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5. Use the algorithm from the textbook to find the four square roots of 37 in \mathbb{Z}_{77}

$$77 = 7 \cdot 11$$

$$x^2 \equiv 37 \pmod{7}$$

$$x^2 \equiv 37 \pmod{11}$$

$$3 \equiv 3 \pmod{4} \quad \checkmark$$

$$11 \equiv 3 \pmod{9} \quad \checkmark$$

$$\pm 37^{(7+1)/4}$$

$$\pm 37^2 \pmod{7}$$

$$\pm 37^{(11+1)/4}$$

$$\pm 37^3 \pmod{11}$$

$$(1369, -1369) \pmod{7} \quad (50653, -50653) \pmod{11}$$

$$(4, 3)$$

$$9, 2$$

$$^a x \equiv 4 \pmod{7}$$

$$x \equiv 9 \pmod{11}$$

$$^c x \equiv 4 \pmod{7}$$

$$x \equiv 2 \pmod{11}$$

$$^b x \equiv 3 \pmod{7}$$

$$x \equiv 9 \pmod{11}$$

$$^d x \equiv 3 \pmod{7}$$

$$x \equiv 2 \pmod{11}$$

$$(a) 4 + 7k \equiv 9 \pmod{11}$$

$$7^{-1} \cdot 7k \equiv 5 \pmod{11}$$

$$k \equiv 55 \pmod{11}$$

$$k = 55 + 11j$$

$$x = 4 + 7(55 + 11j)$$

$$x = 389 + 77j$$

$$389$$

$$(b) 3 + 7k \equiv 9 \pmod{11}$$

$$7k \equiv 6 \pmod{11}$$

$$k \equiv 66 \pmod{11}$$

$$k = 66 + 11j$$

$$x = 3 + 7(66 + 11j)$$

$$x = 465 + 77j$$

$$465$$