Should Do Ch 11 Empirical Estimation of the IS Curve

Zachary Ramirez

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1 Introduction

The IS curve illustrates the negative relationship between interest rates and short-run output. When interest rates rise, investment and output decline, and vice versa. To build this model, we begin with the national income identity and break GDP into its proportional components, as summarized in the table below:

The Original Setup of the Economy for the IS Curve	
Endogenous variables:	$Y_t, C_t, I_t, G_t, EX_t, IM_t$
National income identity:	$Y_t = C_t + I_t + G_t + EX_t - IM_t$
Consumption:	$C_t = \bar{a}_c Y_t$
Government purchases:	$G_t = \bar{a}_g Y_t$
Exports:	$EX_t = \bar{a}_x Y_t$
Imports:	$IM_t = \bar{a}_m Y_t$
Investment:	$\frac{I_t}{Y_t} = \bar{a}_i - \bar{b}(R_t - \bar{r})$
Exogenous variables/parameters:	$Y, \bar{r}, \bar{a}_c, \bar{a}_g, \bar{a}_x, \bar{a}_m, \bar{b}$

This setup forms the foundation of the IS curve by linking output to its determinants, particularly investment, which responds negatively to changes in the real interest rate R_t.

To derive the IS curve, we start with the national income identity:

$$Y_t = C_t + I_t + G_t + EX_t - IM_t$$

Dividing both sides by potential output $\bar{Y}_t$ gives:

$$\frac{Y_t}{\bar{Y}_t} = \frac{C_t}{\bar{Y}_t} + \frac{I_t}{\bar{Y}_t} + \frac{G_t}{\bar{Y}_t} + \frac{EX_t}{\bar{Y}_t} - \frac{IM_t}{\bar{Y}_t}$$

Substituting the equations from our table:

$$\frac{Y_t}{\bar{Y}_t} = \frac{\bar{a}_c \bar{Y}_t}{\bar{Y}_t} + \frac{(\bar{a}_i \bar{Y}_t - \bar{b}\bar{Y}_t R_t + \bar{b}\bar{Y}_t \bar{r})}{\bar{Y}_t} + \frac{\bar{a}_g \bar{Y}_t}{\bar{Y}_t} + \frac{\bar{a}_{ex} \bar{Y}_t}{\bar{Y}_t} - \frac{\bar{a}_{im} \bar{Y}_t}{\bar{Y}_t}$$

Simplifying:

$$\frac{Y_t}{\bar{Y}_t} = \bar{a}_c + \bar{a}_i - \bar{b}(R_t - \bar{r}) + \bar{a}_g + \bar{a}_{ex} - \bar{a}_{im}$$

To express short-run output explicitly, we subtract 1 from both sides:

$$\frac{Y_t}{\bar{Y}_t} - 1 = \bar{a}_c + \bar{a}_i + \bar{a}_g + \bar{a}_{ex} - \bar{a}_{im} - 1 - \bar{b}(R_t - \bar{r})$$

This simplifies to the IS curve equation:

$$\tilde{Y} = \bar{a} - \bar{b}(R_t - \bar{r})$$

Where:

- $\tilde{Y}$ – Output gap
- $\bar{a}$ – Autonomous spending
- $\bar{b}$ – Interest rate sensitivity
- R_t – Real interest rate
- $\bar{r}$ – Natural interest rate

This equation shows that short-run output depends primarily on autonomous spending and the difference between the real and natural interest rates. However, for the experiments we will be conducting in this paper we will be modifying some of the equations:

The Setup of the Economy for the Modified IS Curve	
Endogenous variables:	Y_t, $\tilde{Y}_t$, C_t, I_t, G_t, EX_t, IM_t
National income identity:	$Y_t = C_t + I_t + G_t + EX_t - IM_t$
Consumption:	$\frac{C_t}{Y_t} = a_c - b_c\,(R_t - r_t)$
Investment:	$\frac{I_t}{Y_t} = a_i - b_i\left(R_t - r_t\right)$
Government purchases:	$G_t = \bar{a}_g\, Y_t$
Exports:	$EX_t = \bar{a}_x\, Y_t$
Imports:	$\frac{IM_t}{\tilde{Y}_t} = a_{im} - n\,\tilde{Y}_t$
Exogenous variables/parameters:	a_i, b_i, a_c, b_c, a_{im}, n, $\bar{a}_g$, $\bar{a}_x$, R_t, r_t

The new consumption equation is given by:

$$\frac{C_t}{Y_t} = a_c - b_c\,(R_t - r_t)$$

Where:

- C_t – Total consumption
- Y_t – Total output (GDP)
- a_c – Autonomous consumption (consumption independent of income)
- b_c – Sensitivity of consumption to the interest rate gap
- R_t – Real interest rate
- r_t – Reference (or natural) interest rate

This equation builds on our original IS function by introducing a sensitivity parameter, b_c, which explicitly captures how consumption reacts to changes in interest rates. This modification allows for a more precise analysis of how economic agents adjust their spending behavior in response to monetary policy changes. Now the modified imports function is given by:

$$\frac{IM_t}{\tilde{Y}_t} = a_{im} - n\,\tilde{Y}_t$$

Where:

- IM_t – Total imports
- $\tilde{Y}_t$ – Short-run output (or output gap)
- a_{im} – Autonomous import ratio
- n – Sensitivity of imports to changes in short-run output

This adjustment to the imports equation introduces a dependence on short-run output, $\tilde{Y}_t$, allowing us to examine how import levels fluctuate in response to domestic economic conditions. It provides a clearer framework for assessing how a country adapts its import strategy based on its economic performance, reflecting the dynamic nature of trade flows in response to cyclical fluctuations.

2 Parameter Interpretations for Investment, Consumption, and Imports

Now that we've built our modified IS curve with dynamic functions, it's time to test it using real-world data. We'll focus on the equations for investment, consumption, and imports, analyzing how well they hold up in practice. Our goal is to determine whether the assumptions we've made align with actual economic behavior and to evaluate any discrepancies that emerge.

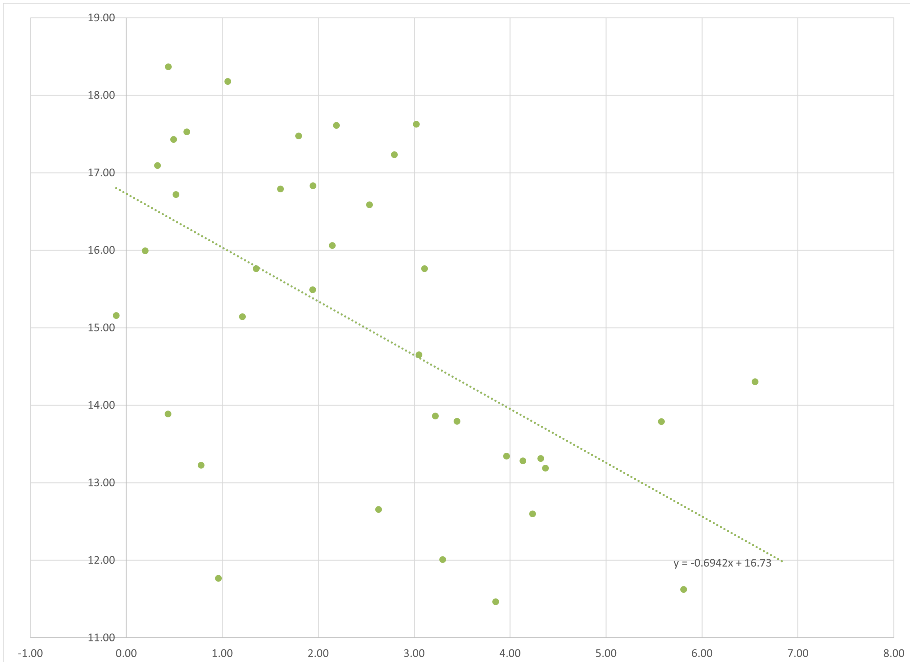

Figure 1: Investment Function

Investment

Above is the graph for investment that we created from real-world data obtained from FRED. At first glance, it appears downward sloping exactly as expected. Running a regression on the model, we obtained the following equation:

$$y = 16.73 - 0.6942x$$

In the context of our investment equation, this can be interpreted as:

$$\frac{I_t}{Y_t} = 16.73 - 0.6942(R_t - \bar{r})$$

This tells us that the baseline level of investment is 16.73 percent of GDP when there is no interest rate gap, and for every 1 percent change in the interest rate gap, investment as a share of GDP decreases by 0.6942 percentage points. The slope matches our economic theory, as we would expect higher interest rates to lower investment, consistent with both the structure of our equation and historical data.

However, something interesting to note is that this intercept seems relatively high. This could be explained by the fact that in the last few years we have seen some very prosperous times for the economy. When economic prosperity, there is a resulting heightened investor sentiment and could explain this inflated a_i parameter.

Of course there are several other factors that are not represented by our model; for example business confidence or technological changes can affect investment and aren't necessarily represented only by the difference in interest rates. We also see in the data that it takes awhile for the market to represent changes in interest rates and that is not necessarily show in this model. However, in general we can see the data reinforces the assumptions we make for investment.

Consumption

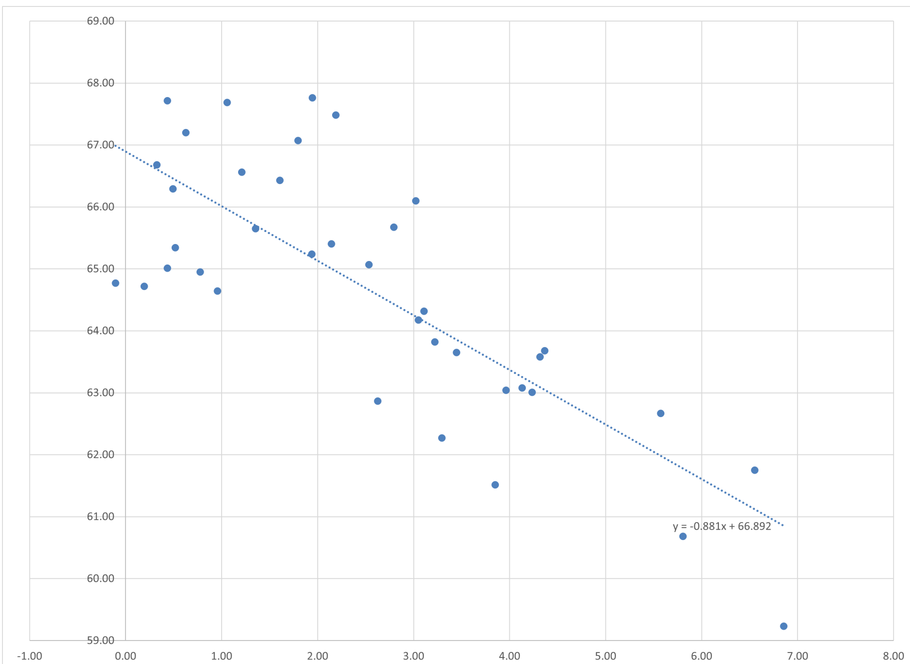

Figure 2: Consumption Function

We performed a similar regression analysis for consumption, yielding the equation:

$$y = 66.892 - 0.881x$$

which translates to:

$$\frac{C_t}{Y_t} = 66.892 - 0.881(R_t - r_t)$$

This implies that when there is no interest rate gap, consumption is 66.892 percent of GDP. Each 1 percent increase in the interest rate reduces consumption by 0.881 percentage points, reinforcing the expected inverse relationship between interest rates and consumer spending.

Notably, the intercept is relatively high, similar to our investment equation. This aligns with recent economic trends where consumption remained strong despite rising interest rates. A potential explanation is pent-up demand from the COVID-19 pandemic and the impact of government stimulus checks, which injected liquidity into households and sustained spending.

Of course, this model simplifies real-world consumption dynamics. Other factors, such as consumer sentiment, income distribution, and delayed responses to interest rate changes, also play a role. While the model does not capture these complexities, our results support that higher interest rates discourage consumption.

Imports

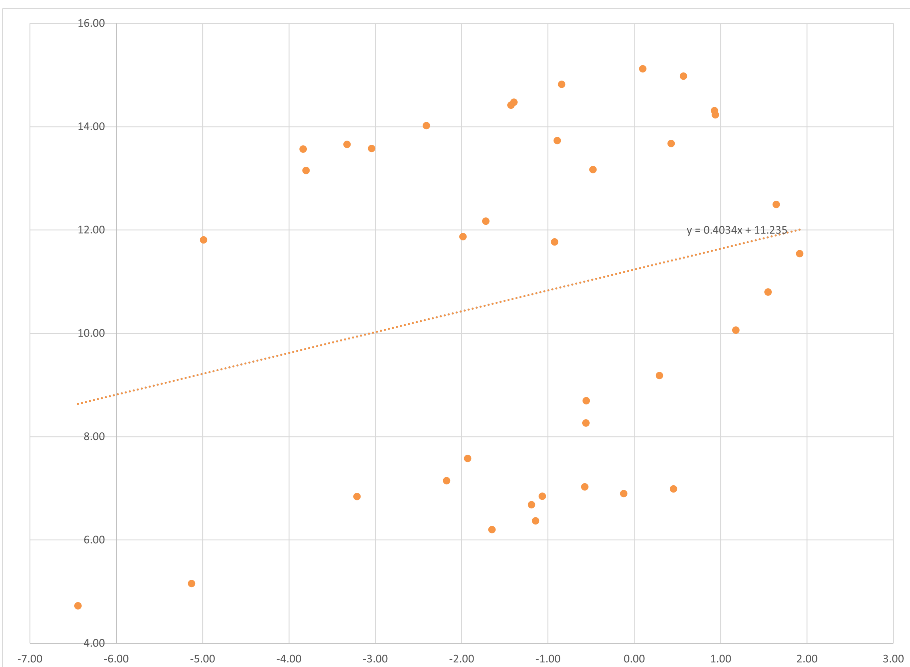

Figure 3: Imports Function

Above is the graph for imports based on the data we collected. The regression produced the following equation:

$$y = 11.235 + 0.4034x$$

In the context of our modified imports function, this gives us:

$$\frac{IM_t}{\tilde{Y}_t} = 11.235 + 0.4034\tilde{Y}_t$$

This equation indicates that when the short-run output gap is zero, imports account for 11.235 percent of GDP. Additionally, for every 1 percentage point increase in the short-run output gap, imports rise by 0.4034 percentage points. This positive relationship is supported by economic theory, as a growing economy typically increases demand for foreign goods and services.

The most important takeaway from this function is its upwards slope, which contrasts with the trends we saw with investments and consumption. While higher interest rates reduce consumption and investment, a booming economy increases raises import levels. This reflects the broader aspect of monetary policy that when the economy is booming, central banks often raise interest rates to curb inflation and prevent overheating.

There are factors not included in our model, such as exchange rate fluctuations and trade policies that may also influence import levels. However, overall, our model serves as an accurate proxy for predicting that as the economy grows, import demand also increases.

3 Parameter Interpretations for Quarterly Data

Using quarterly data from FRED we can estimate the parameter a for each equation, shown by the table below:

Parameter Terms	**Averages**
a_c	64.43
a_i	14.74
a_g	22.28
a_{nx}	-1.90

Table 1: Average Values of Model Parameters

Each of these parameters estimates the baseline activity of a GDP component. They also provide insights into what GDP looks like without the influence of other factors:

- $a_c = 64.43$**:** This parameter tells us that, when there is no influence from changes in interest rates, consumption represents about 64.43 percent of GDP. This high value is justified, as household consumption has historically made up the largest share of GDP.

- $a_i = 14.74$**:** This parameter indicates that when there are no other changes in interest rates, investment represents a share of about 14.74 percent of GDP. This value is also justified, as investment, though smaller than consumption, still represents a significant portion of GDP through production investments.

- a_g = 22.28**:** This parameter indicates that government spending makes up 22.28 percent of GDP as a baseline for the economy. While consumption is generally lower than government spending, this higher value could be reflective of recent substantial public investments, such as COVID-19 responses.

- $a_{nx} = -1.90$**:** This negative value for net exports tells us that, in our economy, net exports act as a subtraction from GDP. However, this is to be expected because, in our economy, imports slightly outweigh exports.

Overall, these parameters align with our prior knowledge about how GDP is divided. Consumption dominates GDP, and investment and government spending make significant contributions, though in smaller amounts. The magnitudes of these parameters allow us to see how our economy is divided and provide further evidence of the IS model's key assumptions.

4 Quarterly and Yearly Comparison for a_c and a_i

So far we have made an estimate for a_c and for a_i using quarterly and yearly data. Below is a table comparing the two results from the calculations we made:

	Quarterly Estimate	Yearly Estimate
a_c	64.43	66.892
a_i	14.74	16.73

Table 2: Quarterly and Yearly Estimates for a_c and a_i

The estimates are similar, but there are still some differences, as the yearly data is notably about two percentage points higher than the quarterly data.

These differences can be explained by the way the data was calculated. Data is generally collected either quarterly or yearly, but using yearly data to calculate quarterly values can lead to different approximations.

When calculating the parameter values with yearly data, we first plotted the data on a graph. We then used the regression equation and made estimates for the parameters based on what the trend lines gave us. However, when using quarterly data, we took the averages and plugged them into the equations.

Overall, both approaches yielded similar values, and the differences were not significant enough to impact our analysis. However, because the yearly data had already been transformed, it was likely the cause of these discrepancies.

5 Does $a = a_c + a_i + a_g + a_{nx}$ - 100 = 0 hold?

When estimating parameters for the economy, it makes sense that these parameters should sum to 100 percent of GDP, as they represent the baseline components of GDP. However, does this assumption hold for the parameters we estimated above? Let's plug them in and see:

$$a = a_c + a_i + a_g + a_{nx} - 100 = 0$$

Substituting the values from our table:

$$a = 64.43 + 14.74 + 22.28 - 1.90 - 100$$

$$a = 99.55 - 100$$

$$a = -0.45$$

Thus, we find that:

$$a \neq 0, \quad a = -0.45$$

The assumption that the sum of all parameters equals 100 percent doesn't hold exactly. However, this result isn't unexpected. All of these parameters were approximated using real-world data so a discrepancy of 0.45 percentage points is relatively minor. Such discrepancies are common in models like these, where simplifications and assumptions are made to represent a complex economy. While this result is not perfect, the differences are not large enough to render the model invalid.