Chapter 7
Pushdown Automata

Vocabulary

- Backus Naur Form (BNF)
- λ–productions
- λ–free languages
- Unit production
- s-grammars

Backus Naur Form

- Variables are enclosed in $< >$ if A was a variable it would be denoted by "$< A >$".
- $::=$ is used instead of $\rightarrow$
- Terminals have no special markup.

Additional Normal forms for CFGs

Chomsky normal form

All productions are of the form

$$A \rightarrow BC$$

or

$$A \rightarrow a$$

Where

$$A, B, C \in V$$

a $\in T$

Additional Normal forms for CFGs

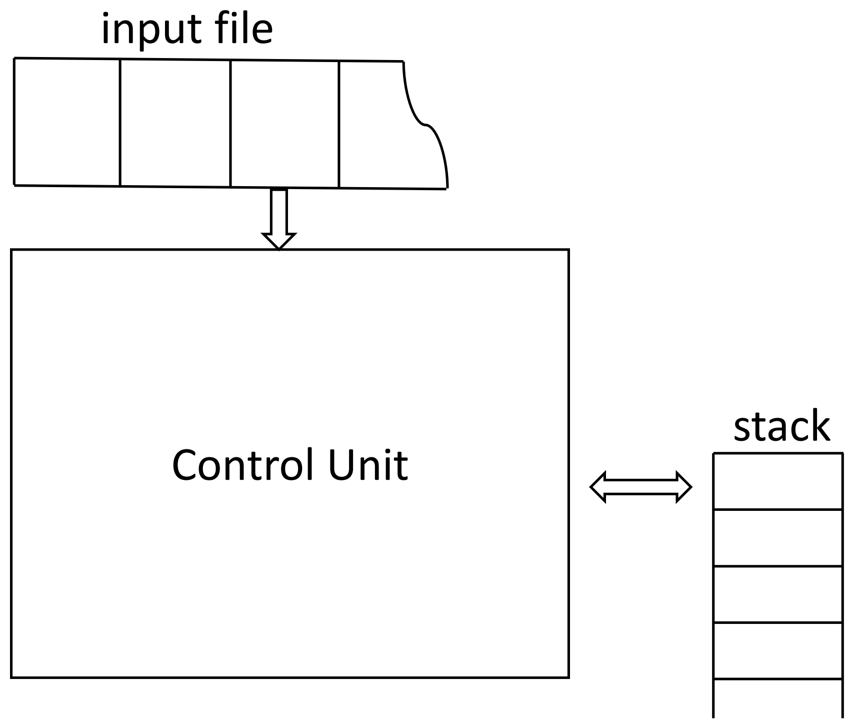
Greibach normal form

Form like s-grammars but no restriction on how many times a variable can produce a particular terminal.

Pushdown Automata

- a class of machines that allows counting with limitless memory.
- nondeterministic pushdown automata accepts context-free languages.
- NO equivalence between nondeterministic pdas and deterministic pdas.
- deterministic pdas accept deterministic context-free languages.

PDA schematic



Nondeterministic PDA

$M = (Q, \Sigma, \Gamma, \delta, q_0, z, F)$

where

Q is a finite set of internal states of the control unit

Σ is the input alphabet

Γ is a finite set of symbols called the stack alphabet

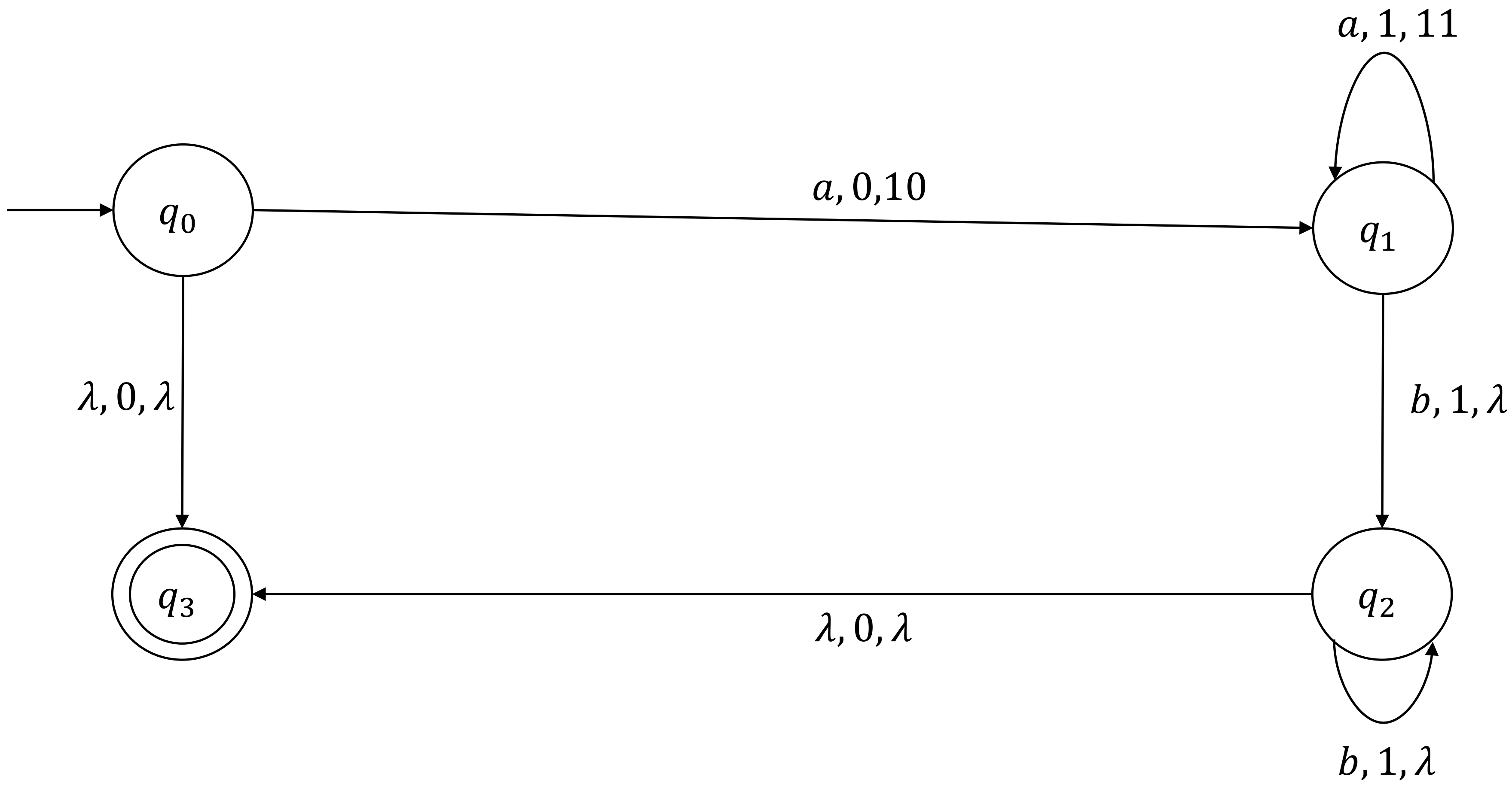
$\delta: Q \times (\Sigma \cup \{\lambda\}) \times \Gamma) \rightarrow$ set of finite states of Q$\times \Gamma^*$ is the transition function

$q_0 \in Q$ *is the initial state of the control unit*

$z \in \Gamma$ *is the stack start symbol*

$F \subseteq Q$ is the set of final states.

Example 1: Is $L = \{a^n b^n : n \geq 0\}$ a CFL?



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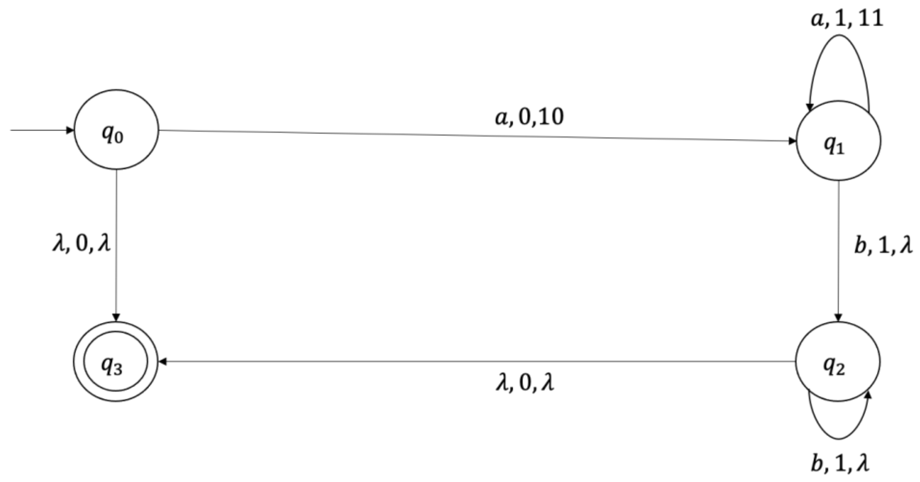

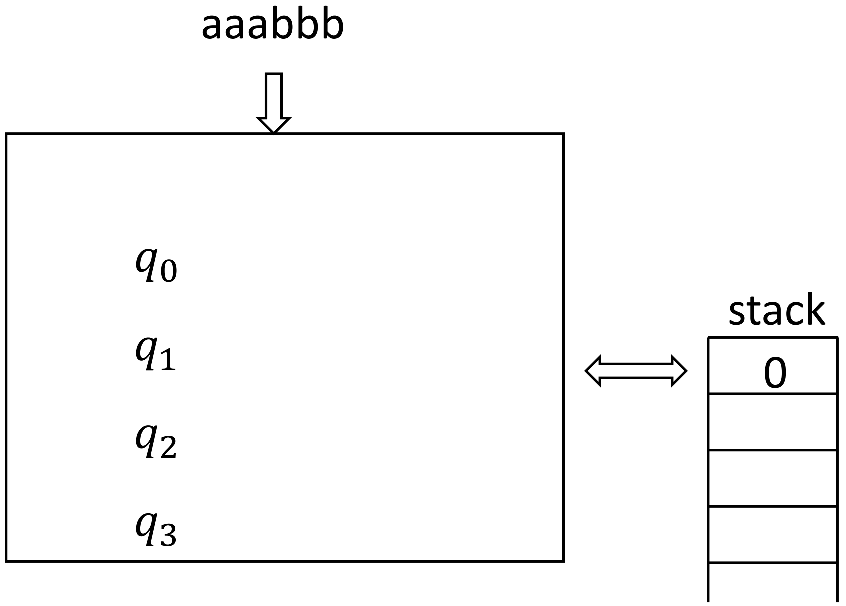

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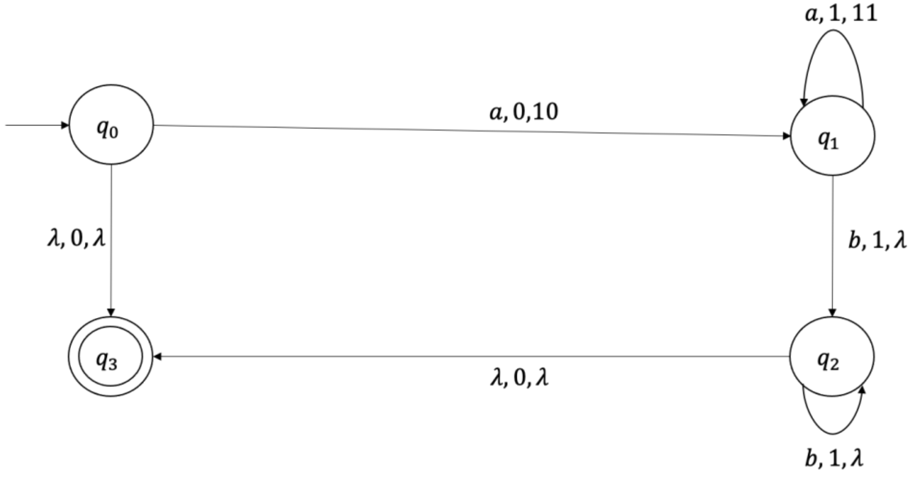

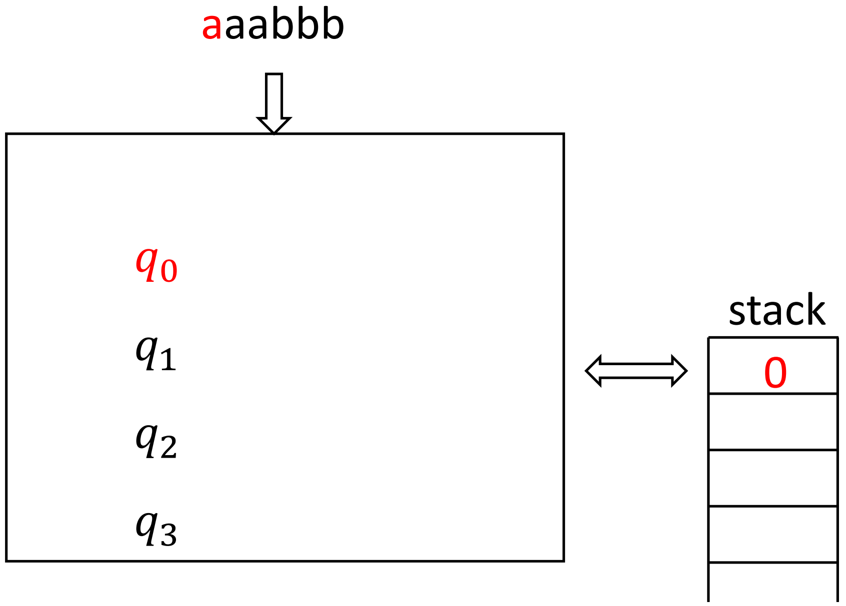

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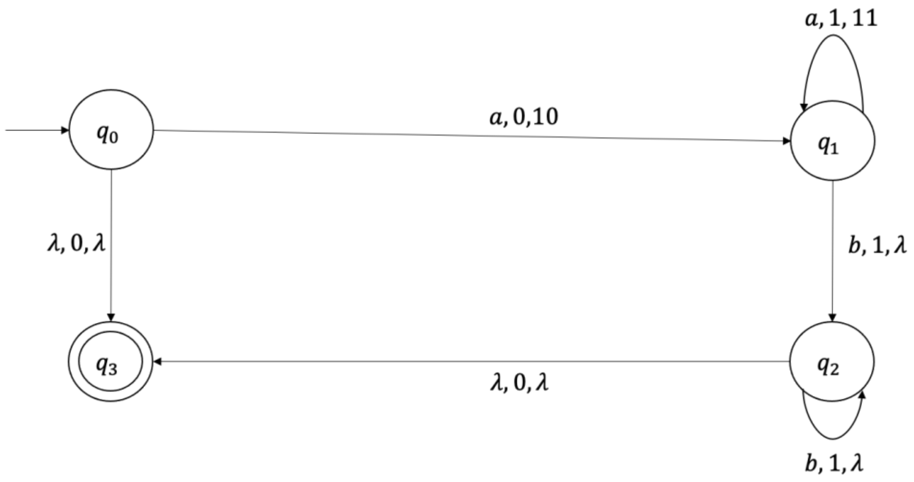

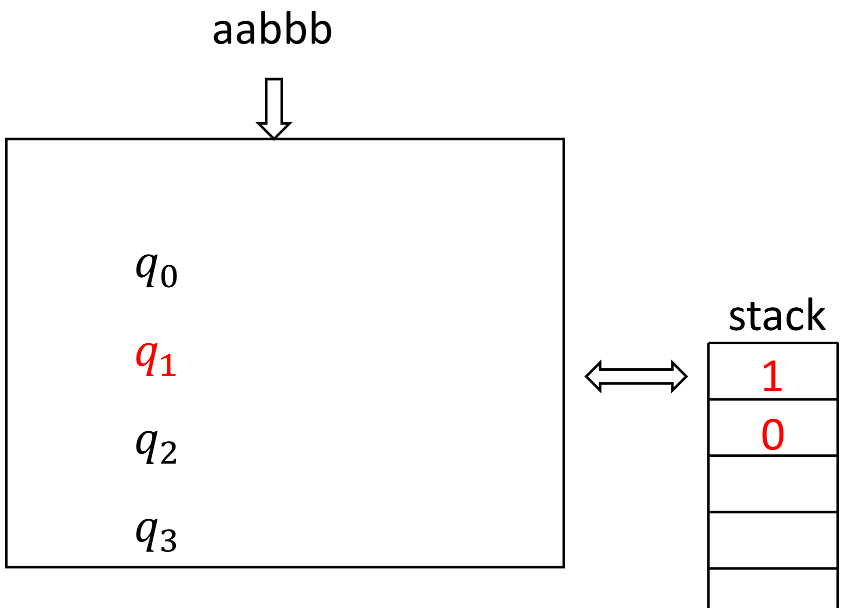

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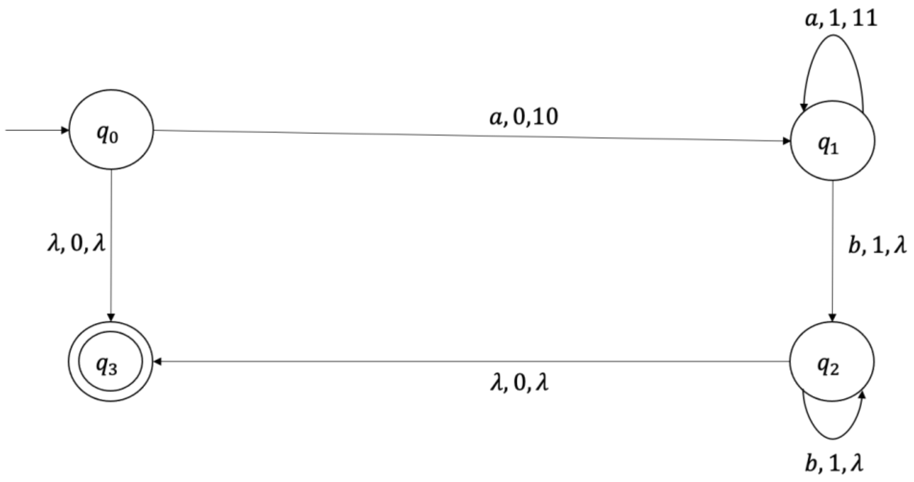

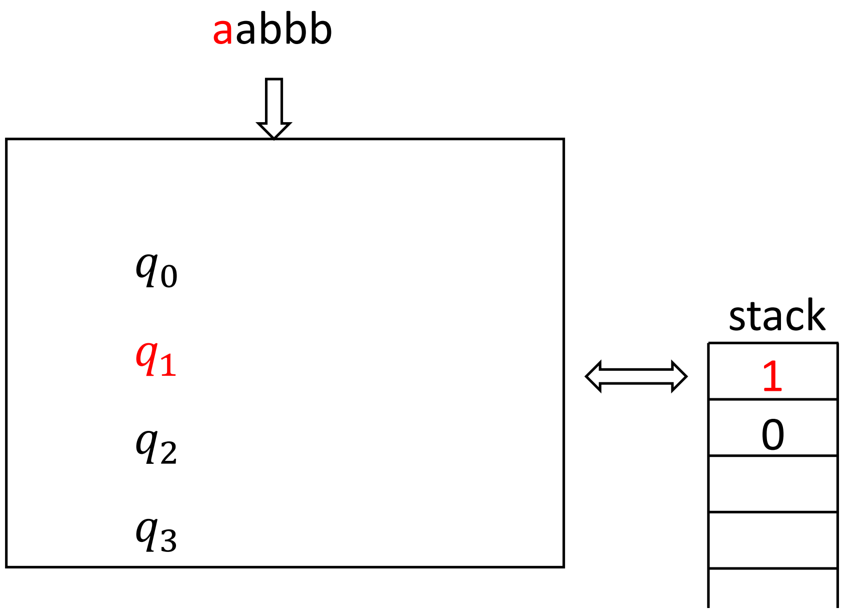

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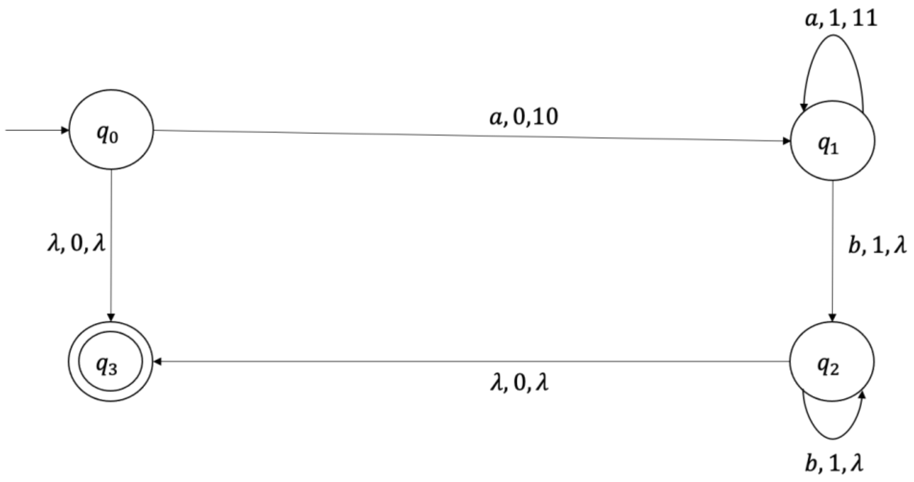

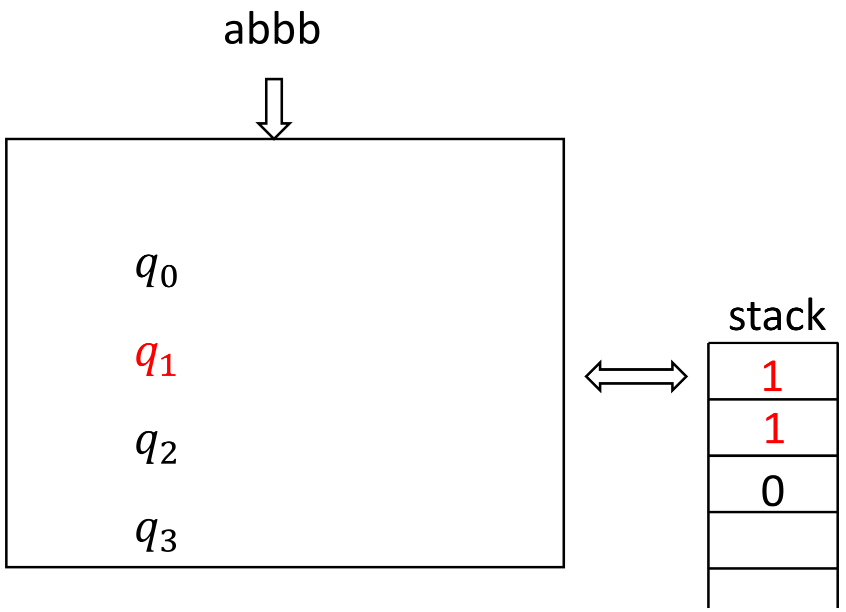

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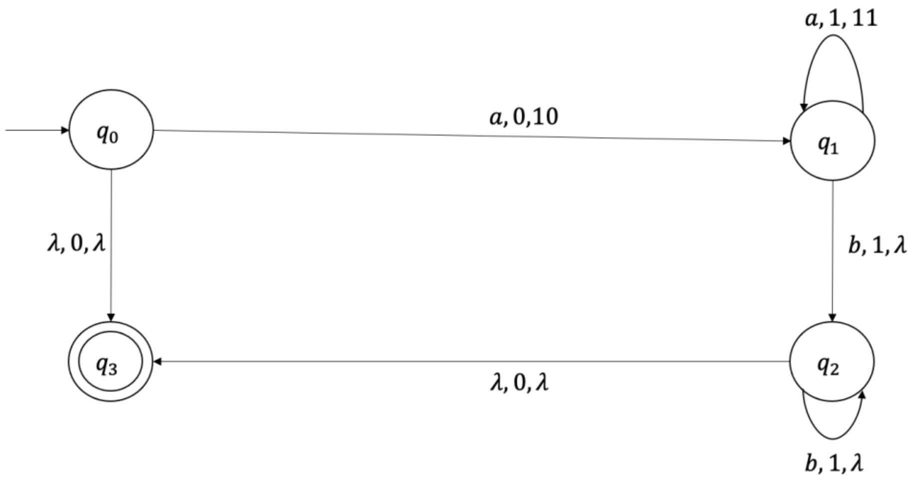

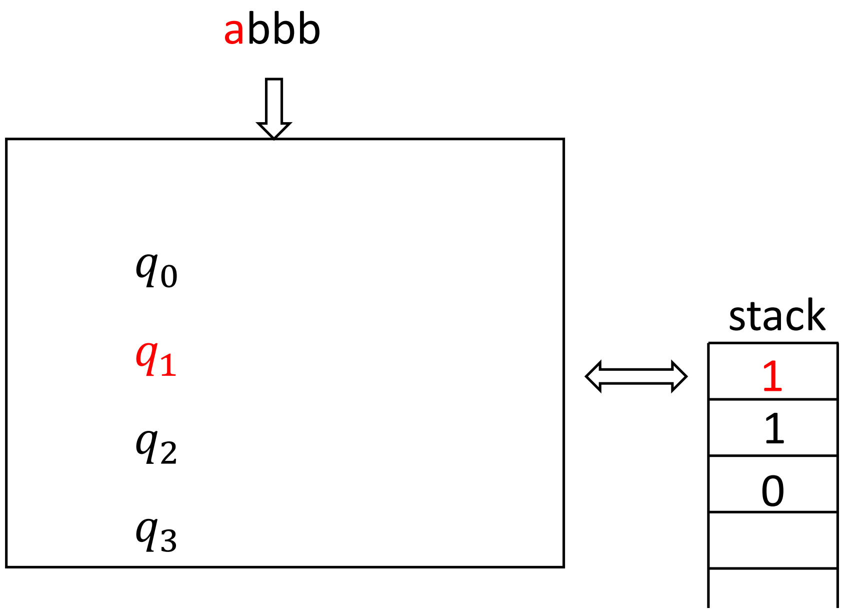

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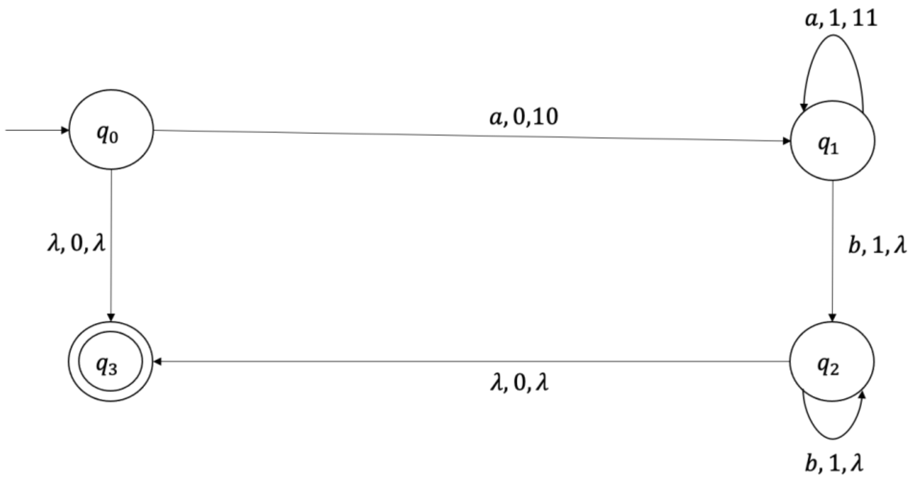

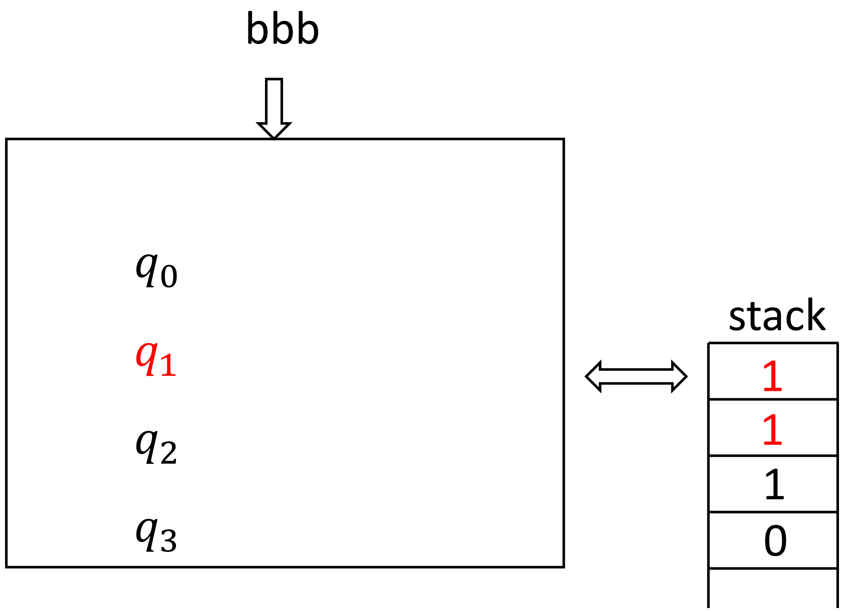

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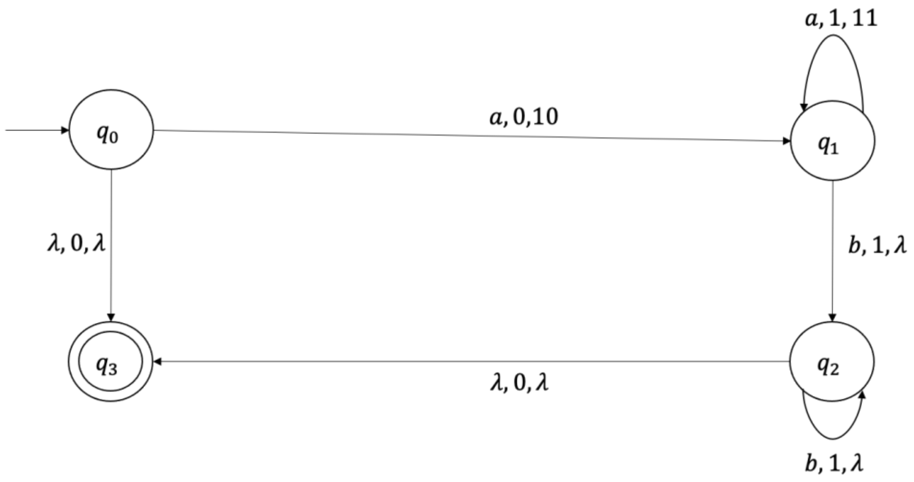

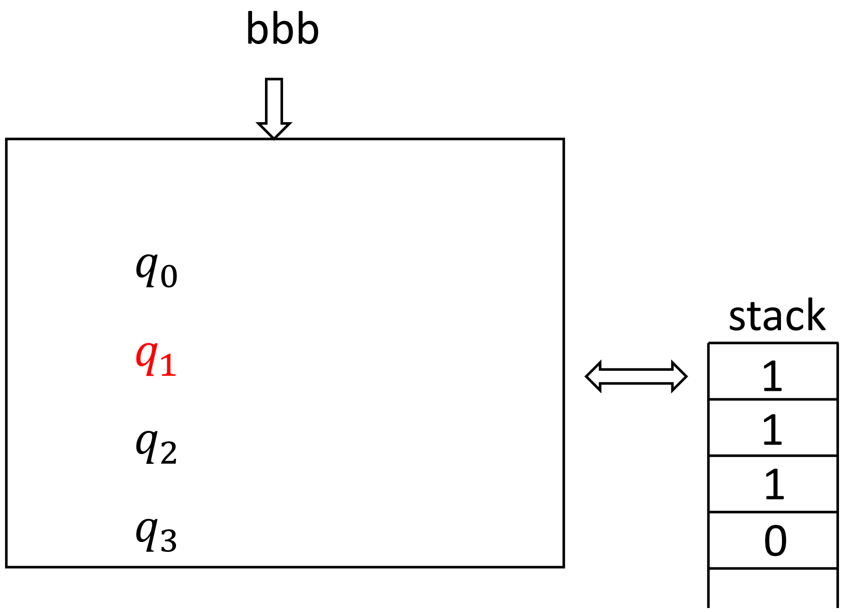

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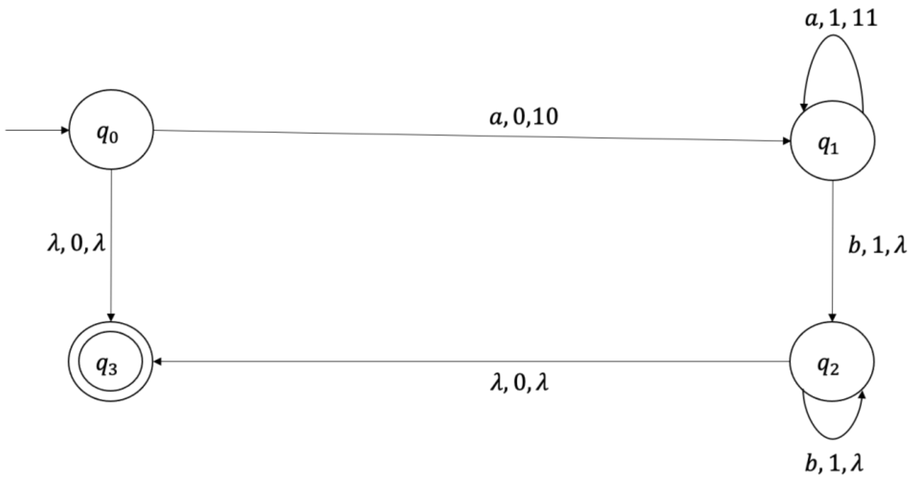

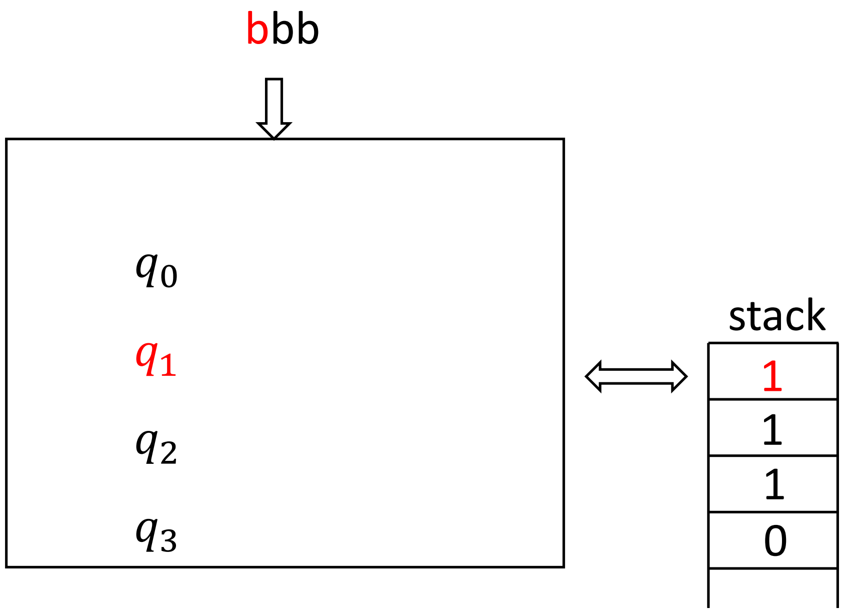

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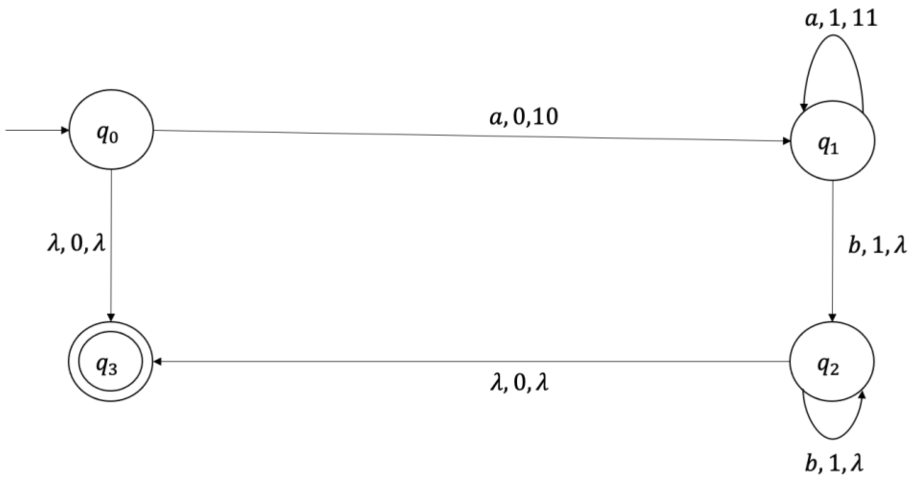

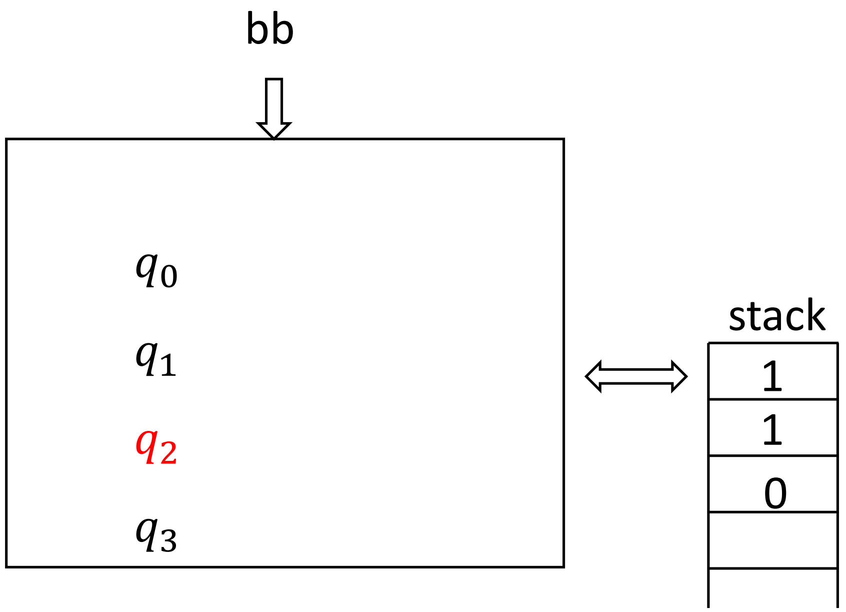

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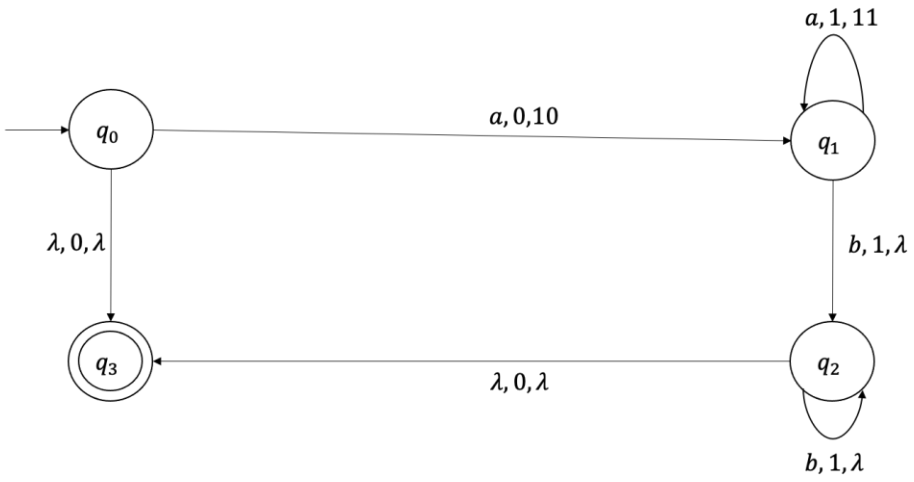

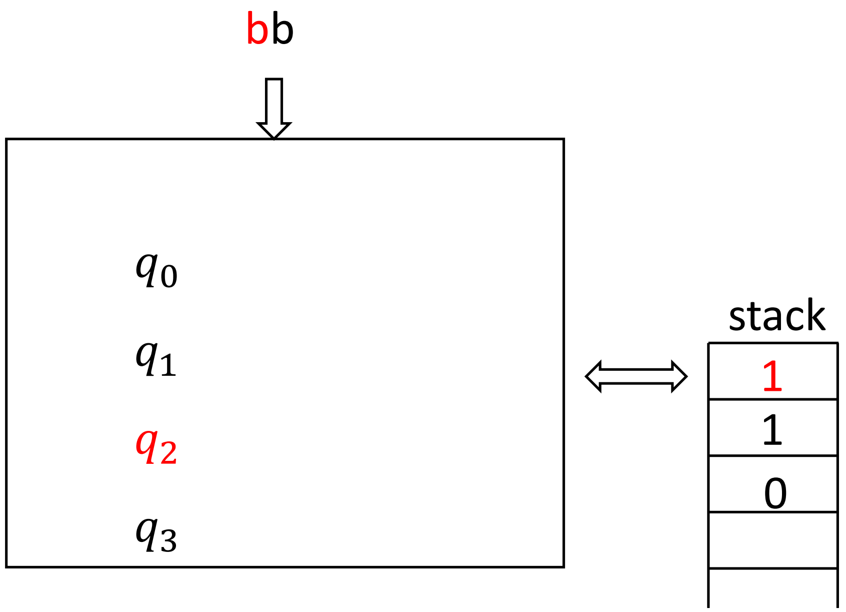

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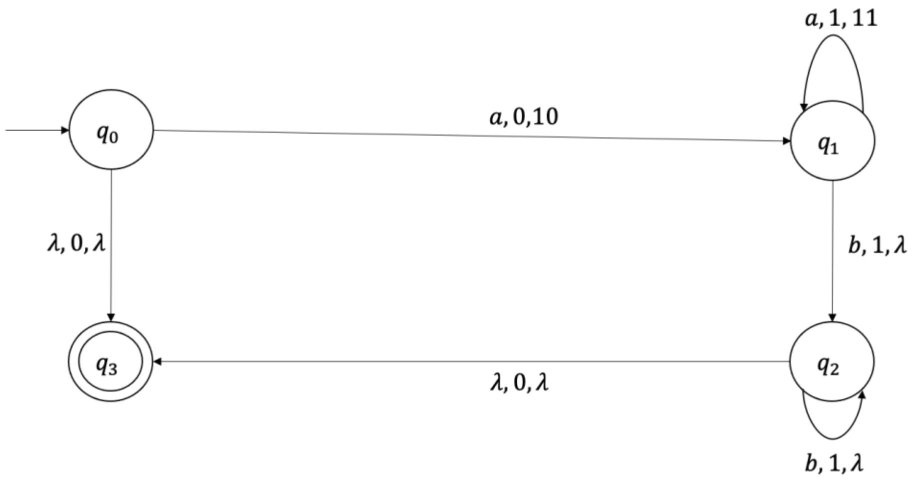

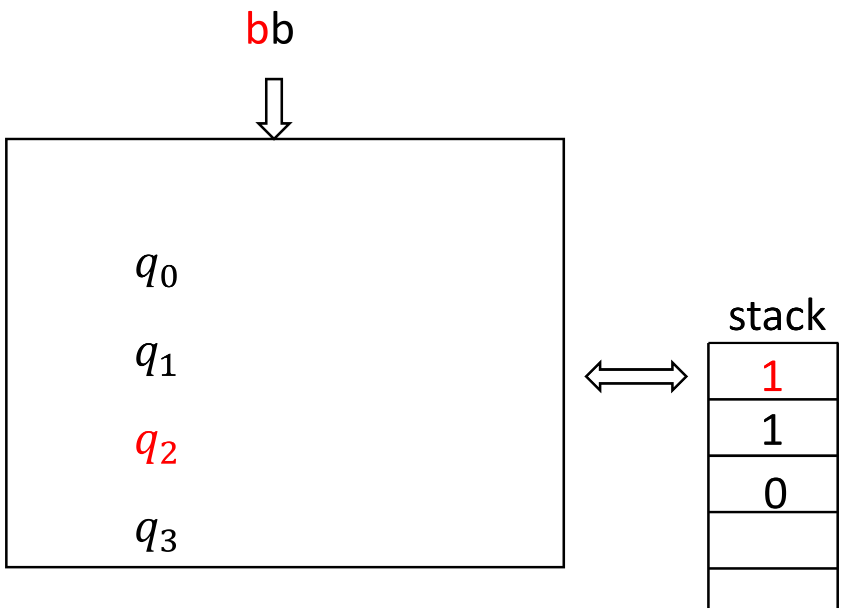

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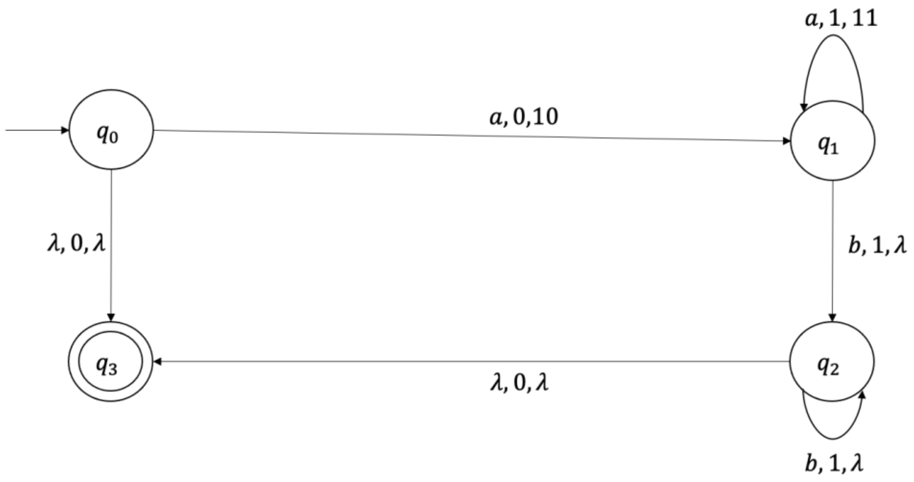

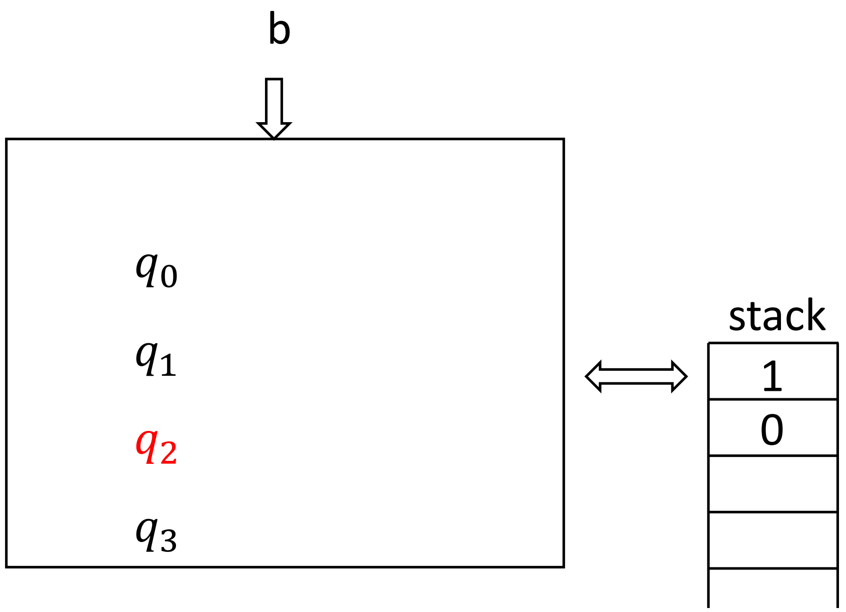

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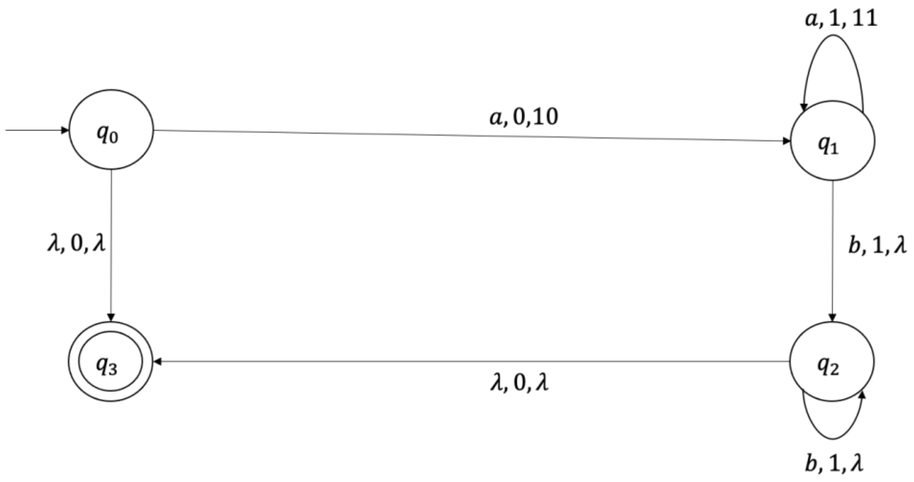

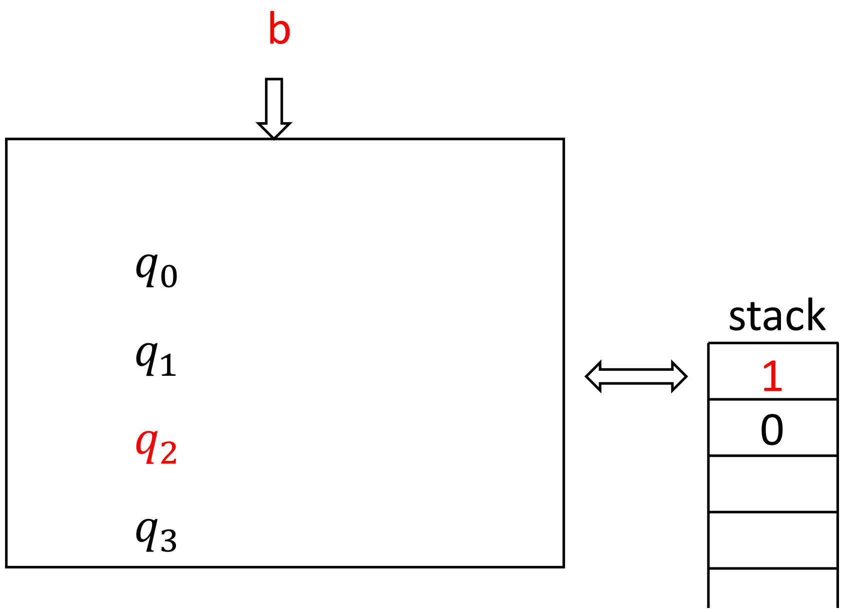

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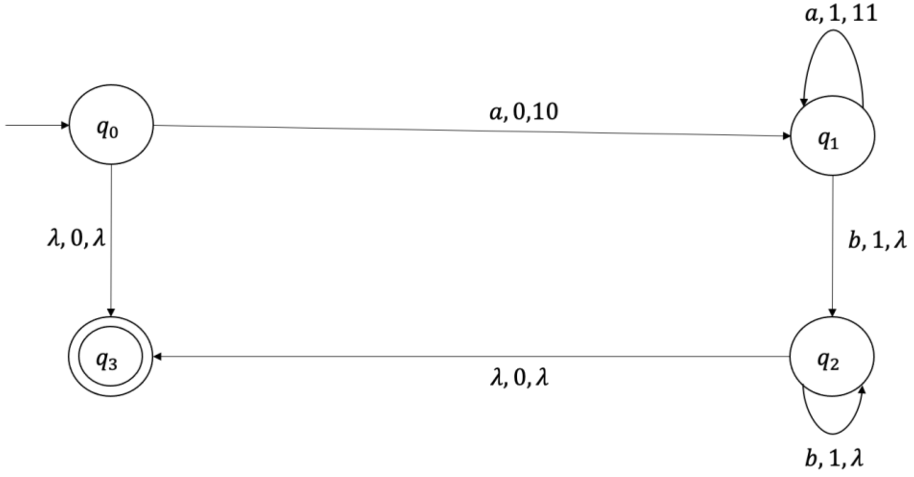

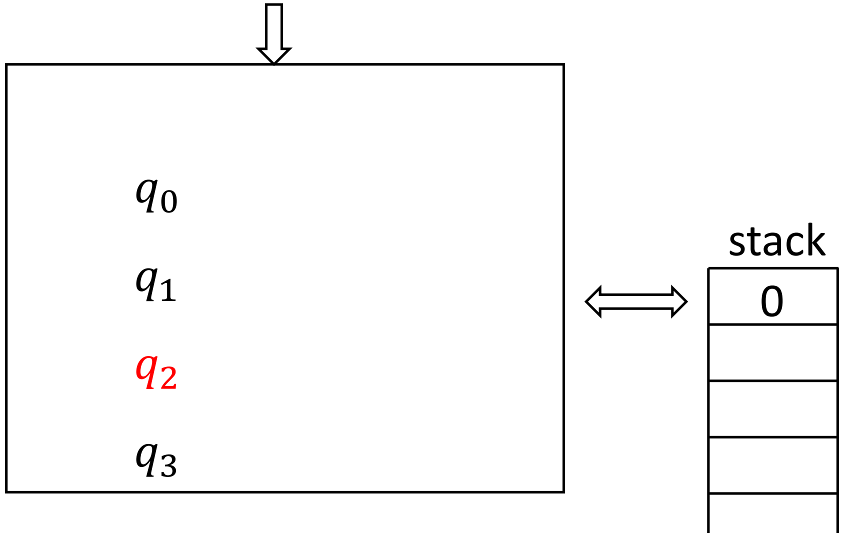

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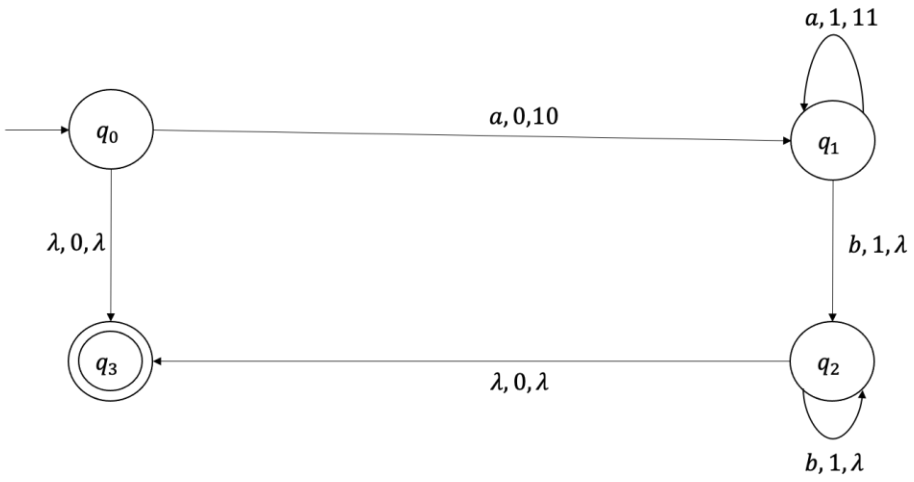

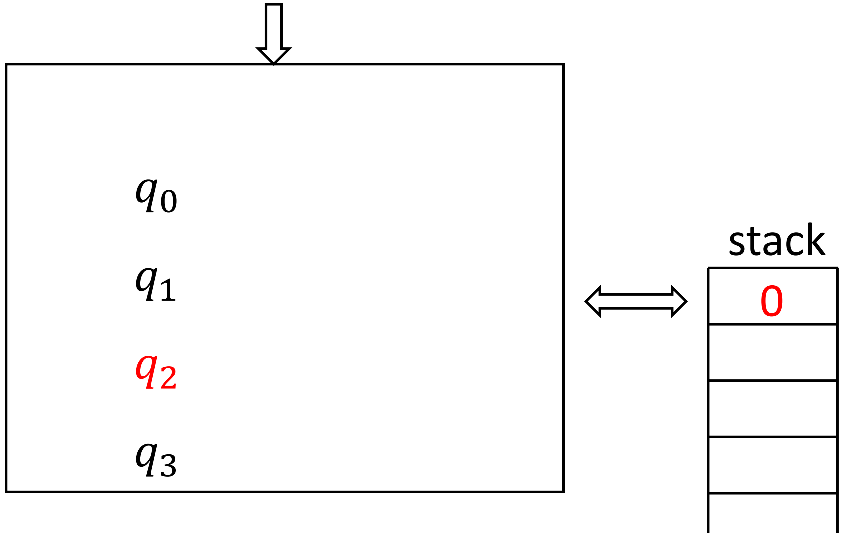

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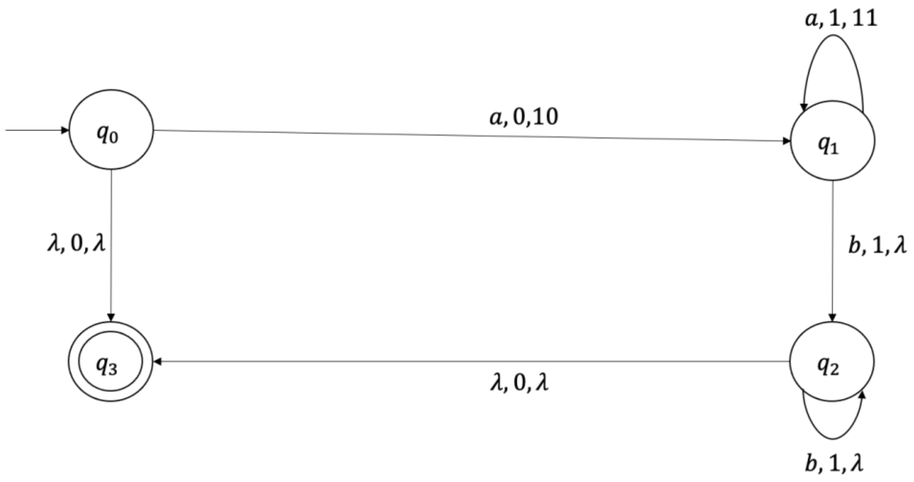

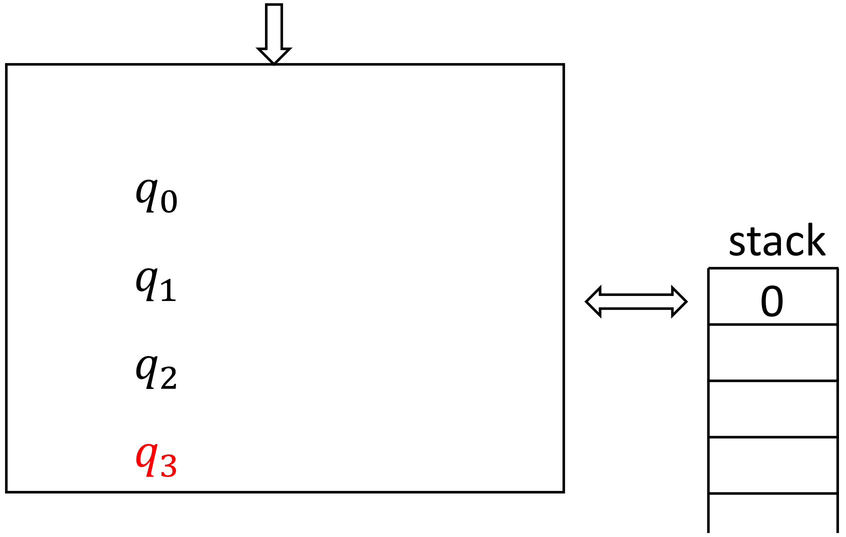

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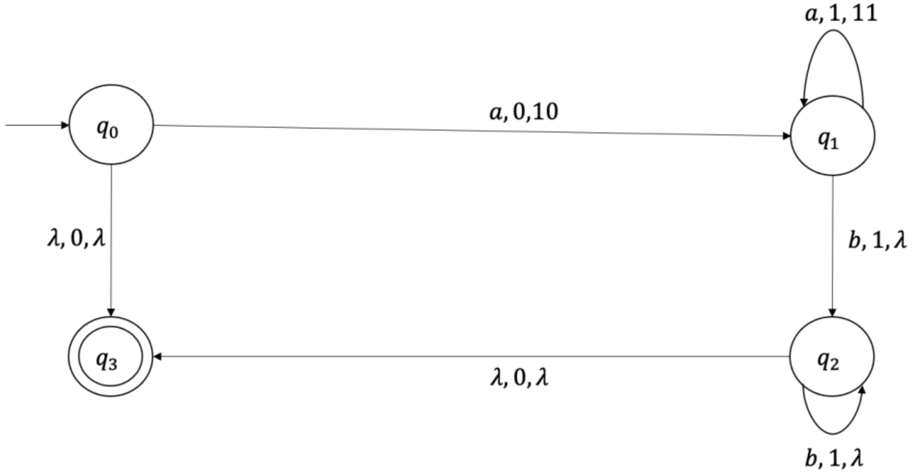

$$M = (\{q_0, q_1, q_2, q_3\},\ \{a, b\},\ \{0, 1\}, \delta, q_0, 0,\ \{q_3\})$$

$$\delta(q_0, a, 0) = \{(q_1, 10)\}$$
$$\delta(q_0, \lambda, 0) = \{(q_3, \lambda)\}$$
$$\delta(q_1, a, 1) = \{(q_1, 11)\}$$
$$\delta(q_1, b, 1) = \{(q_2, \lambda)\}$$
$$\delta(q_2, b, 1) = \{(q_2, \lambda)\}$$
$$\delta(q_2, \lambda, 0) = \{(q_3, \lambda)\}$$

Instantaneous Description

- In the form (current state, unread string, contents of stack)
- $\vdash$ denotes a move from one instantaneous description to another
- $\vdash_M$ denotes a move from a particular pushdown automaton

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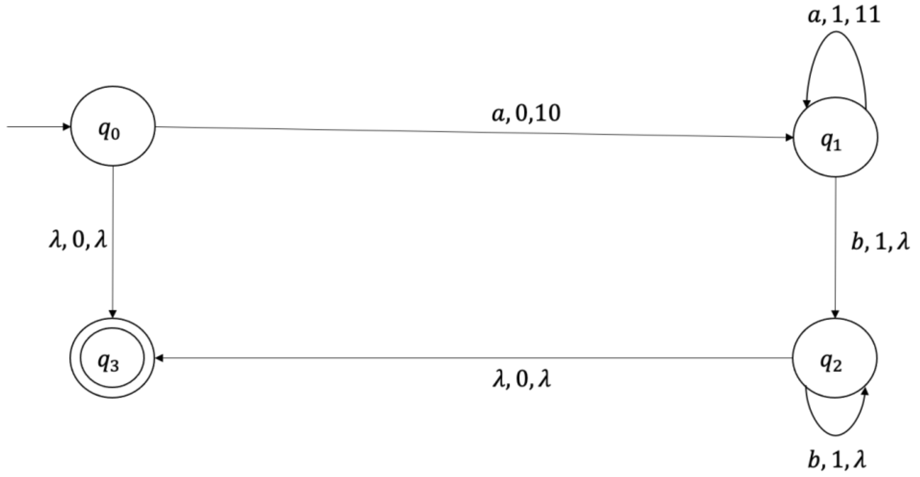

$(q_0, aaabbb, 0) \vdash_M (q_1, aabbb, 10) \vdash_M (q_1, abbb, 110) \vdash_M (q_1, bbb, 1110) \vdash_M (q_2, bb, 110) \vdash_M$
$(q_2, b, 10) \vdash_M (q_2, \lambda, 0) \vdash_M (q_3, \lambda, \lambda)$

$$\delta(q_0, a, 0) = \{(q_1, 10)\}$$
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$$\delta(q_2, \lambda, 0) = \{(q_3, \lambda)\}$$

Relationship between PDA and CFL

- Given a CFG we can construct an NPDA for that grammar
- Given a NPDA for a CFL, we can construct a CFG.

Construction of an NPDA from a Greibach Normal Form grammar

$S \to aSA \,|a$
$A \to bB$
$B \to b$

1. Put Start symbol onto the stack

   $\delta(q_0, \lambda, z) = \{(q_1, Sz)\}$

2. The NPDA will be simulated by taking S from the stack and replacing it with SA and removing a from the beginning of the input.

   $\delta(q_1, a, S) = \{(q_1, SA), (q_1, \lambda)\}$

3. Do all productions the same way

   $\delta(q_1, b, A) = \{(q_1, B)\}$

   $\delta(q_1, b, B) = \{\ (q_1, \lambda)\}$

4. Stack start symbols appears back on top of the stack

   $\delta(q_1, \lambda, z) = \{(q_2, \lambda)\}$

Methods of Transforming Grammars

Substitution rule

Given that we have productions in the form $\boldsymbol{A \to x_1 B x_2}$ in grammar

$$G = (V, T, S, P)$$

$$\boldsymbol{B \to y_1 | y_2 | \dots | y_n}$$

$$L(G) = L(\hat{G})$$

$$\hat{G} = (V, T, S, \hat{P})$$

Delete all production where B is in the right side of productions($\boldsymbol{A \to x_1 B x_2}$) and replace with the sentinel forms ($\boldsymbol{y_1 | y_2 | \dots | y_n}$)of productions where B is the left side($\boldsymbol{B \to y_1 | y_2 | \dots | y_n}$) symbol

$$A \to x_1 B x_2 \implies A \to x_1 y_1 x_2 \,| x_1 y_2 x_2 \,| x_1 \dots x_2 | x_1 y_n x_2$$

Methods of Transforming Grammars

By applying the substitution rule, then we may introduce **useless productions.**

Some useless production can occur by not having a path to a variable during derivation such as

$S \rightarrow aSb \mid ab$
$A \rightarrow aAb \mid \lambda$

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Methods of Transforming Grammars

Other **useless productions.**

When introducing a variable into a derivation that has no way of being removed which means no sentence can be derived.

$S \to aSb \mid \lambda \mid A$
$A \to aA$

Methods of Transforming Grammars

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$S \rightarrow aSb \mid \lambda \mid A$
$A \rightarrow aA$

Methods of Transforming Grammars

Removing λ–production for λ–free languages

$$L1 = \{a^n b^n : \ n \geq 0\}$$

$$L2 = \{a^n b^n : \ n \geq 1\}$$

$$G = (\{S, A\}, \{a, b\}, S, P)$$

$$S \rightarrow aAb$$

$$A \rightarrow aAb \mid \lambda$$

Methods of Transforming Grammars

Removing λ–production for λ–free languages

$$L = \{a^n b^n : n \geq 0\}$$

$$L = \{a^n b^n : n \geq 1\}$$

$$G = (\{S, A\}, \{a, b\}, S, P)$$

$$S \rightarrow aAb$$

$$A \rightarrow aAb$$

$$A \rightarrow \lambda$$

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$$S \rightarrow aAb \mid ab$$

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$$A \rightarrow \lambda$$

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Removing λ–production for λ–free languages

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$G = (\{S, A\}, \{a, b\}, S, P)$

$$S \rightarrow aAb \mid ab$$

$$A \rightarrow aAb \mid ab$$

General Algorithm to remove λ–productions from λ–free languages (Theorem 6.3)

1. For any $A \in V$ and there is a production $A \to \lambda$ put A in a list of nullable variables. A nullable variable is any variable that starting with that variable there is a path that produces the empty string.
2. Repeat the process of replacing on the right side of productions with different combinations of nullable variables. If there is a combination that makes a new variable nullable, then you add it to the list of nullable variables until no new nullable values are added.
3. Now take all productions in the grammar $G = (V, T, S, P)$ and add to grammar $\hat{G} = \left(V, T, S, \hat{P}\right)$. Exclude production in the form of $A \to \lambda$
4. Add productions that replaces each nullable variable in all combinations in a production.

Example 2: Find a equivalent grammar without any λ–productions

$S \rightarrow ABaC$
$A \rightarrow BC$
$B \rightarrow b \mid \lambda$
$C \rightarrow D \mid \lambda$
$D \rightarrow d$

Example 2: Find a equivalent grammar without any λ–productions

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$B \rightarrow \lambda$
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Nullable variables

B, C

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Nullable variables

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Example 2: Find a equivalent grammar without any λ–productions

$S \rightarrow ABaC$

$A \rightarrow BC \mid B \mid C \mid \lambda$

$B \rightarrow b$

$B \rightarrow \lambda$

$C \rightarrow D$

$C \rightarrow \lambda$

$D \rightarrow d$

Nullable variables

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Example 2: Find a equivalent grammar without any λ–productions

$S \rightarrow ABaC$

$A \rightarrow BC \mid B \mid C$

$A \rightarrow \lambda$

$B \rightarrow b$

$B \rightarrow \lambda$

$C \rightarrow D$

C$\rightarrow \lambda$

$D \rightarrow d$

Nullable variables

B, C, A

Example 2: Find a equivalent grammar without any λ–productions

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$A \rightarrow BC \mid B \mid C$
$A \rightarrow \lambda$
$B \rightarrow b$
$B \rightarrow \lambda$
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$C \rightarrow \lambda$
$D \rightarrow d$

Nullable variables

B, C, A

$S \rightarrow ABaC \mid BaC \mid AaC \mid ABa \mid aC \mid Ba \mid Aa \mid a$
$A \rightarrow BC \mid B \mid C$
$B \rightarrow b$
$C \rightarrow D$
$D \rightarrow d$

Methods of Transforming Grammars

Unit-Productions

Any production in the form of $A \ \rightarrow B$ where $A, B \ \in \ V$.

Any production with $A \ \rightarrow A$ can be removed without consequence; however where the left side variable and right side variable are different, we need to be careful.

We need to find all dependencies in the grammar and if

$A \overset{*}{\Rightarrow} B$ where $A, B \ \in \ V$

then we need to add non-unit productions of B to A

Example 3: Remove unit-productions from grammar

$S \to Aa \mid B$
$A \to a \mid bc \mid B$
$B \to \mathrm{A} \mid \mathrm{bb}$

$S \to B$
$S \to A$
$A \to B$
$B \to A$

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Revised Productions ($\hat{P}$)

$S \rightarrow B$
$S \rightarrow A$
$A \rightarrow B$
$B \rightarrow A$

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$S \rightarrow Aa$
$A \rightarrow a \mid bc$
$B \rightarrow \mathrm{bb}$

Example 3: Remove unit-productions from grammar

$S \rightarrow Aa \mid B$
$A \rightarrow a \mid bc \mid B$
$B \rightarrow \mathrm{A} \mid \mathrm{bb}$

$S \rightarrow B$
$S \rightarrow A$
$A \rightarrow B$
$B \rightarrow A$

<u>Revised Productions ($\hat{P}$)</u>

$S \rightarrow Aa \mid bb$
$A \rightarrow a \mid bc$
$B \rightarrow \mathrm{bb}$

Example 3: Remove unit-productions from grammar

$S \rightarrow Aa \mid B$
$A \rightarrow a \mid bc \mid B$
$B \rightarrow \mathrm{A} \mid \mathrm{bb}$

$S \rightarrow B$
$S \rightarrow A$
$A \rightarrow B$
$B \rightarrow A$

<u>Revised Productions ($\hat{P}$)</u>
$S \rightarrow Aa \mid bb$
$A \rightarrow a \mid bc$
$B \rightarrow \mathrm{bb}$

Example 3: Remove unit-productions from grammar

$S \to Aa \mid B$
$A \to a \mid bc \mid B$
$B \to \mathrm{A} \mid \mathrm{bb}$

$S \to B$
$S \to A$
$A \to B$
$B \to A$

Revised Productions $(\hat{P})$

$S \to Aa \mid bb$
$A \to a \mid bc$
$B \to \mathrm{bb}$

Example 3: Remove unit-productions from grammar

$S \rightarrow Aa \mid B$
$A \rightarrow a \mid bc \mid B$
$B \rightarrow \mathrm{A} \mid \mathrm{bb}$

$S \rightarrow B$
$S \rightarrow A$
$A \rightarrow B$
$B \rightarrow A$

<u>Revised Productions ($\hat{P}$)</u>

$S \rightarrow Aa \mid bb \mid a \mid bc$
$A \rightarrow a \mid bc$
$B \rightarrow \mathrm{bb}$

Example 3: Remove unit-productions from grammar

$S \to Aa \mid B$
$A \to a \mid bc \mid B$
$B \to \mathrm{A} \mid \mathrm{bb}$

$S \to B$
$S \to A$
$A \to B$
$B \to A$

<u>Revised Productions ($\hat{P}$)</u>
$S \to Aa \mid bb \mid a \mid bc$
$A \to a \mid bc$
$B \to \mathrm{bb}$

Example 3: Remove unit-productions from grammar

$S \rightarrow Aa \mid B$
$A \rightarrow a \mid bc \mid B$
$B \rightarrow \mathrm{A} \mid \mathrm{bb}$

$S \rightarrow B$
$S \rightarrow A$
$A \rightarrow B$
$B \rightarrow A$

<u>Revised Productions ($\hat{P}$)</u>

$S \rightarrow Aa \mid bb \mid a \mid bc$
$A \rightarrow a \mid bc$
$B \rightarrow \mathrm{bb}$

Example 3: Remove unit-productions from grammar

$S \rightarrow Aa \mid B$
$A \rightarrow a \mid bc \mid B$
$B \rightarrow \mathrm{A} \mid \mathrm{bb}$

$S \rightarrow B$
$S \rightarrow A$
$A \rightarrow B$
$B \rightarrow A$

<u>Revised Productions ($\hat{P}$)</u>

$S \rightarrow Aa \mid bb \mid a \mid bc$
$A \rightarrow a \mid bc \mid bb$
$B \rightarrow \mathrm{bb}$

Example 3: Remove unit-productions from grammar

$S \rightarrow Aa \mid B$
$A \rightarrow a \mid bc \mid B$
$B \rightarrow \mathrm{A} \mid \mathrm{bb}$

$S \rightarrow B$
$S \rightarrow A$
$A \rightarrow B$
$B \rightarrow A$

<u>Revised Productions ($\hat{P}$)</u>
$S \rightarrow Aa \mid bb \mid a \mid bc$
$A \rightarrow a \mid bc \mid bb$
$B \rightarrow \mathrm{bb}$

Example 3: Remove unit-productions from grammar

$S \rightarrow Aa \mid B$
$A \rightarrow a \mid bc \mid B$
$B \rightarrow \text{A} \mid \text{bb}$

$S \rightarrow B$
$S \rightarrow A$
$A \rightarrow B$
$B \rightarrow A$

<u>Revised Productions ($\hat{P}$)</u>
$S \rightarrow Aa \mid bb \mid a \mid bc$
$A \rightarrow a \mid bc \mid bb$
$B \rightarrow \text{bb}$

Example 3: Remove unit-productions from grammar

$S \rightarrow Aa \mid B$
$A \rightarrow a \mid bc \mid B$
$B \rightarrow \mathrm{A} \mid \mathrm{bb}$

$S \rightarrow B$
$S \rightarrow A$
$A \rightarrow B$
$B \rightarrow A$

<u>Revised Productions $(\hat{P})$</u>

$S \rightarrow Aa \mid bb \mid a \mid bc$
$A \rightarrow a \mid bc \mid bb$
$B \rightarrow \mathrm{bb} \mid \mathrm{a} \mid \mathrm{bc}$

Example 3: Remove unit-productions from grammar

$S \rightarrow Aa \mid B$
$A \rightarrow a \mid bc \mid B$
$B \rightarrow \mathrm{A} \mid \mathrm{bb}$

$S \rightarrow B$
$S \rightarrow A$
$A \rightarrow B$
$B \rightarrow A$

<u>Revised Productions ($\hat{P}$)</u>
$S \rightarrow Aa \mid bb \mid a \mid bc$
$A \rightarrow a \mid bc \mid bb$
$B \rightarrow \mathrm{bb} \mid \mathrm{a} \mid \mathrm{bc}$

Example 3: Remove unit-productions from grammar

$S \to Aa \mid B$
$A \to a \mid bc \mid B$
$B \to \mathrm{A} \mid \mathrm{bb}$

$S \to B$
$S \to A$
$A \to B$
$B \to A$

<u>Revised Productions $(\hat{P})$</u>
$S \to Aa \mid bb \mid a \mid bc$
$A \to a \mid bc \mid bb$
$B \to \mathrm{bb} \mid \mathrm{a} \mid \mathrm{bc}$

Example 3: Remove unit-productions from grammar

$S \rightarrow Aa \mid B$
$A \rightarrow a \mid bc \mid B$
$B \rightarrow \mathrm{A} \mid \mathrm{bb}$

$S \rightarrow B$
$S \rightarrow A$
$A \rightarrow B$
$B \rightarrow A$

<u>Revised Productions ($\hat{P}$)</u>
$S \rightarrow Aa \mid bb \mid a \mid bc$
$A \rightarrow a \mid bc \mid bb$

Theorem 7.1: For any CFL L, there exists an npda M such that $L = L(M)$

- The npda will be $M = (\{q_0, q_1, q_f\}, T, V \cup \{z\}, \delta, q_0, z, \{q_f\})$
- The transition function will include $\delta(q_0, \lambda, z) = \{(q_1, Sz)\}$
- Given $A \to ax$, $(q_1, x) \in \delta(q_1, a, A)$

$$\delta(q_1, a, A) = \{\dots, (q_1, x), \dots\}$$

- $\delta(q_1, \lambda, z) = \{(q_f, z)\}$

$$(q_0, w, z\} \vdash (q_1, w, Sz) \vdash \cdots \vdash (q_2, \lambda, z) \vdash (q_f, \lambda, z)$$

Example 4: Consider the grammar with productions

$S \to aA$
$A \to aABC \mid bB \mid a$
$B \to b$
$C \to c$

$$\delta(q_0, \lambda, z) = \{(q_1, Sz)\}$$

Example 4: Consider the grammar with productions

$S \to aA$
$A \to aABC \mid bB \mid a$
$B \to b$
$C \to c$

$$\delta(q_0, \lambda, z) = \{(q_1, Sz)\}$$
$$\delta(q_1, a, S) = \{(q_1, A)\}$$

Example 4: Consider the grammar with productions

$S \rightarrow aA$

$A \rightarrow aABC \mid bB \mid a$

$B \rightarrow b$

$C \rightarrow c$

$$\delta(q_0, \lambda, z) = \{(q_1, Sz)\}$$
$$\delta(q_1, a, S) = \{(q_1, A)\}$$
$$\delta(q_1, a, A) = \{(q_1, ABC)\}$$

Example 4: Consider the grammar with productions

$S \rightarrow aA$

$A \rightarrow aABC \mid bB \mid a$

$B \rightarrow b$

$C \rightarrow c$

$$\delta(q_0, \lambda, z) = \{(q_1, Sz)\}$$
$$\delta(q_1, a, S) = \{(q_1, A)\}$$
$$\delta(q_1, a, A) = \{(q_1, ABC), (q_1, \lambda)\}$$

Example 4: Consider the grammar with productions

$S \rightarrow aA$

$A \rightarrow aABC \mid bB \mid a$

$B \rightarrow b$

$C \rightarrow c$

$$\delta(q_0, \lambda, z) = \{(q_1, Sz)\}$$
$$\delta(q_1, a, S) = \{(q_1, A)\}$$
$$\delta(q_1, a, A) = \{(q_1, ABC), (q_1, \lambda)\}$$
$$\delta(q_1, b, A) = \{(q_1, B)\}$$

Example 4: Consider the grammar with productions

$S \to aA$
$A \to aABC \mid bB \mid a$
$B \to b$
$C \to c$

$$\delta(q_0, \lambda, z) = \{(q_1, Sz)\}$$
$$\delta(q_1, a, S) = \{(q_1, A)\}$$
$$\delta(q_1, a, A) = \{(q_1, ABC), (q_1, \lambda)\}$$
$$\delta(q_1, b, A) = \{(q_1, B)\}$$
$$\delta(q_1, b, B) = \{(q_1, \lambda)\}$$

Example 4: Consider the grammar with productions

$S \to aA$
$A \to aABC \mid bB \mid a$
$B \to b$
$C \to c$

$$\delta(q_0, \lambda, z) = \{(q_1, Sz)\}$$
$$\delta(q_1, a, S) = \{(q_1, A)\}$$
$$\delta(q_1, a, A) = \{(q_1, ABC), (q_1, \lambda)\}$$
$$\delta(q_1, b, A) = \{(q_1, B)\}$$
$$\delta(q_1, b, B) = \{(q_1, \lambda)\}$$
$$\delta(q_1, c, C) = \{(q_1, \lambda)\}$$

Example 4: Consider the grammar with productions

$S \rightarrow aA$
$A \rightarrow aABC \mid bB \mid a$
$B \rightarrow b$
$C \rightarrow c$

$\delta(q_0, \lambda, z) = \{(q_1, Sz)\}$
$\delta(q_1, a, S) = \{(q_1, A)\}$
$\delta(q_1, a, A) = \{(q_1, ABC), (q_1, \lambda)\}$
$\delta(q_1, b, A) = \{(q_1, B)\}$
$\delta(q_1, b, B) = \{(q_1, \lambda)\}$
$\delta(q_1, c, C) = \{(q_1, \lambda)\}$
$\delta(q_1, \lambda, z) = \{(q_f, z)\}$

More general Construction

$A \to Bx$

$$\delta(q_1, \lambda, A) = \{(q_1, Bx)\}$$

$A \to abBx$

$$\delta(q_1, ab, A) = \{(q_1, Bx)\}$$