CS/MATH111 ASSIGNMENT 5

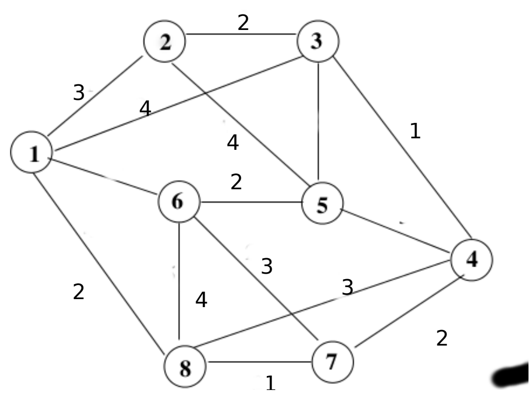
due Friday, March 15 (8AM)

Zachary Şells SID 861013217

Individual assignment: Problems 1 and 2.
Group assignment: Problems 1, 2 and 3.

Problem 1: An *edge coloring* of a graph is an assignment of colors to edges such that any two edges that share an endpoint have different colors.

Here is an example of an edge coloring of a graph with 5 colors (colors represented by numbers):



(a) For the graph above, find an edge coloring with at most 4 colors.

(b) Let Δ denote the maximum vertex degree in a graph G. Prove that if $\Delta \geq 1$ then G can be edge-colored with at most $2\Delta - 1$ colors. You need to give a direct proof, without using any results from the literature. Make your argument complete and rigorous.

Problem 1B Solution

Since Δ represents the maximum degree vertex in the graph, that means that we can color all edges with that same Δ colors.
The reason for this is that since we have at most $\Delta edgesthatareadjacenttoeachother, thereforeweneedatleast \Delta$ colors to make sure that no edge is the same color of any of its neighbours
We want to show that $\Delta \leq 2 * \Delta - 1$ because $\Delta \leq 1$, that means that $\Delta \leq 2 * \Delta - 1$ colors are sufficient to color the graph.
There ends the proof.

Hint: In class we proved that each graph of maximum degree Δ has a vertex coloring with at most $\Delta + 1$ colors. Follow the reasoning from that proof, modifying it slightly to work for coloring edges.

Problem 2: Let G be a planar graph with n vertices and m edges, where $n \geq 3$. Prove that if G is bipartite then $m \leq 2n - 4$.

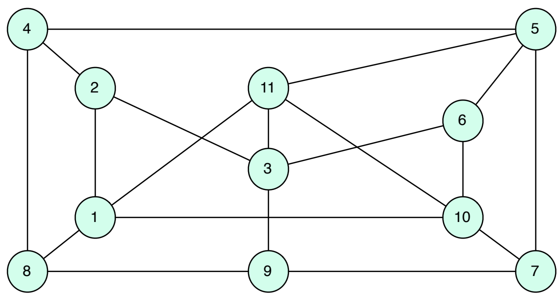
Hint: Follow the proof for Euler's inequality ($m \leq 3n - 6$) that we covered in class. In that proof we used the equation $m = n + f - 2$, and to find a relationship between m and f we estimated the number of "edge sides" by $3f$, by noticing that each face must have at least three edge sides. How can you improve this estimate if G is bipartite? Can a face of G have three edges?

Problem 2 Solution

In a bipartite planar graph, it is true that with $n \geq 3$ then it takes at least 2 added edges to create a face. We can see that this is true for $m = 4$ because with 4 edges you can create no more than $\frac{4}{2}$ faces.

Using this information, we can derive the inequality $2f \leq m$ where f =number of faces, and m =number of edges.
Simplifying the inequality gives: $f \leq \frac{m}{2}$
Simplifying Euler's formula gives: $f = m - n + 2$
Substituting our inequality into Euler's equation gives: $\frac{m}{2} \geq m - n + 2$
Simplification gives: $m \leq 2n - 4$ Which ends our proof.

Problem 3: Determine the minimum number of colors needed to color the vertices of the following graph G:



You need to (a) show a coloring with minimum number of colors and (b) prove that it is not possible to use fewer colors.

Submission. To submit the homework, you need to upload the pdf file into ilearn by 8AM on Friday, March 15, and turn-in a paper copy in class. Pictures should be imported into LaTeX in pdf (see the source file to see how to do that). You can draw them in any drawing software and export in pdf, or draw by hand and scan.