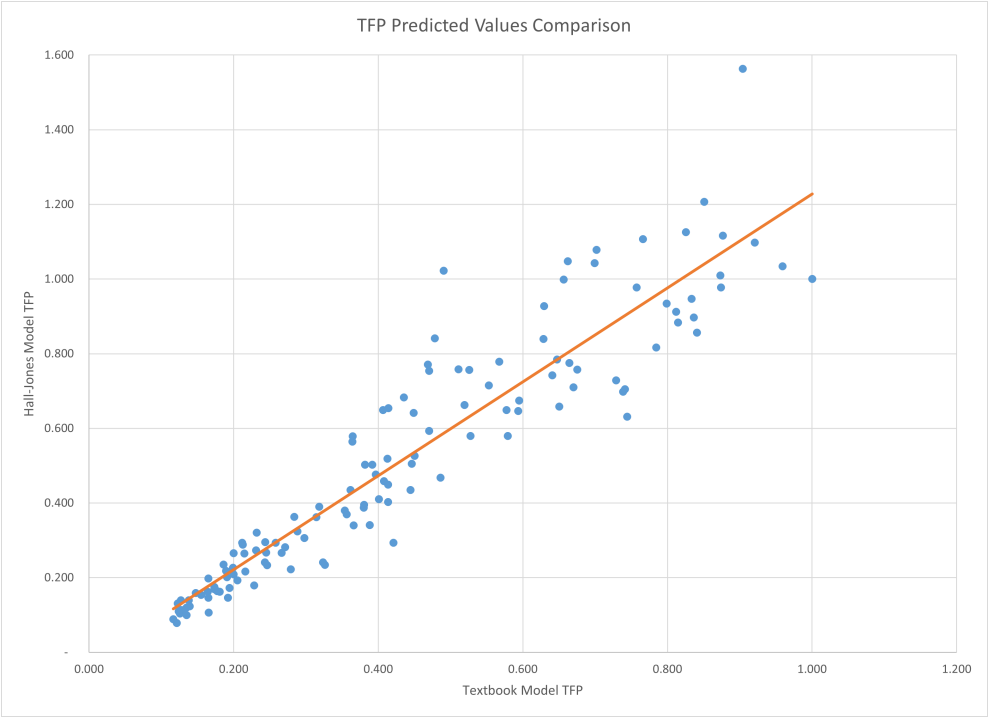
While the Hall and Jones model provides a better fit than the standard Cobb-Douglas function, it still falls short as a precise predictor of GDP. Its primary advantage lies in incorporating human capital, defined as the stock of skills that enhance worker productivity. However, the model measures human capital h solely through education, which presents a significant limitation. The Hall and Jones model assumes that years of education are the primary driver of human capital, but in reality, two countries with the same average years of schooling may have vastly different GDP levels due to differences in institutional quality, technological adoption,

and work experience. For example, two nations with an average of 12 years of schooling may have different productivity levels if one has a more efficient legal system, better infrastructure, or higher investment in development. By excluding these factors, the HJ model still fails to fully explain GDP variations

Measuring human capital solely by years of schooling is insufficient. A more accurate measure would include vocational training and work experience. For example, Germany's workforce excels due to its dual vocational training system, which combines education with hands-on skills. Incorporating such factors into the Hall and Jones model would improve GDP predictions by better reflecting real labor productivity. Incorporating these additional factors into the Hall and Jones model would explain why some countries achieve high GDP per capita despite fewer years of formal education. This would allow for more accurate GDP predictions and give policy makers a complete idea of how different countries are performing.

3 Analyzing the Correlation Between Human Capital and TFP in the Hall-Jones Model

When analyzing the relationship between Human Capital and the HJ model, it is helpful to first begin with comparing the differences between how the Cobb-Douglas function and how the Hall-Jones function represent TFP. This correlation is represented by the figure below:

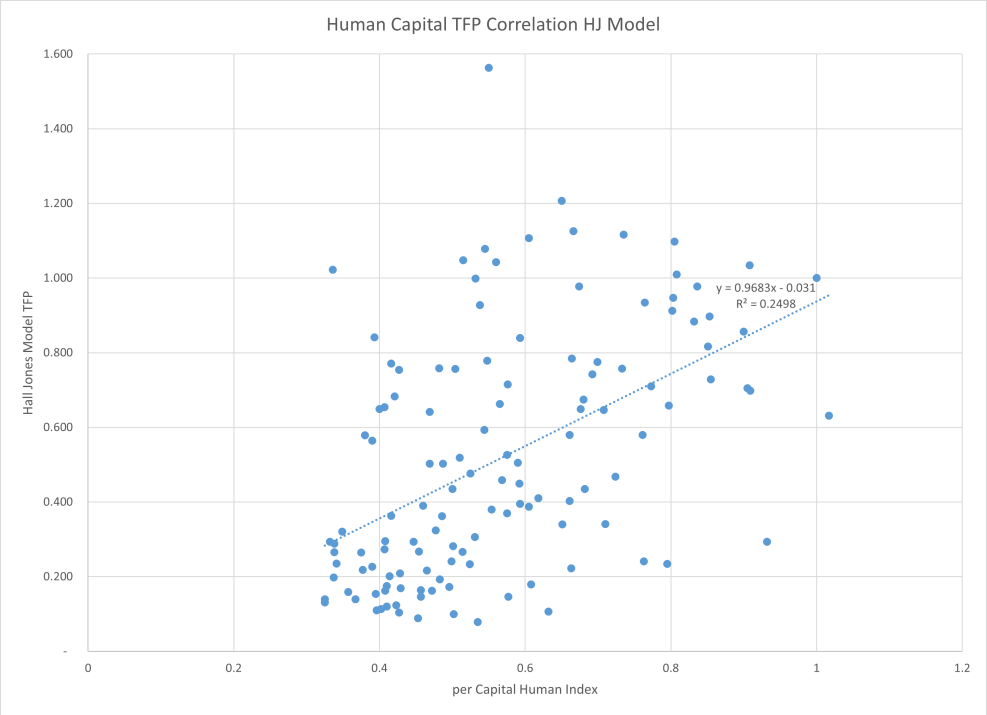


We can see that the prediction for TFP that both models make comes out to about the same. There are some significant outliers, but as a general rule when one model predicts that TFP will be high, the other model makes a similar prediction. This shows that each model uses TFP and does a good job of consistently predicting what TFP is.

Both the Cobb-Douglas function and the Hall and Jones Model utilize Total Factor Productivity (TFP) but the the unique variable that sets apart the equation for the Hall and Jones model is the measurement of Human Capital per Capita Index, which is a measure of the average level of skills, education and health of a population. The intention of measuring this statistic is to give us a more complete picture of the growth of a country and how they produce, as we explored in the previous section.

In the HJ model, TFP serves as a multiplier a multiplicative factor that scales the output that a country produces. Even if the human capital per capita (h) is high a low level of Total Factor Productivity will lead to a lower output. A good example of this is Venezuela; they have a relatively well-educated workforce but suffer from low TFP because of economics mismanagement and corruption. On the other hand, the U.S. has moderate human capital and high TFP, which leads to a higher performance. This gives a good idea of the correlation between the two variables and how important it is in analyzing economic performance.

When analyzing the accuracy of the Hall and Jones model, it is is helpful to do an analysis on how exactly the TFP and the human capital factor are correlated. This regression is represented by the figure below:



The regression equation from this graph is:

$$y = 0.9683x - 0.031 \tag{8}$$

The coefficient of determination is:

$$R^2 = 0.2498 \tag{9}$$

This analysis shows a relatively weak correlation between human capital and TFP. The positive slope suggests a relationship, but the R^2 value indicates that only about 25 percent of the variation in TFP can be explained by differences in human capital per capita. This implies that while human capital is important, other factors significantly influence TFP.

In economies with high TFP, productivity tends to be self-reinforcing circle that continues to grow. As individual companies become more efficient in the economy, they attract more investment, better workers, and technological improvements, which further increases TFP as a whole for the country. A great example of this circle, South Korea in the 1960s had low productivity but invested heavily in education and technology adoption. This led to a rapid increase in productivity, making its firms globally competitive. This self-reinforcing cycle is why high-TFP economies tend to stay ahead, while low-TFP economies struggle to catch up.

When thinking about why human capital specifically may have little effect on TFP, it is key for us to analyze the other factors that make up TFP and how they affect a country. A big example of this is institutions, which make a huge change in how firms can flourish economically.

In countries with well established intuitions, firms face relatively low barriers to entry and only have to focus on the productivity of their business. However in countries with less effective institutions, these same firms face a large amount of other inhibitors of productivity such as corruption, bribes, government seizure in the case of a productive business, high taxes etc. All of these factors can affect business greatly and hamper productivity.

Another factor that contributes a large amount to TFP is misallocation. Misallocation happens when there is an inefficient distribution of resources across firms in the economy in a country. In countries where the financial markets are health and functioning how they should, goods naturally flow the most productive firms which increase TFP. However, with misallocation due to corruption or other factors, this doesn't happen and hampers the total factor productivity.

All in all, TFP and human capital are related but are not directly correlated. While human capital plays a large role in the productivity of a country, there are a wealth of other factors that play into calculating the productivity of a country's economy. When making decisions on policy, policymakers should not only focus on helping people to become more educated but also to focus on improvements in institutions and trying to improve misallocation.