CSCI 450 Fall 2024
Homework 1

Problem 1: (2 pts) On the set of positive integers, we can define an equivalence relation
$$x \equiv y$$
if and only if $\quad$ x mod 3 = y mod 3.

Use the above equivalence to partition the set {2, 4, 5, 6, 9, 23, 24, 25, 31, 37} into equivalence sets (i.e. put the numbers with the same remainder in one set).

Problem 2: (2 pts) Draw a picture of the graph with vertices $\{v_1, v_2, v_3\}$
and edges $\{(v_1, v_1), (v_1, v_2), (v_2, v_3), (v_2, v_1), (v_3, v_1)\}$.
Enumerate all cycles that goes from v_1 to v_1 ($v_1 \rightarrow v_1$).

Problem 3: (2 pts) Use proof by induction to show that $\frac{1}{n^2} \leq \frac{1}{n-1} - \frac{1}{n}$ $(n \geq 2)$

Problem 4: (2 pts) Find a grammar that generates the language

$L = \{ww^R : w \in \{a, b\}^+\}$.

Problem 5: (2 pts) Are the following two grammars with respective productions equivalent? Assume that S is the start symbol in both cases.

$S \rightarrow aSb \mid ab \mid \lambda,$

and

$S \rightarrow aAb \mid ab,$
$A \rightarrow aAb \mid \lambda$