Project Euler Proofs

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1. $\sum_{k=1}^{n} k = \frac{n*(n+1)}{2}$ for all $n \geq 1$.

Proof. The formula clearly holds when $n = 1$ and $n = 2$. Now suppose the formula holds when $n = m \geq 2$. Then

$$\sum_{k=1}^{m+1} k = \sum_{k=1}^{m} k + m + 1 = \frac{m * (m+1)}{2} + m + 1 = \frac{(m+1) * (m+2)}{2}$$

Thus the formula holds for $n = m + 1$, so by the principle of induction it holds for all $n \geq 1$. $\square$

2. Let $a, b, c \in \mathbb{Z}$. Then $b \mid a$ and $c \mid a$ if and only if $\mathrm{lcm}(b, c) \mid a$.

Proof. Set $n = \mathrm{lcm}(b, c)$, and suppose $b \mid a$ and $c \mid a$. By the division algorithm, $a = pn + r$ for some $p, r \in \mathbb{Z}$, where $0 \leq r < n$. Thus, since $b \mid a$ and $b \mid n$, we have $b \mid r$. By the same reasoning, we also have $c \mid r$. Since r is a common multiple of b and c and $r < n$, r must be 0. Thus $a = pn$, that is, $n \mid a$. Conversely, let $k, j \in \mathbb{Z}$ such that $n = kb$ and $n = jc$. Then if $n \mid a$, we have, for some $m \in \mathbb{Z}$, $a = mn = (mk)b = (mj)c$, and thus $c \mid a$ and $b \mid a$. $\square$