# Should Do Ch 5 Solow Simulations Report

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## 1 Introduction to the Solow Model

The Solow growth model is a predictive framework that explains long-term economic growth through quantifying that growth mathematically. The model uses five equations to accomplish this goal; the production function, capital accumulation, labor force, resource constraint, and an equation for allocation of resources.

Below we breakdown every component of the Solow Model and describe how every equation impacts our predictions that we are able to make about long run economic growth. First we have the production function, typically a Cobb-Douglas function, given by:

$$Y = K^{\alpha}(AL)^{1-\alpha} \tag{1}$$

- $A$ represents labor-augmenting technological progress (TFP), capturing improvements in labor efficiency,
- $K$ is capital,
- $L$ denotes labor, and
- $0 < \alpha < 1$ is the capital share of output.

This equation determines how much the economy is producing, given by a function of capital and labor. This allows us to predict where output is going in the long run and is the foundation of the model. The next two equations of the model are what we look at to determine capital and labor in the above function, starting with the capital accumulation function below:

$$\Delta K_{t+1} = I_t - \delta K_t, \tag{2}$$

where:

- $I_t$ denotes investment at time $t$,
- $\delta$ is the depreciation rate,
- $K_t$ represents the capital stock at time $t$, and

- $\Delta K_{t+1}$ is the change in capital stock from time $t$ to $t+1$.

This functions shows us that the capital we accumulate over time is the capital affected by depreciation, subtracted from investment. Essentially, the change of capital is what stays as investment subtracted by what the economy uses. Below is the equation for labor:

$$L_t = \bar{L}, \tag{3}$$

where:

- $L_t$ is the labor at time $t$,
- $\bar{L}$ denotes the constant (time-invariant) level of labor.

Labor is represented endogenously in the model. The next equation we add is the equation for the budget constraint represented below:

$$Y_t = C_t + I_t \tag{4}$$

- $C_t$ is consumption at time $t$
- $I_t$ is investment at time $t$

An economy's consumption can be divided between these two categories as output is either consumer or invested to create more production. Investment for the model is calculated by the equation below:

$$I = \bar{s}Y \tag{5}$$

where:

- $I$ is total investment,
- $\bar{s}$ is the savings rate,
- $Y$ is total output.

Bringing these elements together, we present the table below.

Table 1: The Solow Model

| **Unknowns / Endogenous Variables** | $Y, K, L, C, I$ |
|---|---|
| **Production Function** | $Y = K^{\alpha}(AL)^{1-\alpha}$ |
| **Capital Accumulation** | $\Delta K_{t+1} = I_t - \delta K_t$ |
| **Labor Force** | $L_t = \bar{L}$ |
| **Resource Constraint** | $Y_t = C_t + I_t$ |
| **Allocation of Resources** | $I = \bar{s}Y$ |
| **Parameters** | $A, \bar{s}, \delta, \bar{L}, K_0, \alpha$ |

Now that we have established the components of the Solow model, we can use it to analyze economic growth and examine how changes in one variable impact the others. To begin, we define the initial parameter values as follows:

$$\bar{s} = 0.2, \quad \delta = 0.15, \quad \alpha = \frac{1}{2}, \quad A = 1, \quad L = 1.$$

For each quantitative experiment, we will adjust one parameter and observe its effect on the endogenous variables of the model, allowing us to understand the model's dynamics and long-run implications.

## 2 Solow Model Savings Rate Experiment

Suppose the United States implements a tax policy that discourages investment, leading to an immediate and permanent reduction in the savings rate from:

$$\bar{s} = 0.2 \quad \text{to} \quad \bar{s} = 0.1 \quad \text{at} \quad t = 10.$$

All other initial parameters remain unchanged. Let's analyze the impact of this policy change on output, capital, investment, and then explore the economic implications.

**Impact on Output**

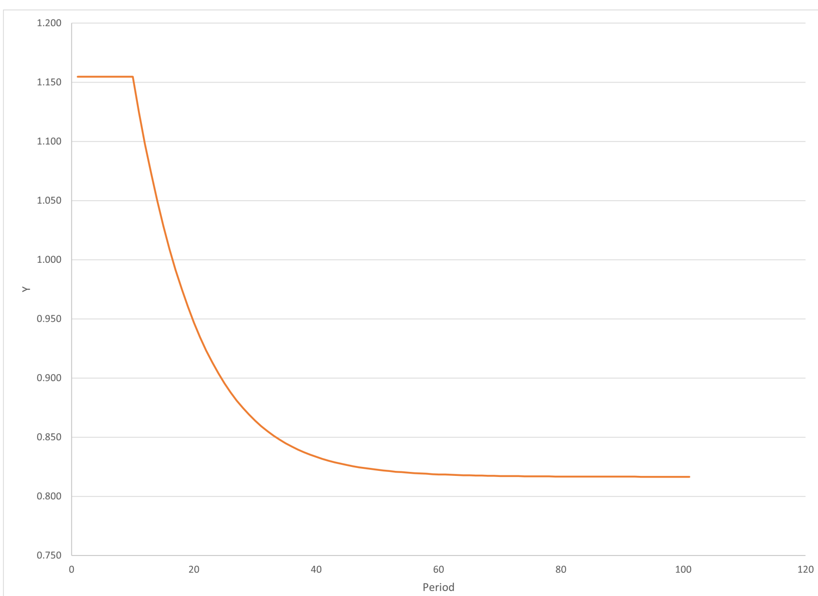



Figure 1: Change in $(Y)$

The graph above illustrates the immediate impact of a reduction in the savings rate on output. Initially, we observe a sharp decline in production as firms adjust to the lower savings rate. Since less income is allocated to investment, the capital stock begins to decline over time, reducing output ($Y$). However, as the economy adjusts, the model converges toward a new steady-state level of production but at a lower level than before.

**Impact on Capital**

To better understand the effect of our change in the savings rate for capital we would utilize the equation for finding $\Delta K$ referenced in the introduction. However, let's start by using the other equation for capital accumulation represented below:

$$K_{t+1} = K_t + I_t - \delta K_t \tag{6}$$

The difference between this equation and the equation for $\Delta K$ is that it allows us to calculate the total change in capital from period to period, rather than just considering how investment affects that change. Below is the graph illustrating how the savings rate affects the total capital amount, utilizing this equation:

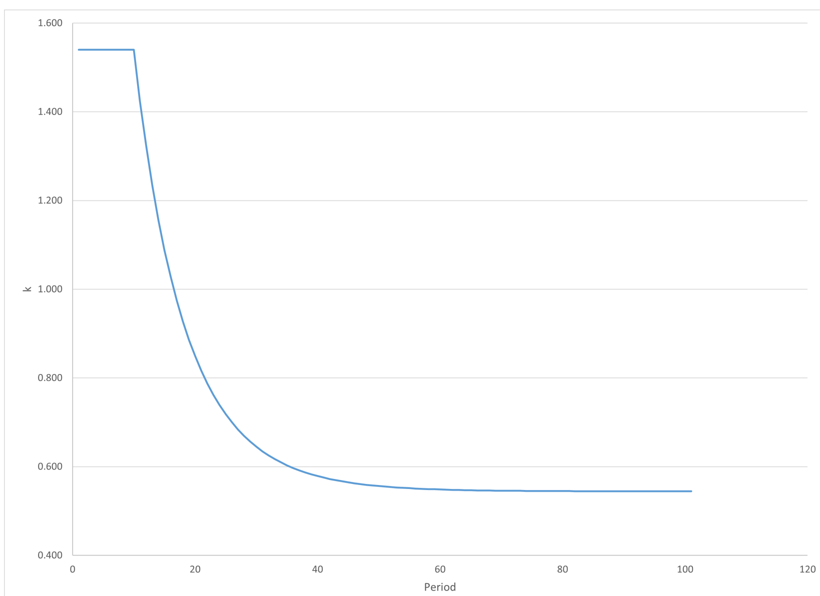



Figure 2: Change in $K$

What we can see is that the drop of capital is initially very large, even more extreme than the change for output. This make sense, as the capital accumulation equation is directly correlated with investment. When the saving's rate goes down, investment in production becomes more expensive, leading to an overall in the firm's capital. This assumption is further substantiated by the graph below:

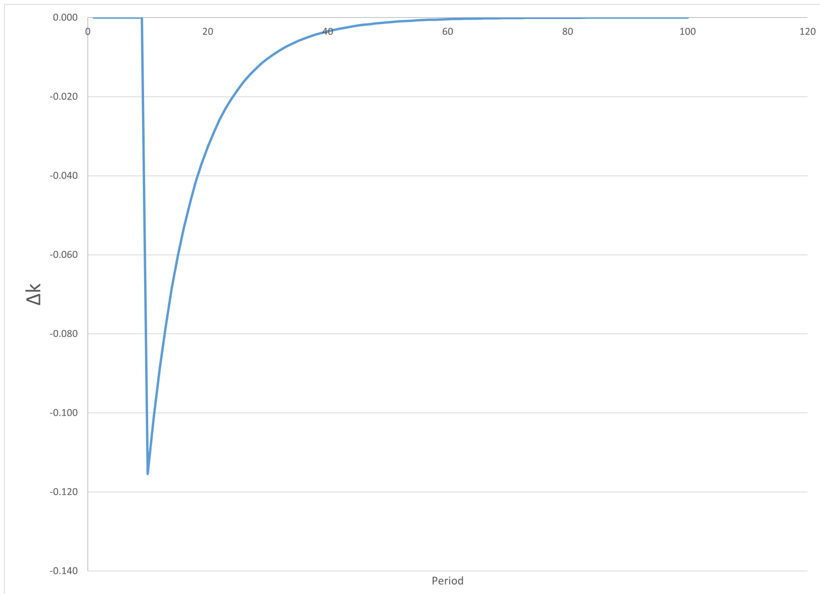



Figure 3: Change in $\Delta K$

As we can see, that initial dip is very large but returns to a baseline $\Delta K$, showing us that the return of capital to it's steady state, albeit a significantly lower steady state than before.

### Impact on Investment and the Solow Phase Diagram

Out of all the variables that are impacted by the savings rate, investment is the variable that has the most extreme change. As we can remember from above investment is equal to:

$$I = \bar{s}Y$$

Since investment depends entirely on the savings rate, a sharp drop leads to a rapid decline in investment, as shown in Figure 4 below. Around period ten, firms no longer find it profitable to invest, redirecting resources elsewhere. This demonstrates how effective such a policy can be in discouraging investment—firms will always seek to optimize profits and will withdraw funds if investment becomes nonviable. However, like other variables in this experiment, investment eventually stabilizes at a new steady state:

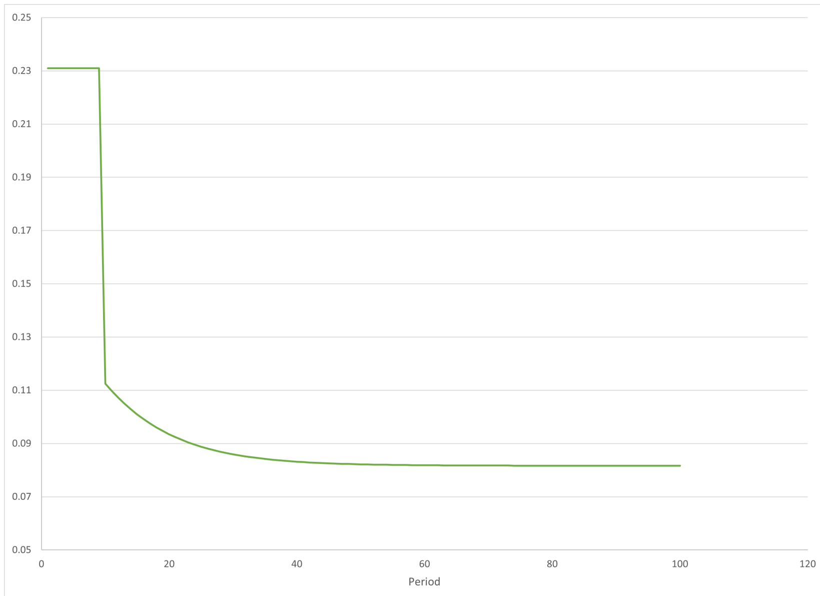
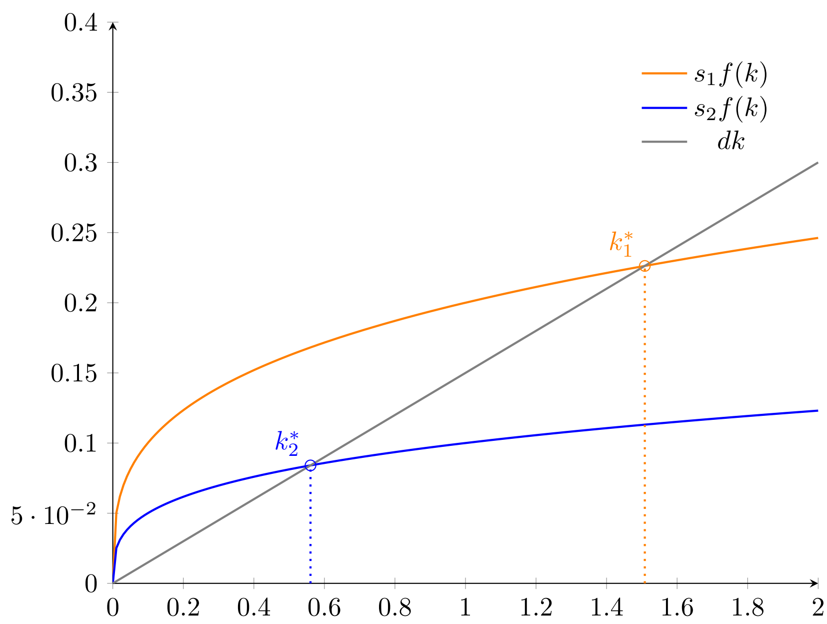



Figure 4: Change in $I$




Above is the Solow Model that substantiates the claim above and explains the transition dynamics between all the variables. As we can see, the lowering of the savings rate moved the steady state level from $K_1*$ to $K_2*$. $s_1 f(K)$ on the graph represents the level of capital with the original savings rate, and $s_2 f(K)$ represents the the level of capital with the lowered rate. As we can see, the lowering of the savings rate causes the investment curve to shift downward, as less income is saved and reinvested, reducing the capital available for production. Initially, the capital stock declines rapidly due to the reduced savings, but over time, it stabilizes at a lower steady-state level, where investment matches depreciation. This results in a lower capital stock in the long run, which reduces output, as less capital is available for production. Investment also decreases, reflecting the new equilibrium with lower capital and output compared to the initial steady state.

### The Optimal Savings Rate

The Solow model provides a framework to predict how long-term economic growth is affected by changes in key economic variables. One of the most important concepts derived from this model is the *golden rule* level of savings, which identifies the savings rate that maximizes steady-state consumption. While a higher savings rate increases investment and capital accumulation, excessive savings reduce consumption since more output is allocated to investment rather than immediate use. Conversely, a lower savings rate may fail to sustain sufficient investment, leading to stagnation or decline in economic output over time. Thus, an optimal savings rate exists that balances these trade-offs and ensures maximum long-term consumption, referred to as the *olden rule.

Consumption is defined in terms of output and the savings rate as:

$$C_t = (1 - \bar{s})Y_t$$

Output in the model follows the Cobb-Douglas production function:

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}$$

At steady state, capital per worker ($k^*$) is given by:

$$k^* = \left( \frac{\bar{s}}{\delta + n} \right)^{\frac{1}{1-\alpha}}$$

Substituting $k^*$ into the production function allows us to determine steady-state output per worker ($Y^*$). To find the golden rule savings rate that maximizes steady-state consumption, we express consumption per worker as:

$$C^* = Y^* - s^* Y^*$$

or equivalently,

$$C^* = (1 - s^*) f(k^*)$$

To maximize consumption, we take the derivative with respect to $s^*$ and set it equal to zero:

$$\frac{d}{ds}\left[f(k^*) - s^* f(k^*)\right] = 0$$

Using the fact that, at the steady state, the marginal product of capital (MPK) must equal depreciation plus population growth:

$$MPK = f'(k^*) = \delta$$

For the *golden rule* savings rate, we assume no population growth ($n = 0$), which simplifies the condition to:

$$MPK = \delta$$

This result implies that the optimal savings rate is the level at which the marginal product of capital equals the depreciation rate, usually represented by 1/3. At this point, capital accumulation neither leads to excessive diminishing returns nor insufficient investment, ensuring maximum sustainable consumption. Thus is this the savings rate that I would recommend.

# 3 Technology Transfers Experiment

For our next experiment, we will be jumping across the world to China. In recent decades, China has implemented several policies that encourage a phenomenon called "technology transfer." These policies include incentives that promote the adoption of new technologies and ideas within the country as a side effect of international trade. In our analysis, assuming all other initial values remain the same, we will examine how these policies affect long-term economic growth by introducing a change in TFP from:

$$A = 1 \quad \text{to} \quad A = 2 \quad \text{at} \quad t = 10.$$

Much like the first experiment, below we'll analyze how output, capital, and investment are affected by this change over time.

**Impact on Output**

In Figure 5 below, we can see that with an immediate increase in $A$, output rises rapidly after period ten. It continues to grow until it reaches a new, higher steady-state level of output when the model stabilizes. This occurs because, in the Solow model, output is a function of TFP, capital, and labor. A direct increase in TFP leads to rapid growth in output. Economically speaking, an increase in the skills and technology of production allows a country to achieve a much more efficient production process almost immediately, enabling them to produce significantly more with fewer resources.

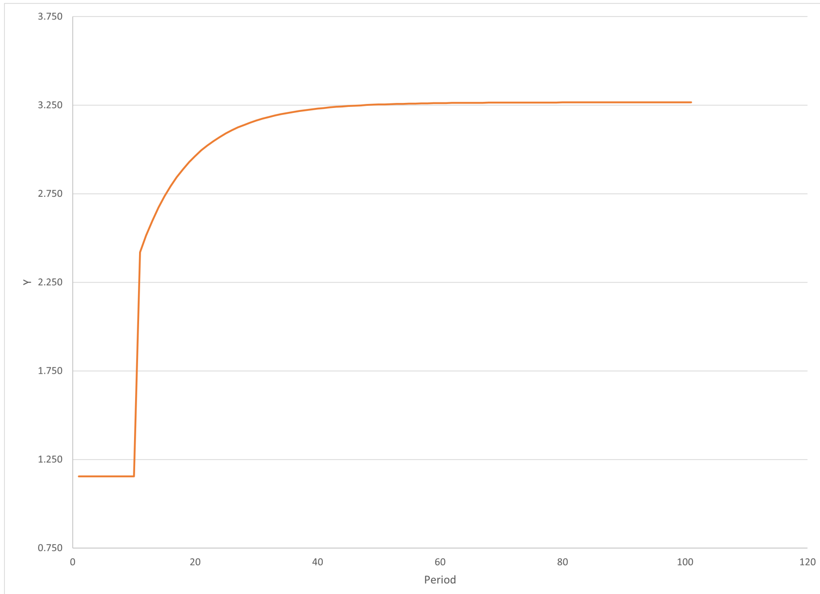



Figure 5: Change in $Y$

**Impact on Capital**

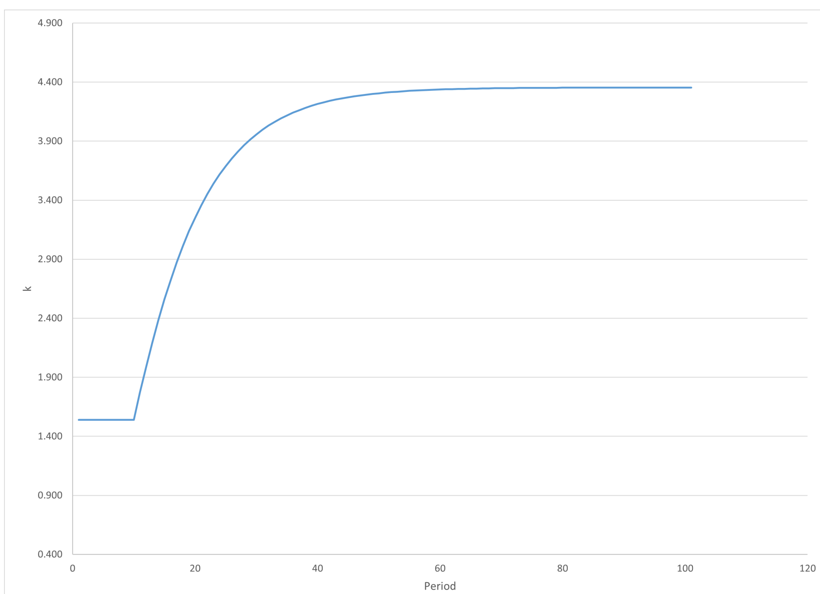



Figure 6: Change in $K$

Figure 6 above illustrates the changes in capital $K$. We observe that a change in total factor productivity (TFP) leads to an increase in capital, although not as pronounced as the rise in output. This increase in capital stems from the same underlying factor that drives the rise in output: enhanced production efficiency. As the economy becomes more efficient, it is able to produce more with less, thus freeing up capital in the short run. Over time however, the level of capital will converge toward its steady state as firms continue to integrate the newly freed capital into the production process. This process occurs as investment continues, driven by higher profitability from increased productivity, but gradually slows due to diminishing returns to capital. This concept is further shown by Figure 8 down below:

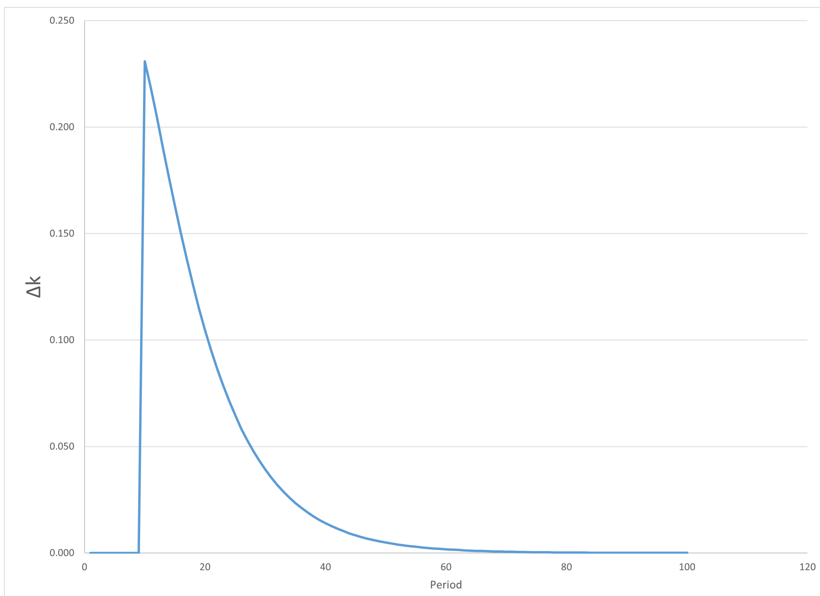



Figure 7: Change in $\Delta K$

**Impact on Investment and the Solow Phase Diagram**

Figure 8 below illustrates the changes in investment resulting from an increase in total factor productivity (TFP). As TFP rises, production efficiency improves, leading to higher output with fewer inputs, as shown in the previous graphs. This increase in productivity is attractive to firms, as it results in higher returns. In response, firms are encouraged to gather more capital and invest it, seeing it as an opportunity to earn greater returns. Consequently, the economy as a whole sees a rise in investment. However, over time, due to diminishing returns, the point will be reached where further investment in production no longer yields the same level of economic benefit. As a result, the investment curve gradually flattens, settling into a new, higher steady state.

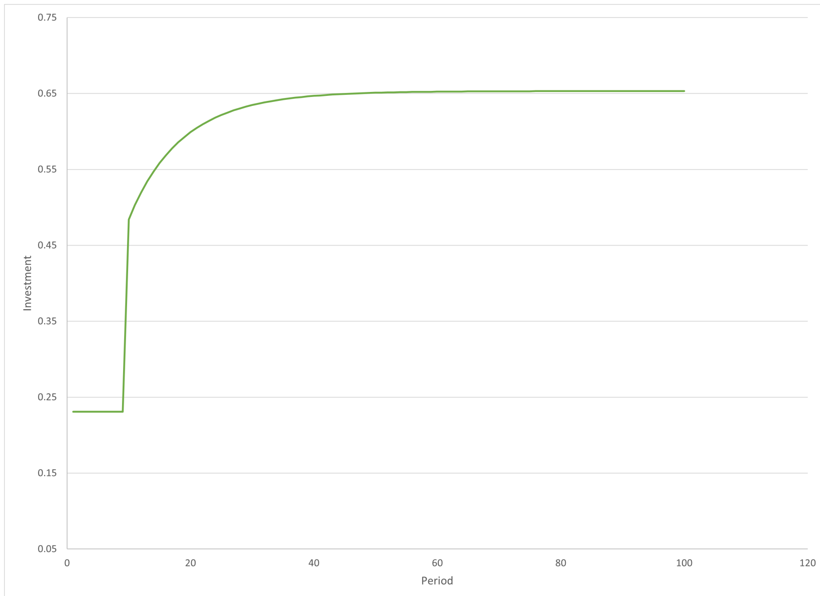
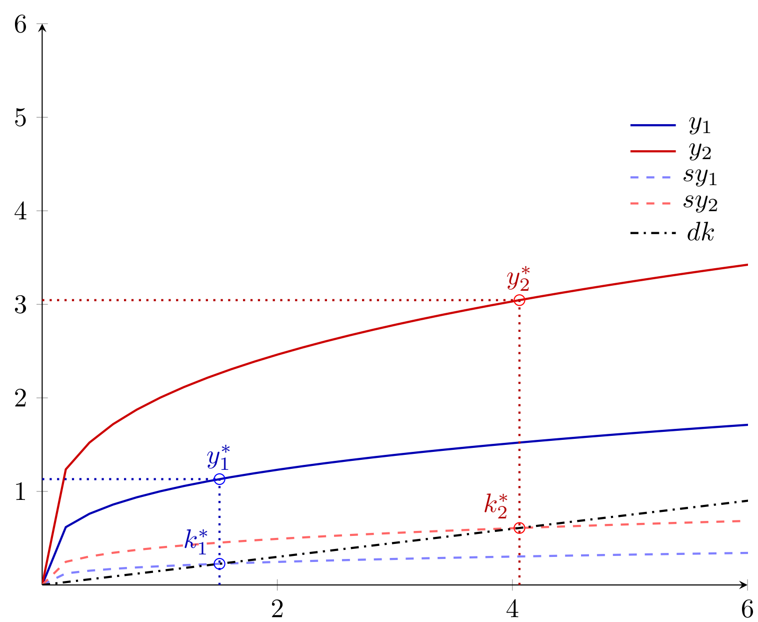



Figure 8: Change in $I$




The phase diagram above illustrates these changes more effectively and explains the transition dynamics. An increase in total factor productivity (TFP) raised the country to a higher steady state for both capital and output, shifting from $K_1^*$ to $K_2^*$ and from $y_1^*$ to $y_2^*$. This increase in output is significant because the country can now produce more efficiently with its available capital. Similarly, capital accumulation rises as productivity improvements allow for greater output with fewer resources, enhancing overall efficiency. Ultimately, this shift leads to a higher steady state for the country's economy, reflecting substantial long-term gains.

### An Advisor's Policy Recommendations

As a member of the U.S. President's Council of Economic Advisors, I would argue that a one-time technological transfer is ineffective in addressing poverty and fostering long-term economic growth. While continuous increases in total factor productivity (TFP) lead to sustained improvements, a single technological boost only provides temporary gains. Without ongoing advancements, economies revert to previous steady-state levels of production. Sustained growth is essential for poverty reduction, as it creates jobs, raises wages, and improves living standards. Without consistent productivity increases, temporary gains fail to create long-term opportunities, leaving poverty unchanged.

According to the Solow Growth Model, a temporary increase in TFP results in a short-term rise in output, but the effect quickly fades, and the economy returns to its original equilibrium. To illustrate this, consider a scenario where a country's TFP is raised from $A = 1$ to $A = 5$ for one period, with all other parameters constant. The following graph demonstrates the fluctuations in output, emphasizing the temporary nature of growth without sustained technological progress:

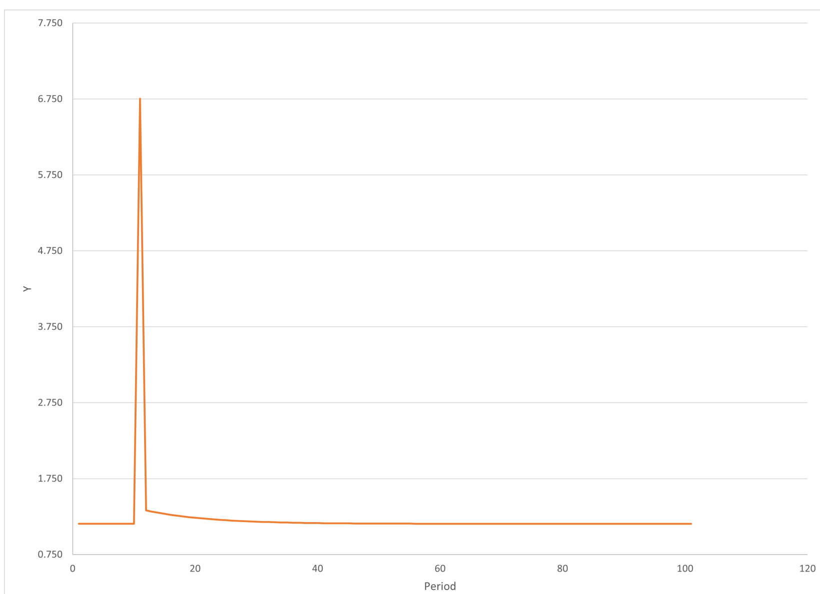



Figure 9: Change in $Y$ With a One Time Increase in $A$

As we can see from Figure 9 above, all changes from this one-time transfer were quickly lost, and the country reverted back to its previous steady state, making no changes to the poverty level in the country. The rapid return to the initial steady state occurs because capital accumulation alone is insufficient to sustain growth without continuous technological progress.

A real-world example of how a temporary increase in TFP does not result in meaningful long-term effects is Venezuela in the early 2000s. The country's economy was highly dependent on oil, and due to high oil prices worldwide, the economy enjoyed a significant increase in production and TFP. The government, under Hugo Chávez, took this increased revenue and invested in infrastructure and social welfare programs.

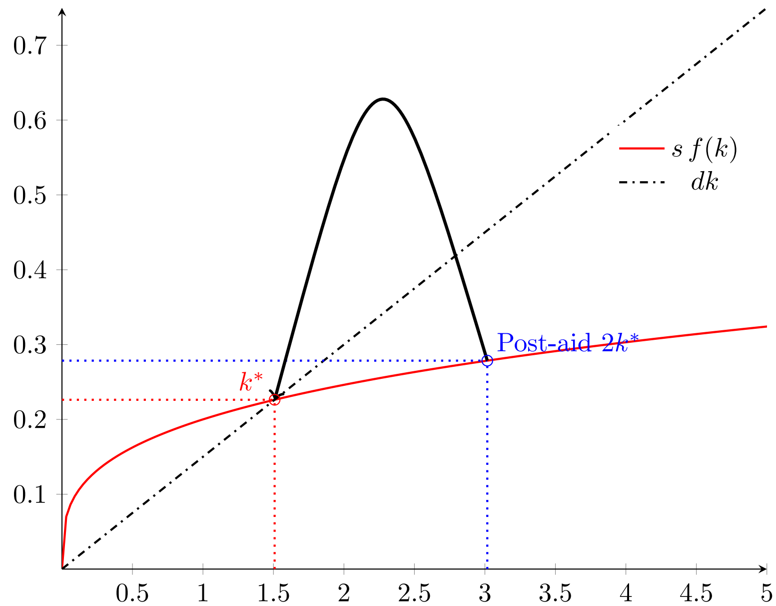
However, when oil prices began to decline, Venezuela's economy declined with it, leading to the economic crisis the country faces today. The economy was tied to a single resource, making its increase in production resemble the graph above. As oil revenues collapsed, Venezuela faced skyrocketing inflation, mass unemployment, and severe shortages of essential goods, plunging millions into extreme poverty. Had the government used oil wealth to diversify its economy—investing in education, manufacturing, and technology—Venezuela could have achieved a sustainable growth path, avoiding its current economic crisis. This example highlights why policymakers should prioritize long-term investments in innovation and productivity, rather than relying on short-term gains.

## 4 Solow Model Foreign Aid Experiment

For our final experiment with the Solow model, let's consider a scenario where Ethiopia is severely impacted by Covid-19. In response to their downturn, they receive a foreign aid package that doubles their current capital stock at $t = 10$. However, the fundamental parameters of the model remain unchanged—only the capital level experiences a sudden increase. Below we will examine the effects of this change.

**The Ethiopia Solow Phase Model**

Below is the phase diagram illustrating the impact of these increases. The economy begins at $k^*$, the original steady-state level. The jump to $2k^*$ represents the boost from the increase in capital stock; as a result, output and investment rise. However, because the fundamental parameters of the economy remain unchanged, this change is not sustainable. There is no shift in the steady state, and the economy gradually moves back to $k^*$, restoring long-run equilibrium. The arrow on the graph illustrates this process. At $2k^*$, depreciation exceeds investment, causing capital to decline over time. As a result, capital naturally depreciates, bringing it back to its steady-state level.




**Impact on Output**

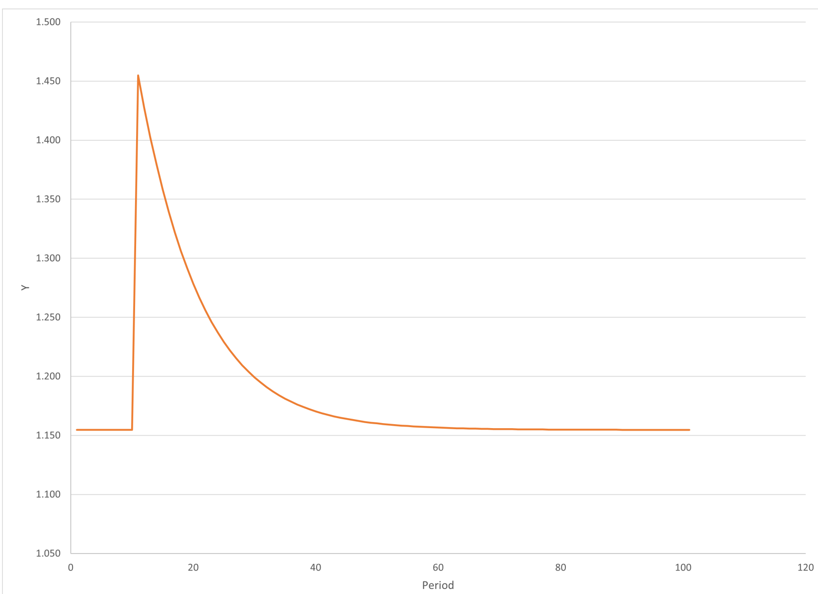



Figure 10: Change in $Y$

As shown in the graph above, the initial surge in output is short-lived. The influx of new capital allows firms to produce significantly more, causing a sharp increase in $Y$. However, since the fundamental parameters of the model remain unchanged, output quickly returns to its original steady-state level, erasing the temporary gains from the capital boost. At the higher capital level, depreciation also rises, and because savings and investment have not permanently increased, they are insufficient to maintain the elevated capital stock.

**Impact on Capital**

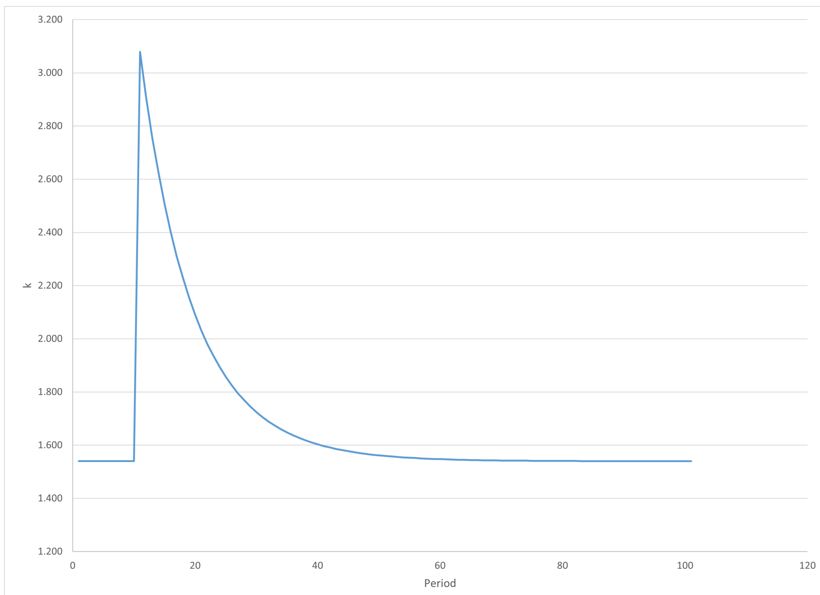



Figure 11: Change in $k$

Figure 11 illustrates the changes in capital and aligns with our expectations. Around period 10, the influx of foreign aid significantly increases the capital stock. It continues to rise briefly before rapidly declining back to its original steady-state level—an even sharper decline than what we observed in the output graph. This outcome was predicted in our phase diagram, which showed that while an elevated capital level leads to higher short-run production, it does not have a lasting impact on the economy, as capital naturally converges back to $k$.

Figure 12 further reinforces this point. Although the initial change in $k$ is dramatic, it quickly dissipates, returning to its long-run equilibrium.

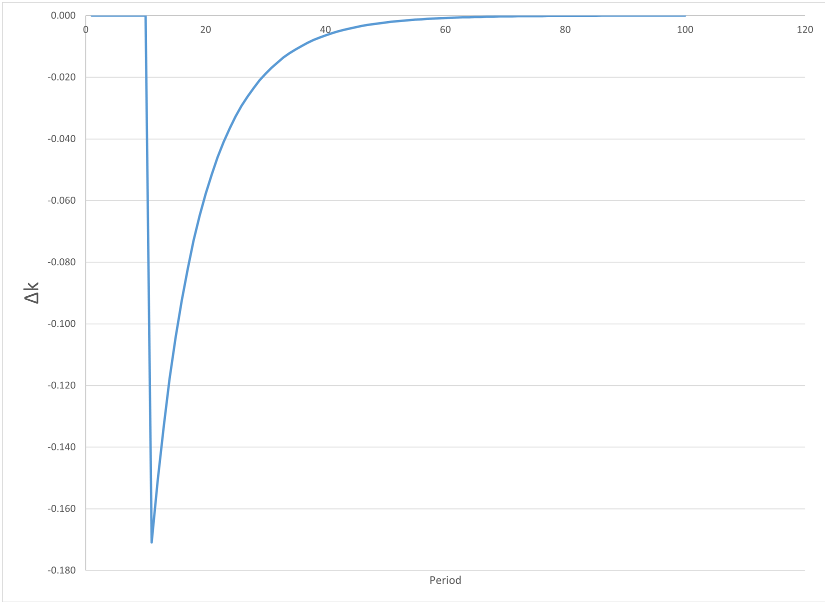



Figure 12: Change in $\Delta k$

**Impact on Investment**

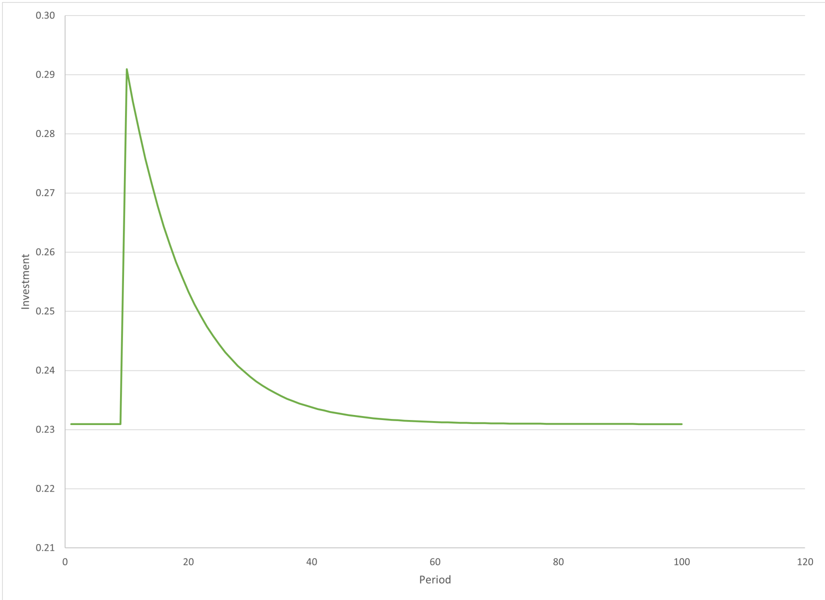



Figure 13: Change in $I$

Figure 13 further shows the point that the growth from the introduction of this capital is unsustainable. The graph begins with an increase because of the doubling of capital stock but in the end returns back to it's steady state. The increase and drop is not as significant as the other factors we analyzed because investment is not a direct function of capital. However it is still effected by the raising of capital, but because no other essential parameters of the model are changed it returns to the steady state level of investment.

### Impact on Consumption

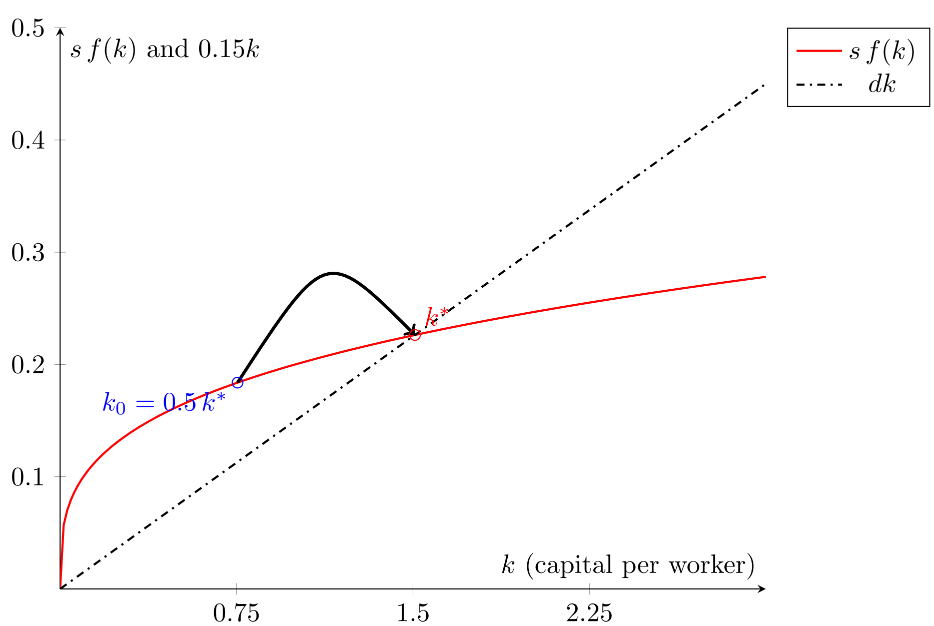
When analyzing how consumption is affected by the increase in capital, we first need to revisit the equation for consumption:

$$C_t = (1 - \bar{s})Y_t$$

Consumption is a function of the savings rate and output. Thus, with an increase in capital, there is a resulting increase in consumption. As a result, the graph of consumption closely mirrors that of output: there is a sharp increase following the introduction of capital, and in the long run, consumption returns to its original steady-state level.

### Ethiopia Experiment Effects With a Capital Decrease

Let's suppose that instead of the original level of capital Ethiopia starts with, it starts off with half of that level. Let's analyze the effects of such a change with a phase diagram:



What we can see from the graph is what would happen is that is that economy would return to it's original steady state of capital in the long run. Output, capital, investment, and consumption in the short run for their curves would show the effects of these shocks in their curves; starting at half the level and doubling with the introduction of the foreign aid. However in the long run there would be no significant difference, as they would only return to the original steady state level as demanded by the original parameters of the model.

### Conclusion - The Consequences of Foreign Aid

The experiments we conducted using the Solow growth model taught us that while foreign aid increases production and economic output in the short run, its impact is extremely short-lived. Our experiments showed that an injection of capital creates a boost in output and consumption initially, but over the long run, the economy simply returns to its original steady-state level. These results imply that cash aid, as a stand-alone policy, does not have lasting merit for improving welfare in developing countries.

Although the immediate benefits of foreign aid are undeniably important, especially in cases of war or extreme economic downturns, the net benefit from such interventions is quickly offset by depreciation and the unchanged parameters of the economy. Therefore, while foreign aid provides an economic shock and some temporary relief, it does not have a long-term impact on the fundamental welfare of the country unless deeper structural changes are made.

If rich countries truly want to help developing nations become more financially stable and foster sustained growth, the effective strategy would be to invest in areas that boost total factor productivity. This includes investing in technology, human capital development, and infrastructure improvements. The key takeaway is that policymakers should focus on creating sustainable development instead of relying on short-term fixes. Only with this approach will real long-term improvements in the economy and welfare of developing countries be possible.