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CS 111 ASSIGNMENT 3
due Friday, February 22

CS/MATH111 ASSIGNMENT 3
due Thursday, February 21 (8AM)

Individual assignment: Problems 1 and 2.
Group assignment: Problems 1,2 and 3.

Problem 1: Let W_n be the number of strings of length n formed from letters A, B, C, that do not contain AB or CC. For example, for $n = 3$, all the strings with this property are:

$$AAA, AAC, ACA, ACB, BAA, BAC, BBA, BBB,$$
$$BBC, BCA, BCB, CAA, CAC, CBA, CBB, CBC,$$

and thus $W_3 = 16$. (Note that $W_0 = 1$, because the empty string satisfies the condition.)
(a) Derive a recurrence relation for the numbers W_n. Justify it.
(b) Find the formula for the numbers W_n by solving this recurrence. Show your work.

Problem 1 Solution
Using the fact that $W_0 = 1, W_1 = 4$, and $W_2 = 13$ We try to find a formula using these knowns that both agrees with the initial conditions, and gives us the correct number for any given n.

The number of words that satisfy the conditions is the cardinality of the setA,B,C times the value of $W_(n-1)$ plus the value of $W_(n-2)$ giving us a final recurrence equation of $W_n = 3W_(n-1) + W_(n-2)$. We can verify this equation by plugging in $n = 3$. Using counting techniques, we find that the number of words that meet the conditions is 43. And if find this value by using or recurrence equation, we get: $W_4 = 3 * (13) + 4$ which also equals 43.

Now we must solve this recurrence relation.
The characteristic equation of this relation is given by: $x^2 - 3x - 1$
Solving this gives us roots of: $r_1 = \frac{3+\sqrt{13}}{2}$ and $r_2 = \frac{3-\sqrt{13}}{2}$
Giving us the general equation of: $W_n = \alpha_1 * (\frac{3+\sqrt{13}}{2})^n + \alpha_2 * (\frac{3-\sqrt{13}}{2})^n$
Plugging in the initial conditions... $\alpha_1 = \frac{1}{2} + \frac{5}{2*\sqrt{13}}$ and $\alpha_2 = \frac{\sqrt{13}-5}{2*\sqrt{13}}$

Our final solution is... $W_n = (\frac{1}{2} + \frac{5}{2*\sqrt{13}}) * (\frac{3+\sqrt{13}}{2})^n + (\frac{\sqrt{13}-5}{2*\sqrt{13}}) * (\frac{3-\sqrt{13}}{2})^n$

Problem 2: Solve the following recurrence equation:

$$\begin{aligned} f_n &= 3f_{n-1} + 15f_{n-2} + 2n + 3 \\ f_0 &= 0 \\ f_1 &= 1 \end{aligned}$$

Show your work, step by step: the associated homogeneous equation, the characteristic polynomial and its roots, the general solution of the homogeneous equation, computing a particular solution, the general solution of the non-homogeneous equation, using the initial conditions to compute the final solution.

Problem 2 Solution To solve this recurrence we will break it up into two separate components(Homogeneous and Non-homogeneous), then add them together to get the final solution.
$f(n) = 3*f(n-1)+15*f(n-2)$ is the homogeneous part of the equation, which we will find the characteristic

equation of.
The characteristic equation is $x^2 - 3x - 15$.
Solving this gives roots of: $\frac{3+\sqrt{69}}{2}$ and $\frac{3-\sqrt{69}}{2}$
This gives us the general solution of $f(n) = \alpha_1 * (\frac{3+\sqrt{69}}{2})^n + \alpha_2 * (\frac{3-\sqrt{69}}{2})^n$

Now we will find the solution to the non-homogeneous part of the recurrence.
Lets guess the solution: $\beta_1 * n + \beta_2$
Plugging back into the original recurrence and simplifying gives the two equations: $33 * \beta_1 - 17 * \beta_2 - 3 = 0$ and $-17\beta_1 - 2 = 0$
Solving for β_1 and β_2 gives $\frac{-2}{17}$ and $\frac{-117}{289}$, respectively.
Adding this solution to the homogeneous solution gives: $f(n) = \alpha_1 * (\frac{3+\sqrt{69}}{2})^n + \alpha_2 * (\frac{3-\sqrt{69}}{2})^n - \frac{2}{17} * n - \frac{117}{289}$
Now we must solve for the two alphas using the initial conditions
This gives $\alpha_1 = \frac{529+117*\sqrt{69}}{578*\sqrt{69}}$ and $\alpha_2 = \frac{351-23*\sqrt{69}}{1734}$

Making our final solution: $f(n) = (\frac{529+117*\sqrt{69}}{578*\sqrt{69}}) * (\frac{3+\sqrt{69}}{2})^n + (\frac{351-23*\sqrt{69}}{1734}) * (\frac{3-\sqrt{69}}{2})^n - \frac{2}{17} * n - \frac{117}{289}$

Problem 3: Solve the following recurrence equation:

$$
\begin{aligned}
t_n &= 3t_{n-1} + 2t_{n-2} \\
t_0 &= 0 \\
t_1 &= 2
\end{aligned}
$$

Show your work.

Submission. To submit the homework, you need to upload the pdf file into ilearn by 8AM on Thursday, February 21, and turn-in a paper copy in class (or slip it under my door, no later than 10AM).