Homework 1 Zachary Hightower

2. Let $d, a, b \in \mathbb{Z}$ If $d | a$ and $d | b$, prove that $d | (ax + by)$ for every $x, y \in \mathbb{Z}$

If d divides both a and b, it must be a common divisor of a and b. There fore, anything that is a multiple of a and b is divisible by d. Anything written in the form $(ax + by)$, will produce multiples of a and b, no matter the chosen x and y. Thus, the statement is true for every $x, y \in \mathbb{Z}$

3. For each pair of integers a and b given below, find the integers q and r, such that $a = bq + r$ and $0 \leq r < b$ and also compute a div b and a mod b.

a. $a = 100$, $b = 3$ 100 div 3 = 33

$100 = 3(33) + r$ 100 mod 3 = 1

$100 = 3(33) + 1$

$Q = 33$ $r = 1$

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b. $a = -100$, $b = 3$ ~~-100~~ -100 div 3 = -34

-100 mod 3 = 2

~~-100 = 3(-34)~~

$-100 = 3(-34) + 2$

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c. $a = 99$, $b = 3$ 99 div 3 = 33

$99 = 3(33) + 0$ 99 mod 3 = 0

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d. $a = 0$, $b = 3$ 0 div 3 = 0

0 mod 3 = 0

$0 = 3(0) + 0$

4. Find all positive integers N such that the following equation is true:

$20 \bmod N = 4$

$20 = N(b) + 4$

$20 = 8(2) + 4$

List of satisfactory integers $N = 8$

5. Prove that the sum of 3 consecutive integers is divisible by 3

Let us call our three consecutive integers $n-1$, n, and $n+1$

So $(n-1) + n + (n+1) = n + n + n = 3n$

$3n$ must be a multiple of 3, and this process can be repeated with any integers.

H_1

6. a.) Use the Euclidian algorithm to find $gcd(1015, 231)$

$1015 \bmod 231 = 91$
$231 \bmod 91 = 49$
$91 \bmod 49 = 42$
$49 \bmod 42 = 7$
$42 \bmod 7 = 0$

$gcd = 7$

b.) Find two integers x, y such that $1015x + 231y = gcd(1015, 231)$

$7 = 1(49 - 1)(42)$
$7 = 1(49 - 1)(91 - 1(49))$
$7 = -1(91 + 2(49))$
$7 = -1(91 + 2)(231 - 2(91))$
$7 = 2(231 - 5(91))$
$7 = 2(231 - 5)(1015 - 4(231))$
$7 = -5(1015) + 22(231)$

So $x = -5$ and $y = 22$

7. Let $a, n \in \mathbb{Z}$ and suppose that a and n are relatively prime. Prove that there is an integer b such that $ab \equiv 1 \pmod{n}$

If a and n are relatively prime,
their gcd is $= 1$
If $\gcd(a, n) = 1$ then there are integers
x and y such that $ax + ny = 1$
Rewritten this is $ax \equiv 1 \pmod{n}$
And we can rewrite x in the equation
as b to get
$ab \equiv 1 \pmod{n}$