

15)

3. (a) What are quadratic residues and quadratic non-residues $(\bmod 13)$

$$\begin{aligned} 1^2 &= 1, \quad 2^2 = 4, \quad 3^2 = 9, \quad 4^2 = 3, \quad 5^2 = 12 \\ 6^2 &= 10, \quad 7^2 = 10, \quad 8^2 = 12, \quad 9^2 = 3, \quad 10^2 = 9 \\ 11^2 &= 9, \quad 12^2 = 1 \end{aligned}$$

Qu. Res: 1, 3, 4, 9, 10, 12

Non Res: 2, 5, 6, 7, 8, 11

(b) For each quadratic residues in \mathbb{Z}_{13}^* what are its square roots

+ check

$$\mathbb{Z}_{13}^* \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

$$1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144$$

$$x^2 = x \otimes x \equiv 13 = 1 \otimes 1 = 12 \otimes 12$$

$$y^2 = " \equiv 3 = 4 \otimes 4 = 9 \otimes 9$$

$$z^2 = " = 4 = 2 \otimes 2 = 10 \otimes 10$$

$$w^2 = " , \equiv 9 = 3 \otimes 3 = 11 \otimes 11$$

$$v^2 = " , \equiv 10 = 6 \otimes 6 = 7 \otimes 7$$

$$u^2 = " = 12 = 5 \otimes 5 = 8 \otimes 8$$

$$\sqrt{1} = 1, 12$$

$$\sqrt{12} = 5, 8$$

$$\sqrt{3} = 9, 9$$

$$\sqrt{4} = 2, 11$$

$$\sqrt{9} = 3, 10$$

$$\sqrt{10} = 6, 7$$