Zachary Hightower Homework #1

1. Consider the permutations

$$\pi = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 3 & 4 & 5 & 2 & 7 & 8 & 9 & 6 \end{bmatrix}$$

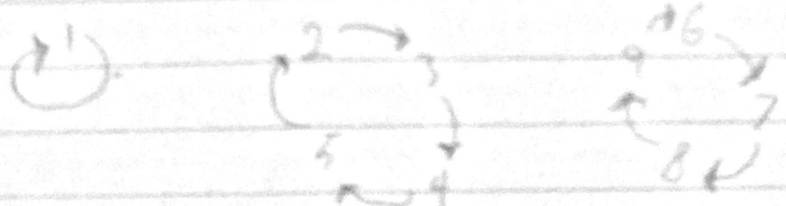
and

$$\sigma = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 4 & 5 & 6 & 7 & 8 & 9 & 1 & 2 \end{bmatrix}$$

a. The permutations π and σ belong to S_n, for what value of n?

There are 9 distinct integers in the set, so π and σ belong to S_9. This means that there are $9!$ permutations in the set.

b. Draw a picture depicting how π acts on the elements from 1 to 9.



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c. Write π and σ in disjoint cycle notation

$\pi = (1)(2,3,4,5)(6,7,8,9)$

$\sigma = (1,3,5,7,9,2,4,6,8)$

d. Write π^{-1} and σ^{-1} in disjoint cycle notation

$\pi^{-1} = (1)(5,4,3,2)(9,8,7,6)$

$\sigma^{-1} = (8,6,4,2,9,7,5,3,1)$

e. Write $\pi \circ \sigma$ and $\sigma \circ \pi$ in disjoint cycle notation

$\pi \circ \sigma = (1,4,7,6,9,3,2,5,8)$

$\sigma \circ \pi = (1,3,6,9,8,2,5,4,7)$

f. Is $\pi \circ \sigma = \sigma \circ \pi$

No, we proceed along a different basis with each problem and arrive at different conclusions

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2. Write the following permutations in S_5 as a composition of transpositions

a. $\pi = (1,2,3,4,5)$

$\pi = (1,5) \circ (1,4) \circ (1,3) \circ (1,2)$

b. $\sigma = (1,3)(2,4,5)$

$\sigma = (1,3) \circ (2,4) \circ (4,5)$

3. If π is an even permutation and σ is an even permutation, then explain why $\pi \circ \sigma$ and $\sigma \circ \pi$ are an even permutations.

An even permutation is one with an even number of inversions. Pairs (x,y) where $x<y$ and $f(x) > f(y)$

$$P(f) = \prod_{x<y} \frac{f(x) - f(y)}{x - y}$$

From our definition of even we can conclude $P(f) = 1$, when f is even and is $P(f) = -1$ in all other cases.

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3. conc. So if $P(f) = 1$
and $P(g) = 1$

$P(fg) = 1$, which is even.

4. In class we showed that any cycle of length 4 is odd since $(a,b,c,d) = (a,b)(b,c)(c,d)$.

a. Prove that any cycle π of length 3 is even.
Hint: Assume π has the form $\pi = (a,b,c)$

We have a cycle of length 3.
We can assume that any cycle of length 3 can be represented as
$\pi = (a,b,c)$
So, we can turn our disjoint cycle into a set of transpositions
$\pi = (a,b) \circ (b,c)$
This is the shortest configuration that is a valid transposition.
We know that even with longer valid transpositions the parity remains the same.
So, since our transposition here is even, we can conclude that all cycles of length 3 are even.

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4 cont.

b.) Can a cycle of length 4 be written as a composition of cycles of length 3? Explain your answer.
Hint: What does the problem (3) say about the compositions of cycles of length 3?

No, because the 3 cycles cannot evenly divide the elements in the 4 cycle.
With elements (a, b, c, d) in the 4 cycle we can break them down into a 3 cycle of composition (a, b, c), or another configuration, but the different configurations do not change our conclusion. the only thing we can do to properly complete the cycle once it decomposes from 4 to 3 is add the missing element as a 1 cycle (a, b, c) (d). So the cycle of length 4 cannot be written as a composition of cycles with length 3.

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5. Disprove the following statement:
"If τ and σ are transpositions, then $\tau \circ \sigma = \sigma \circ \tau$"
Hint: Give a counter-example. Come up with an example of two transpositions in S_n for some n where the above statement is not true.

Counter-example:
In S_3
$\tau = (1,2)$
$\sigma = (2,3)$

$\sigma \circ \tau = (1,3)(2,1)$
$\tau \circ \sigma = (1,2,3)$

The results are not equal here, So it cannot be true that "If τ and σ are transpositions, then $\tau \circ \sigma = \sigma \circ \tau$"