Project Report in partial fulfillment of Spring 2021 EC/ME/SE 543 Course Requirements

Title: Demand Scheduling of EV Charging

Date Submitted: 5/6/21

Acknowledgment:

As the sole author of this report I certify that I am aware of the following citation/referencing requirements and that failure to abide by them may result in academic misconduct disciplinary action and a failing grade.

* All passages, figures, tables, graphs and any other material lifted from a web site document, an existing report whether archived or informally made available to me, must be included in quotes and referenced in the text by exact bibliographic form and page number.
* The above holds for paraphrased or modified information or ideas taken from any source. In this case the reference should appear at the bottom of a table or graph or end of a paragraph without quotes.

Furthermore, I certify that I have carefully followed the above requirements in putting together the paper that follows.

Name: Zachary Weiss

Signature:

**Abstract**

As electric vehicles (EVs) continue to gain market share, the question of charging becomes an important one at the transmission-grid level. Owners of EVs wish to charge their vehicles when electric prices are lowest while still ensuring sufficient charge by the time of utilization, and system operators wish to level the demand curve, and, where possible, leverage additional storage connected to the grid to abate supply fluctuations that might force more expensive generation online. In an ideal market, this is fully coordinated and modulated by price; this project sets out to determine the optimal demand schedule of EV cohorts over the course of a day with synthetic data, and seeks to be abstractable to modeling of both greater and lesser complexity.

**1. Introduction**

**1.1 Problem Formulation**

In its simplest form, the problem of scheduling EV demand is for a single cohort, available to charge during any hour, with the additional load having negligible impact on clearing price / grid-level constraints, and with accurate price forecasts. Discretizing the problem into hourly chunks, this can be formalized as:

Subject to:

or equivalently,

Where J is the cost function, S is the stored energy, D is the demand from the EV cohort, R is the charging rate, and Π is the base forecast clearing price. This makes some key assumptions, namely: the EV must be fully charged at the end of the scheduled window, the EV can sell back to the grid at the clearing price of purchasing energy, the EV load has negligible impact on the grid and clearing price, the minimum storage capacity is zero, and that the maximum charge and discharge rates are symmetric. Given perfect knowledge ahead of time of the hourly clearing price, one can intuitively see demand will be scheduled during hours in order of increasing clearing price, and energy will be sold back to the grid during hours in order of decreasing clearing price, with the rates of (dis)charge and scheduled hours determined by the constraint of being fully charged by the final hour. As this is fairly trivially solved in a linear manner, we increase the complexity of the problem to allow the load scheduling from the EV to impact the clearing price. In the most naïve form, we can formalize this as the clearing price equaling the base price forecast in addition to our load multiplied by some coefficient determining the degree of impact.

Where P is our price impact coefficient. This is of course overly simplistic, but at least turns the problem into a nonlinear one (referred to as NLP from here on out), and requires the optimal schedule to consider the amount it schedules by hour on more than just the order of hourly price forecasts. We can complicate the model further by restricting the set of hours during which an EV is allowed to charge: a realistic and important constraint given the hours during which vehicles would be parked at home, or parked at an office with EV outlets, for example. This changes the formalized problem to:

Subject to:

Where H is a set containing the active (available for charging and discharging) hours. Lastly, we can add complexity to the problem by attempting to find the optimal schedule across multiple cohorts of EVs, all with their own capacities, initial charges, maximum charging rates, and hours of activity. This rewrites the problem to be:

Subject to:

With the problem fully described, the next challenge is finding the optimal solution subject to the given constraints (with the presumption that that fulfills the given initial conditions and constraint parameters). To do so, we turn to Python, its applicable libraries, and compatible NLP solver frameworks.

**1.2 Prior Work**

**1.2.1 Pyomo**

The first python library to be leveraged is pyomo. Pyomo is, “a Python-based, open-source optimization modeling language with a diverse set of optimization capabilities”. It flexibly interfaces with 3rd party solvers, supporting linear, nonlinear, mixed-integer, stochastic, disjunctive, differential, and bilevel programming, alongside mathematical programs with equilibrium constraints. Here it is used to express the above stated formalization of the demand scheduling problem, for subsequent use in the NLP solver. The use of pyomo in this project falls under its custom license agreement, copywrite and redistribution terms. [1] [2]

**1.2.2 Interior Point Optimizer (Ipopt)**

Interior Point Optimizer is, “an open source software package for large-scale nonlinear optimization. It can be used to solve general nonlinear programming problems of the form

” [3]

This form is compatible with the problem formulation of the previous section, and as such it was selected for this project. Ipopt is one of the many solvers pyomo can interface with, and even provides a wrapper for. In later code one will see references to the “multistart” solver—this is simply a wrapper for the Ipopt solver with multiple starting points for the optimization. This package’s use falls under the terms of the Eclipse Public License.

**1.2.3 NumPy**

Finally, the NumPy library is utilized. NumPy is a scientific computing package for python, simplifying many numeric computations and matrix operations, and as such is considered a standard in Python. It sees use here throughout, handling any matrix operations outside the NLP solver itself. NumPy is distributed under the BSD license. [4]

**2. Methods**

**2.1 Overview**

After selecting the aforementioned libraries, the formulated problem was solved in steps of increasing complexity. The first iteration solely optimized for a single cohort, available to charge whenever during the day. The second iteration optimized a single EV cohort with limitations to the charging hours. The third iteration abstracted to multiple EV cohorts, made minor changes to the synthetic data used, and implemented human-interpretable readouts; by modifying the configuration of the third iteration, it can be made to optimize the simpler scenarios of the first two project iterations, but with its interface improvements.

**2.2 Single Cohort, Always Online**

In the case of the simplest optimization problem formulated, upon which the later problems are built, the code (listed in full in the appendix: *File 1*) begins by defining global parameters for the number of hours, and the configuration of the EV cohort to be scheduled. The EV configuration entails its starting charge, “S\_0”, its maximum capacity “S\_max”, its maximum charging rate “R\_max”, and its price influence coefficient “P” (as defined in *Section 1.1*). Additionally, a base clearing price forecast signal is generated over the duration of the hours specified—in this iteration this takes the form of scaled random numbers, in later iterations the synthetic data is made more realistic.

After defining all parameters, the model is instantiated in pyomo, with decision variables created for the charge, charging rate, and adjusted clearing price. The objective function, *Equation 1*, and constraints, *Equations 2* through *5*, are applied, with all non-boundary condition constraints being applied over each hour in the set of hours specified. This problem, with applied decision variables, objective function, and constraints is passed to Ipopt through the multistart wrapper, and returned results are displayed.

**2.3 Single Cohort, Limited Hours**

To modify the single cohort, always online case for the limited hours problem formulation, a new field for active hours, “H”, is added to the EV config global setting. To avoid issues when passing to the solver, we check to ensure all hours specified active are within the window of total hours to be optimized over, and if not, throw an error requesting the user modify the configuration such that active hours are within the overall duration they selected.

To modify the constraints to agree with *Equation 9*, within the loop adding constraints for each hour, a check is added that compares the current hour to the configuration; if the current hour does not appear in the configuration, the charge rate is constrained to zero for that hour block, else it is bounded, as before, by its upper and lower limits specified.

**2.4 Multiple Cohorts, Limited Hours**

For the final project iteration, the optimization is expanded to multiple cohorts of EVs, all individually configurable. The EV config global variable is expanded to an array of dicts, of arbitrary length greater than or equal to one (within the limit of one’s computing power), each entry containing the same fields as in the former versions.

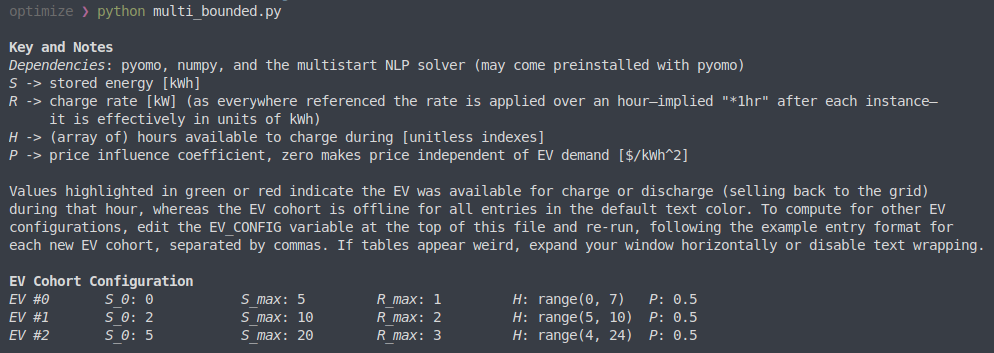
To improve useability, a set of notes, and a key to understanding variable names is printed on the screen at the start of the program execution. It additionally lists the current EV configuration, for ease of reference when interpreting the results, and to allow users to ensure they have not misconfigured it. The input sanitization checks remain the same, now applied iteratively to each EV listed within the config.

As random noise does not particularly well represent standard hourly pricing, the base clearing price forecast signal is modified to be a simple gaussian, scaled and shifted, centered at 6pm, a standard summer peak hour, approximating the hourly profile provided by the U.S. Energy Information Administration (EIA) [5]. To accommodate the iteration over an arbitrary number of EV cohorts, the dimensionality of the decision variables is increased, with one axis holding the hours as before, and the other: the EV cohort to which it corresponds. Constraint instantiation is similarly iterated over per EV, as per *Equations 14* through *18*. Due to technicalities of pyomo variable handling, the inequality constraints, previously handled by limiting the bounds of the relevant decision variables, now are applied directly as inequalities at each hour.

Once solved, the results are passed to a custom data wrangling function. Here the solutions are extracted from pyomo, basic stats calculated, and the results printed in a pleasant human-readable form via the console. The storage values, charging rates, and clearing price are listed by hour, with the storage and charging listed on a per-EV basis, to the number of EVs specified. Within the results, active hours (those during which the given EV can charge or discharge) are represented by colored readouts: green for positive values, red for negative. Finally, the total cost per EV cohort, and the average price of energy each received, is listed. All code can be found in the *Appendix* below, and in more pleasantly displayed and downloadable form, at <https://github.com/zacharyweiss/demandscheduling>.

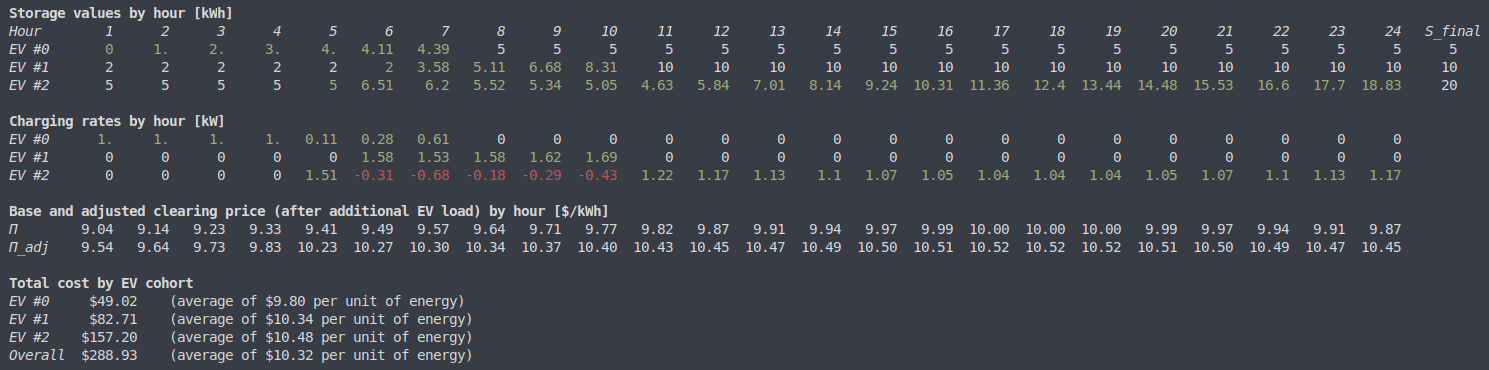
**3. Results**

As described in Section 1.2.3, running the final program iteration first yields the key and EV configuration specified.



*Figure 1*. Key and configuration readout from multi\_bounded.py

This is followed by the found solutions:



*Figure 2*. Sample console-logged solutions from multi\_bounded.py

As demonstrated in the sample results, pictured above, we can see the optimization seeking to schedule more charging during hours of lower prices, up until the impact of the additional EV load brings the clearing price near equal by hour, in hours where other constraints do not prevent this behavior. Additionally, as seen in hours 7 through 10, when multiple EVs are online, and one EV has significant charge already stored, the EV finds the optimal schedule to involve one of the cohorts selling back to the grid, while the others charge in their more limited charging windows.

**4. Future Work**

More realistic synthetic data, can be gathered from an ISO for results that provide insight rather than simply demonstrate the model working

Assumption of perfect forecast -> how to make a robust model? Input noise?

Assumption of desire to charge to max capacity: not all EV users care about this, just sufficient to get home

Assumption of symmetric maximum charge and discharge rates

Minimization of number of charge discharge cycles due to wearing out battery?

Different price for selling back to the grid?

Implementation of more intelligent price variance caused by changes in load: generic demand and supply curves, now stepwise price changes due to demand. One level higher: modeling a grid of a given size with loads and generators at each bus, so as to capture pricing impacts due to binding constraints.

Modular math: doesn’t currently account for

**5. Appendix**

#!/usr/bin/env python

\_\_author\_\_ = "Zachary Weiss"

**import** **pyomo.environ** **as** **pyo**

**import** **numpy** **as** **np**

*# global settings*

N\_HOURS = 24

EV\_CONFIG = {"S\_0": 0, "S\_max": 5, "R\_max": 1, "P": 1}

*# S -> stored energy*

*# R -> charge rate (per hour)*

*# P -> price influence coefficient, zero makes price independent of EV demand*

*# price / load signals*

base\_prices = np.random.rand(N\_HOURS) \* 10

**def** main():

model = pyo.ConcreteModel()

*# index*

hours = range(N\_HOURS)

*# decision vars*

model.S = pyo.Var(range(N\_HOURS+1), bounds=(0, EV\_CONFIG["S\_max"]), within=pyo.NonNegativeReals)

model.R = pyo.Var(hours, bounds=(-EV\_CONFIG["R\_max"], EV\_CONFIG["R\_max"]), within=pyo.Reals)

model.P = pyo.Var(hours, within=pyo.NonNegativeReals)

*# objective function*

cost = sum(model.P[t] \* model.R[t] **for** t **in** hours)

model.cost = pyo.Objective(expr=cost, sense=pyo.minimize)

*# constraints*

model.cons = pyo.ConstraintList()

*# boundary condition: after final hour, storage must equal maximum charge*

model.cons.add(model.S[N\_HOURS] == EV\_CONFIG["S\_max"])

*# boundary condition: storage begins at initial value*

model.cons.add(model.S[0] == EV\_CONFIG["S\_0"])

*# constraints applied each hour (bounds already handled in pyomo variable declaration)*

**for** t **in** hours:

*# update rule, storage at next time point is current storage plus amount charged*

model.cons.add(model.S[t+1] == model.S[t] + model.R[t])

*# price at each hour is sum of base price and amount of price increase from the load scheduled*

model.cons.add(model.P[t] == base\_prices[t] + (EV\_CONFIG["P"] \* model.R[t]))

*# cbc, glpk, gurobi, cplex, pico, scip, xpress: LP/MIP solvers*

*# conopt, cyipopt, ipopt: NLP*

*# path: MCP*

*# more can be found via "pyomo help --solvers"*

results = pyo.SolverFactory('multistart').solve(model, suppress\_unbounded\_warning=True)

*# display results*

model.display()

**print**("done")

**if** \_\_name\_\_ == '\_\_main\_\_':

main()

*File 1*. single\_unbounded.py: simplest form of model

*#!/usr/bin/env python*

*"""*

*Single EV cohort, limited hours of (dis)charge, able to influence price*

*Presumes EV should be fully charged after final hour connected to the grid, in addition to all assumptions stated in*

*the earlier single\_unbounded case.*

*"""*

\_\_author\_\_ = "Zachary Weiss"

**import** **pyomo.environ** **as** **pyo**

**import** **numpy** **as** **np**

*# global settings*

N\_HOURS = 24

EV\_CONFIG = {"S\_0": 0, "S\_max": 5, "R\_max": 1, "H": range(4, 18), "P": 1}

*# S -> stored energy*

*# R -> charge rate (per hour)*

*# H -> (array of) hours available to charge during*

*# P -> price influence coefficient, zero makes price independent of EV demand*

*# price / load signals*

base\_prices = np.random.rand(N\_HOURS) \* 10

base\_loads = np.random.rand(N\_HOURS) \* 2

**def** main():

model = pyo.ConcreteModel()

*# check valid hour configuration (no online hours specified beyond N\_HOURS)*

**if** max(EV\_CONFIG["H"]) >= N\_HOURS **or** min(EV\_CONFIG["H"]) < 0:

**raise** **SystemExit**("Hours specified for EV (dis)charge must be between zero and N\_HOURS. Modify the EV”

"config and rerun.")

*# index*

hours = range(N\_HOURS)

*# decision vars*

model.S = pyo.Var(range(N\_HOURS + 1), bounds=(0, EV\_CONFIG["S\_max"]), within=pyo.NonNegativeReals)

model.R = pyo.Var(hours, bounds=(-EV\_CONFIG["R\_max"], EV\_CONFIG["R\_max"]), within=pyo.Reals)

model.P = pyo.Var(hours, within=pyo.NonNegativeReals)

*# objective function*

cost = sum(model.P[t] \* model.R[t] **for** t **in** hours)

model.cost = pyo.Objective(expr=cost, sense=pyo.minimize)

*# constraints*

model.cons = pyo.ConstraintList()

*# boundary condition: storage begins at initial value*

model.cons.add(model.S[0] == EV\_CONFIG["S\_0"])

*# boundary condition: after final hour, storage must equal maximum charge*

model.cons.add(model.S[N\_HOURS] == EV\_CONFIG["S\_max"])

*# constraints applied each hour (bounds already handled in pyomo variable declaration)*

**for** t **in** hours:

*# if the EV is not able to (dis)charge during the current hour, the rate must be zero*

**if** t **not** **in** EV\_CONFIG["H"]:

model.cons.add(model.R[t] == 0)

*# update rule, storage at next time point is current storage plus amount charged*

model.cons.add(model.S[t + 1] == model.S[t] + model.R[t])

*# price at each hour is sum of base price and amount of price increase from the load scheduled*

model.cons.add(model.P[t] == base\_prices[t] + (EV\_CONFIG["P"] \* model.R[t]))

*# cbc, glpk, gurobi, cplex, pico, scip, xpress: LP/MIP solvers*

*# conopt, cyipopt, ipopt: NLP*

*# path: MCP*

*# more can be found via "pyomo help --solvers"*

results = pyo.SolverFactory('multistart').solve(model, suppress\_unbounded\_warning=True)

*# display results*

model.pprint()

**print**("**\n**" + "#"\*150 + "**\n**")

results.write()

**if** \_\_name\_\_ == '\_\_main\_\_':

main()

*File 2*. single\_bounded.py: limited hours of scheduling

*#!/usr/bin/env python*

*"""*

*Multiple EV cohorts, limited hours of (dis)charge, able to influence price*

*To install all dependencies with conda, run "conda install numpy && conda install -c conda-forge pyomo ipopt=3.11.1"*

*"""*

\_\_author\_\_ = "Zachary Weiss"

**import** **pyomo.environ** **as** **pyo**

**import** **numpy** **as** **np**

*# global settings*

N\_HOURS = 24

EV\_CONFIG = [{"S\_0": 0, "S\_max": 5, "R\_max": 1, "H": range(7), "P": 0.5},

{"S\_0": 2, "S\_max": 10, "R\_max": 2, "H": range(5, 10), "P": 0.5},

{"S\_0": 5, "S\_max": 20, "R\_max": 3, "H": range(4, N\_HOURS), "P": 0.5},

]

*# Example entry: {"S\_0": 0, "S\_max": 5, "R\_max": 1, "H": range(7), "P": 0.5}*

*# S -> stored energy [kWh]*

*# R -> charge rate [kW] (as everywhere referenced the rate is applied over an hour--implied "\*1hr" after each instance--*

*# it is effectively in units of kWh)*

*# H -> (array of) hours available to charge during [unitless indexes]*

*# P -> price influence coefficient, zero makes price independent of EV demand [$/kWh^2]*

**def** main():

notes()

model = pyo.ConcreteModel()

*# price signal: array of prices at each hour [$/kWh], peak value at 6pm*

*# base\_prices = np.random.rand(N\_HOURS) \* 10*

base\_prices = 5 \* gaussian(np.linspace(0, N\_HOURS - 1, N\_HOURS), 17, 26) + 5

*# check valid hour configuration (no online hours specified beyond N\_HOURS)*

**for** ev **in** EV\_CONFIG:

**if** max(ev["H"]) >= N\_HOURS **or** min(ev["H"]) < 0:

**raise** **SystemExit**("Hours specified for EV (dis)charge must be between zero and N\_HOURS. Modify the EV "

"config and rerun.")

*# index*

hours = range(N\_HOURS)

model.i = pyo.Set(initialize=[i **for** i, ev **in** enumerate(EV\_CONFIG)])

model.t = pyo.Set(initialize=hours)

model.t\_1 = pyo.Set(initialize=range(N\_HOURS + 1))

**def** ij\_init(m):

*# key pairs for S and R matrices w/i pyomo*

*# i is the EV number, j is the hour (t)*

**return** ((i, j) **for** i **in** m.i **for** j **in** m.t)

*# same as above, but initializes for one extra hour to be compatible with the update rule*

**def** ij\_init\_1(m):

**return** ((i, j) **for** i **in** m.i **for** j **in** m.t\_1)

*# decision vars*

model.S = pyo.Var(pyo.Set(dimen=2, initialize=ij\_init\_1), within=pyo.NonNegativeReals)

model.R = pyo.Var(pyo.Set(dimen=2, initialize=ij\_init), within=pyo.Reals)

model.P = pyo.Var(hours, within=pyo.NonNegativeReals)

*# objective function*

cost = sum(sum(model.P[t] \* model.R[i, t] **for** t **in** hours) **for** i, ev **in** enumerate(EV\_CONFIG))

model.cost = pyo.Objective(expr=cost, sense=pyo.minimize)

*# constraints*

model.cons = pyo.ConstraintList()

**for** i, ev **in** enumerate(EV\_CONFIG):

*# boundary condition: storage begins at initial value*

model.cons.add(model.S[i, 0] == ev["S\_0"])

*# boundary condition: after final schedule-able hour, storage must equal maximum charge*

model.cons.add(model.S[i, max(ev["H"]) + 1] == ev["S\_max"])

*# constraints applied each hour (bounds already handled in pyomo variable declaration)*

**for** t **in** hours:

**for** i, ev **in** enumerate(EV\_CONFIG):

*# if the EV is not able to (dis)charge during the current hour, the rate must be zero. Else, bounded by max*

*# and min (now added as constraint as cannot be easily added in variable bounds at time of declaration)*

**if** t **not** **in** ev["H"]:

model.cons.add(model.R[i, t] == 0)

**else**:

model.cons.add(pyo.inequality(-ev["R\_max"], model.R[i, t], ev["R\_max"]))

*# stored energy must be between 0 and the maximum for each EV cohort*

model.cons.add(pyo.inequality(0, model.S[i, t], ev["S\_max"]))

*# update rule, storage at next time point is current storage plus amount charged for each EV cohort*

model.cons.add(model.S[i, t + 1] == model.S[i, t] + model.R[i, t])

*# price at each hour is sum of base price and amount of price increase from the load scheduled*

model.cons.add(model.P[t] == base\_prices[t] + sum([ev["P"] \* model.R[i, t] **for** i, ev **in** enumerate(EV\_CONFIG)]))

results = pyo.SolverFactory('multistart').solve(model, suppress\_unbounded\_warning=True)

readout(model, base\_prices)

**def** lrange(\*args):

*"""drop in replacement for 'range()', if one wishes to easily concatenate ranges with '+' in the 'EV\_CONFIG'"""*

**return** list(range(\*args))

**def** gaussian(x, mu, sig):

**return** np.exp(-np.power(x - mu, 2.) / (2 \* np.power(sig, 2.)))

**def** notes():

**print**("""**\n\033**[1mKey and Notes**\033**[0m

**\033**[3mDependencies**\033**[0m: pyomo, numpy, ipopt, and multistart (NLP solver, should come pre-installed with pyomo)

If not fully installed, run "conda install numpy && conda install -c conda-forge pyomo ipopt=3.11.1" without quotes.

**\033**[3mS**\033**[0m -> stored energy [kWh]

**\033**[3mR**\033**[0m -> charge rate [kW] (as everywhere referenced the rate is applied over an hour**\u2014**implied "\*1hr" after each instance**\u2014**

it is effectively in units of kWh)

**\033**[3mH**\033**[0m -> (array of) hours available to charge during [unitless indexes]

**\033**[3mP**\033**[0m -> price influence coefficient, zero makes price independent of EV demand [$/kWh^2]

Values highlighted in green or red indicate the EV was available for charge or discharge (selling back to the grid)

during that hour, whereas the EV cohort is offline for all entries in the default text color. To compute for other EV

configurations, edit the EV\_CONFIG variable at the top of this file and re-run, following the example entry format for

each new EV cohort, separated by commas. If tables appear weird, expand your window horizontally or disable text wrapping.""")

**print**("**\n\033**[1mEV Cohort Configuration**\033**[0m")

**for** i, ev **in** enumerate(EV\_CONFIG):

**print**("**\033**[3m{:<12s}**\033**[0m".format(f"EV #{i}") + "".join("{:<25s}".format(f"**\033**[3m{k}**\033**[0m: {ev[k]}") **for** k

**in** ev))

**def** readout(model, base\_prices):

S\_sol = np.array([v.value **for** i, v **in** model.S.items()]).reshape(len(EV\_CONFIG), N\_HOURS + 1)

R\_sol = np.array([v.value **for** i, v **in** model.R.items()]).reshape(len(EV\_CONFIG), N\_HOURS)

P\_sol = np.array([v.value **for** i, v **in** model.P.items()])

costs = np.array([np.array(ev\_R \* P\_sol) **for** ev\_R **in** R\_sol])

ev\_tot\_cost = costs.sum(axis=1)

ev\_avg\_price = ev\_tot\_cost / R\_sol.sum(axis=1)

**def** pretty\_print(arr):

*# zero out floating point errors within tolerance*

tol = 1e-16

arr[(arr < tol) & (-tol < arr)] = 0

**def** trimmer(n):

**return** np.format\_float\_positional(n, 2, trim="-")

**for** i, row **in** enumerate(arr):

num\_str = ''.join(

(f'**\033**[92m{trimmer(val): >7s}**\033**[0m' **if** val >= 0 **else** f'**\033**[91m{trimmer(val): >7s}**\033**[0m')

**if** t **in** EV\_CONFIG[i]["H"] **else** f'{trimmer(val): >7s}' **for** t, val **in** enumerate(row))

**print**("**\033**[3m{:<6s}**\033**[0m{}".format(f"EV #{i}", num\_str))

**print**("**\n\033**[1mStorage values by hour [kWh]**\033**[0m")

hour\_arr = np.append(np.add(lrange(N\_HOURS), 1), "S\_final")

**print**("**\033**[3m{:<9s}".format("Hour") + ''.join("{:^7s}".format(hr) **for** hr **in** hour\_arr) + "**\033**[0m")

pretty\_print(S\_sol)

**print**("**\n\033**[1mCharging rates by hour [kW]**\033**[0m")

pretty\_print(R\_sol)

**print**("**\n\033**[1mBase and adjusted clearing price (after additional EV load) by hour [$/kWh]**\033**[0m")

**print**("**\033**[3m{:<6s}**\033**[0m".format("Π") + "".join(f"{item: >7.2f}" **for** item **in** base\_prices))

**print**("**\033**[3m{:<6s}**\033**[0m".format("Π\_adj") + "".join(f"{item: >7.2f}" **for** item **in** P\_sol))

**print**("**\n\033**[1mTotal cost by EV cohort**\033**[0m")

**for** i, ev **in** enumerate(EV\_CONFIG):

**print**("**\033**[3m{:<8s}**\033**[0m{:>8s} {}".format(f"EV #{i}", f"${ev\_tot\_cost[i]:.2f}", "(average "

f"of ${ev\_avg\_price[i]:.2f} per unit of energy)"))

**print**(f"**\033**[3mOverall**\033**[0m ${model.cost():.2f} (average of ${model.cost() / sum(sum(R\_sol)):.2f} per unit "

f"of energy)")

**if** \_\_name\_\_ == '\_\_main\_\_':

main()

*File 3*. multi\_unbounded.py: full model with custom readout

# **Works Cited**

|  |  |
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| [1] | M. L. a. H. G. A. a. H. W. E. a. L. C. D. a. N. B. L. a. S. J. D. a. W. J.-P. a. W. D. L. Bynum, Pyomo--optimization modeling in python, Third ed., vol. 67, Springer Science & Business Media, 2021. |
| [2] | W. E. a. W. J.-P. a. W. D. L. Hart, "Pyomo: modeling and solving mathematical programs in Python," *Mathematical Programming Computation,* vol. 3, no. 3, pp. 219--260, 2011. |
| [3] | A. Wächter and L. T. Biegler, "On the implementation of a primal-dual interior point filter line search algorithm for large-scale nonlinear programming," *Mathematical Programming,* vol. 106, no. 1, p. 25–57, 2006. |
| [4] | C. R. Harris, K. J. Millman, S. J. van der Walt and et al., "Array programming with NumPy," *Nature,* vol. 585, no. 7825, pp. 357--362, 2020. |
| [5] | T. Hodge, "Hourly electricity consumption varies throughout the day and across seasons - Today in Energy," U.S. Energy Information Administration, 21 February 2020. [Online]. Available: https://www.eia.gov/todayinenergy/detail.php?id=42915. [Accessed 5 May 2021]. |