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Homework 6 Zachary Hightower

1. In class we showed that $(\mathbb{Z}_4, \oplus)$ and $(\mathbb{Z}_{10}^*, \otimes)$ are isomorphic. The isomorphism we found was $f(0)=1$, $f(1)=3$, $f(2)=9$, and $f(3)=7$. There is another isomorphism (a different) (function from $(\mathbb{Z}_4, \oplus)$ to $(\mathbb{Z}_{10}^*, \otimes)$. Find it.

$\mathbb{Z}_4, \oplus$

$\oplus$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

$\mathbb{Z}_{10}^*$

$\otimes$	1	3	7	9
1	1	3	7	9
3	3	9	1	7
7	7	1	9	3
9	9	7	3	1

Pair identity elements and any elements that are their own inverses.
This gives us pairs for $f(0)=1$ and $f(2)=9$

$0 \leftrightarrow 1$
$1 \leftrightarrow 7$
$2 \leftrightarrow 9$
$3 \leftrightarrow 3$

The final two pairs are left to our discretion, so we choose $f(1)=7$ and $f(3)=3$

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2\. (a) Prove that ($\mathbb{Z}_7^*$, $\otimes$ is cyclic by finding a generator.

$\mathbb{Z}_7^*$ {1,2,3,4,5,6}

1 composed only gives 1

$2^1 = 2$, $2^2 = 4$, $2^3 = 1$, $2^4 = 2$ Not generator

$3^1 = 3$, $3^2 = 2$, $3^3 = 6$, $3^4 = 4$, $3^5 = 5$, $3^6 = 1$

3 is a generator in ($\mathbb{Z}_7^*$, $\otimes$)

(b) Show that ($\mathbb{Z}_6$, $\oplus$) and ($\mathbb{Z}_7^*$, $\otimes$ are isomorphic

$\oplus$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

$\otimes$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

Pair identity elements
$f(0) = 1$

$0 \leftrightarrow 1$
$1 \leftrightarrow 2$
$2 \leftrightarrow 3$
$3 \leftrightarrow 6$
$4 \leftrightarrow 5$
$5 \leftrightarrow 4$

Pair remaining elements

Pair elements that are their own inverses
$f(3) = 6$

2(i) cont.

Both sets have the same number of elements.

The function is one to one, we can see that each input gives a different output.

The function is onto, there is at least one element in set $(\mathbb{Z}_6, \oplus)$ matching with set $(\mathbb{Z}_7^*, \otimes)$

$\forall g, h \in G, f(g \oplus h) = f(g) \otimes f(h)$

$$f(1 \oplus 2) = f(1) \otimes f(2)$$
$$6 = 2 \otimes 3$$
$$6 = 6$$

So, we can conclude that the two groups are isomorphic.

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3. The group S_4, the permutations of the numbers $\{1,2,3,4\}$, has $4! = 24$ elements.
Is it isomorphic to $(\mathbb{Z}_{24}, \oplus)$?
Explain your answer.

Considering the permutations as individual elements, we can say that they both have the same number of elements, since $\mathbb{Z}_{24}$ covers $0-23$.

Given prior work, we know that with an equal number of unique elements in each group, we can find a function that is one to one and onto.

We also know that $(\mathbb{Z}_{24}, \oplus)$ is cyclic.
This means that for the two groups to be isomorphic, they must both be cyclic.
However, S_4 is non abelian
We show this with $A(1,2,3,4)=(2,3,4,1)$ and $B(1,2,3,4)=(4,3,2,1)$

$AB(1,2,3,4) = A(4,3,2,1) = (1,4,3,2)$

$BA(1,2,3,4) = B(2,3,4,1) = (3,2,1,4)$

So, S_4 is non-abelian, and we can also say it is non-cyclic.
So, S_4 is NOT isomorphic to $(\mathbb{Z}_{24}, \oplus)$

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4. Find all the subgroups of $(\mathbb{Z}_6, \oplus)$

* Check These

$\oplus$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

$\{0\}$
$\{0, 3\}$
$\{0, 2, 4\}$
$\{0, 1, 2, 3, 4, 5\}$

5. Find all the subgroups of $(\mathbb{Z}_9, \oplus)$

$\oplus$	0	1	2	3	4	5	6	7	8
0	0	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8	0
2	2	3	4	5	6	7	8	0	1
3	3	4	5	6	7	8	0	1	2
4	4	5	6	7	8	0	1	2	3
5	5	6	7	8	0	1	2	3	4
6	6	7	8	0	1	2	3	4	5
7	7	8	0	1	2	3	4	5	6
8	8	0	1	2	3	4	5	6	7

$\{0\}$
$\{0, 2, 7\}$
$\{0, 1, 2, 3, 4, 5, 6, 7, 8\}$

g 6

6. Find all the Sub groups of $(\mathbb{Z}_8^*, \otimes)$

$\mathbb{Z}_8^* \{1,3,5,7\}$

$\otimes$	1	3	5	7
1	1	3	5	7
3	3	1	7	5
5	5	7	1	3
7	7	5	3	1

$\{1\}$
$\{1,3\}$
$\{1,3,5,7\}$