

141

Zach Hightower Fall 7

1. Find the order of each element in the group $(\mathbb{Z}_{15}^*, \otimes) = \{1, 2, 4, 7, 8, 11, 13, 14\}$

1 is order 1

$2^1=2, 2^2=4, 2^3=8, 2^4=1, 2^5=2, 2^6=4 \mid 2$ is order 4

$4^1=4, 4^2=1, 4^3=4 \mid 4$ is order 2

$7^1=7, 7^2=4, 7^3=13, 7^4=1, 7^5=7 \mid 7$ is order 4

$8^1=8, 8^2=4, 8^3=2, 8^4=1, 8^5=8 \mid 8$ is order 4

$11^1=11, 11^2=1, 11^3=11 \mid 11$ is order 2

$13^1=13, 13^2=4, 13^3=7, 13^4=1, 13^5=13 \mid 13$ is order 4

$14^1=14, 14^2=1, 14^3=14 \mid 14$ is order 2

2. Suppose that $(G, *)$ is a group with 10 elements

9. Let H be a subgroup of G . What are the possible number of elements that H can have? What result are you using?

Since H is a subgroup of $(G, *)$ it is $(H, *)$.

We know from Lagrange's Theorem that

$$|H| \mid |G|$$

So the possible element numbers are

1, 2, 5, 10, 25, 50