5.

Note that 101 is prime

(a) Without using a computer or calculator, evaluate $3^{102} \pmod{101}$

$3^{102} \pmod{101} = 3\,(3^{101} \pmod{101}) = 3(3) = 9$

Using Fermat's Little Theorem

(b) Without using a computer or calculator, evaluate $2^{g} \pmod{101}$ where $g = 10^{2023}$

$2^{100} \bmod (101) = 1$

Because $\phi\, 101 = (100)$ and 2^{100} is a multiple of 100

We know that $2^{10^{x}}$ power, where x is any positive integer greater than 1, will give 1 mod 101, because all the numbers will be multiples of 100.

So $2^{g} \pmod{101} = 1$