

pp1

Zach Lightower Hwk 7

- b. Find the order of each element in the group $(\mathbb{Z}_{15}^*, \otimes) = \{1, 2, 4, 7, 8, 10, 13, 14\}$

1. is order 1

$$2^1=2, 2^2=4, 2^3=8, 2^4=1, 2^5=2, 2^6=4 \quad | 2 \text{ is order } 4$$

$$4^1=4, 4^2=1, 4^3=4 \quad | 4 \text{ is order } 2$$

$$7^1=7, 7^2=4, 7^3=13, 7^4=1, 7^5=7 \quad | 7 \text{ is order } 4$$

$$8^1=8, 8^2=4, 8^3=2, 8^4=1, 8^5=8 \quad | 8 \text{ is order } 4$$

$$11^1=11, 11^2=1, 11^3=11 \quad | 11 \text{ is order } 2$$

$$13^1=13, 13^2=4, 13^3=7, 13^4=1, 13^5=13 \quad | 13 \text{ is order } 4$$

$$14^1=14, 14^2=1, 14^3=14 \quad | 14 \text{ is order } 2$$

2. Suppose that $(G, *)$ is a group with 60 elements

- a. Let H be a subgroup of G . What are the possible number of elements that H can have? What result are you using?

Since H is a subgroup of $(G, *)$ it is $(H, *)$.

We know from Lagrange's Theorem that

$$|H| \mid |G|$$

So the possible element numbers are

$$1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60$$