Functional Analysis Exploration 2

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The purpose of the first section is to show uniqueness of the degree function. What I gathered from **Proposition 1.1** is that if $A \subset \mathbb{R}^n$ is compact and $f : A \to \mathbb{R}^n$ is continuous, then we can extend f (continuously) to $\mathbb{R}^n$ with an inclusion type mapping $\tilde{f}$. To gain some intuition I drew the auxillary function used in the proof for constructing $\tilde{f}$. I suspect the

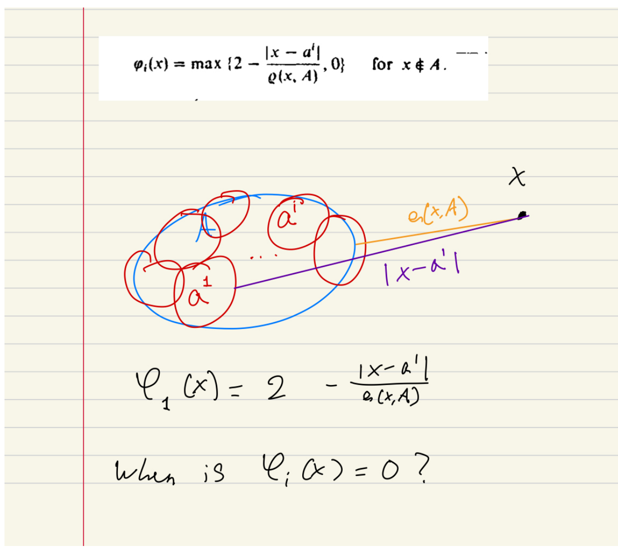

Figure 1: Illustration of $\varphi_1(x)$.

2 used in the definition is to work out nicely with the definition of $\tilde{f}$ which is

$$\tilde{f}(x) = \begin{cases} f(x) & \text{for } x \in A \\ \left(\sum_{i \geq 1} 2^{-i} \varphi_i(x)\right)^{-1} \sum_{i \geq 1} 2^{-i} \varphi_i(x) f(a^i) & \text{for } x \notin A \end{cases}$$

where a^i is the ith subset of the subcover for the compact set A. The "mollifier" function $\varphi_\alpha : \mathbb{R}^n \to \mathbb{R}$ defined as

$$\varphi_1 = \begin{cases} c \cdot \exp\left(-\frac{1}{1-|x|^2}\right) & \text{for } |x| < 1 \\ 0 & \text{otherwise} \end{cases}$$

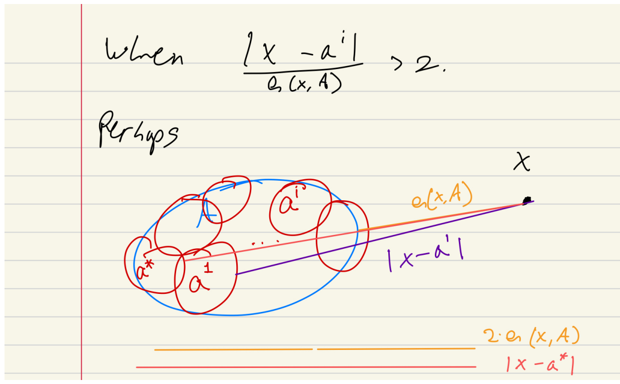

Figure 2: Illustration of when $\varphi_*(x) = 0$.

where $c > 0$ such that $\int_{\mathbb{R}^n} \varphi_1(x)dx = 1$, and $\varphi_\alpha(x) = \alpha^{-n}\varphi_1(x/\alpha)$. This was used in **Proposition 1.2**. Again, I wanted to get an idea of what these functions look like that are used in the proofs.

A lot of the proofs this section seemed to come out of thin air and then Deimling will say, "don't worry as a more general proof will be given later in the book." Because of this, I am sort of taking these results at "face value" for now. I noticed there is often another function set up as a convex combination of two functions of interest for some proofs. I wonder if this is a strategy commonly used in homotopy proofs for deformations. ♣

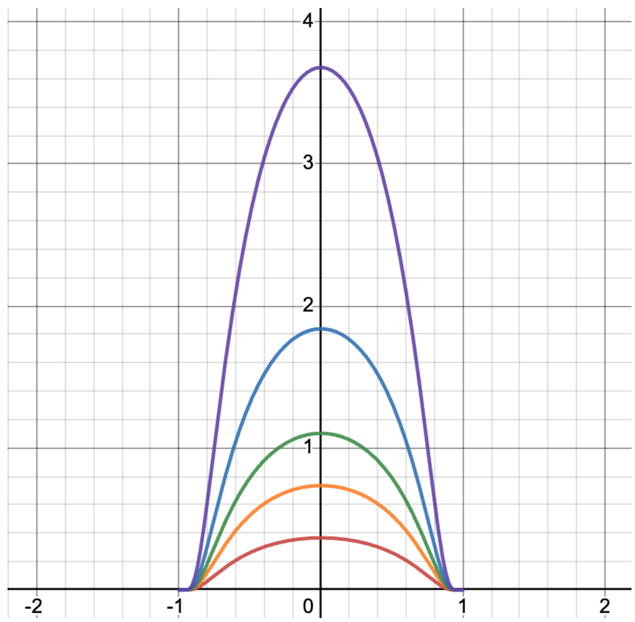

Figure 3: Mollifier function (Deimling page 7) for $c = 1, 2, 3, 5, 10$ for curves red, orange, green, blue, and pruple, respectively.