#2

2. Without using a computer or a calculator, find the last two digits of 7^{442}

$7^{442} \pmod{100}$

$\varphi(100) = \varphi(2^2 \cdot 5^2) = \varphi(2^2)\,\varphi(5^2) = 2 \cdot 20 = 40$

7 and 100 are Relatively prime

$7^{40} = 1 \pmod{100}$

$7^{442} = 7^{40 \cdot 11 + 2} = (7^{40})^{11} \cdot 7^2 = 1^{11} \cdot 7^2 \pmod{100}$

$7^2 = 49 \pmod{100} = 49$