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9. For $a \in \mathbb{Z}_{13}$, what are the possible values of a^{12} and a^{13}?
Hint: you do not have to do 26 calculations!

We know Fermat's Little Theorem which gives $a^p \equiv a \pmod{p}$

So every $a^{13} \equiv a \pmod{13}$

$1^{13} = 1$ $2^{13} = 2$ $3^{13} = 3$ $4^{13} = 4$ $5^{13} = 5$ $6^{13} = 6$
$7^{13} = 7$ $8^{13} = 8$ $9^{13} = 9$ $10^{13} = 10$ $11^{13} = 11$ $12^{13} = 12$
$13^{13} = 13$

In mod 13

For a^{12}, we can do the calculations and give the residues in mod 12

$1^{12} = 1 = 1$ $2^{12} = 4096 = 4$ $3^{12} = 531441 = 9$
$4^{12} = 16777216 = 4$ $5^{12} = 244140625 = 1$ $6^{12} = 2176782336 = 0$
$7^{12} = 13841287201 = 1$ $8^{12} = 68719476736 = 4$ $9^{12} = 282429536481 = 9$
$10^{12} = 1000000000000 = 4$ $11^{12} = 3138428376721 = 1$ $12^{12} = 8916100448256 = 0$
$13^{12} = 2.329808512 \times 10^{13} = 1$