What is a Number?

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1 Introduction

The author is unaware of an universally accepted definition of "number" that unifies all of the standard notions. The tempting definition of *a set of mathematical objects with which one can do arithmetic with* seems too large (likely including all fields) and simultaneously too restricting (likely excluding the natural numbers), depending on what one means by "arithmetic." Instead, we will define the standard sets: the natural numbers ($\mathbb{N}$), the integers ($\mathbb{Z}$), the rational numbers ($\mathbb{Q}$), the real numbers ($\mathbb{R}$), and the complex numbers ($\mathbb{C}$).

2 The Natural Numbers

In order to define the natural numbers, we must have a notion of what a *set* is.

Definition 1 (set)**.** A set is a (possibly empty) collection of mathematical or abstract objects.

This definition is completely unsatisfactory in general, but it will serve our purposes for now. [1] We will define the natural numbers by the Peano axioms.

Axiom 1. Let $\mathbb{N}$ denote the set of all natural numbers. We take (axiomatically) that:

i) $1 \in \mathbb{N}$

ii) If $a \in \mathbb{N}$, then a has a *successor* $S(a)$ and $S(a) \in \mathbb{N}$

iii)

List of Symbols

$\mathbb{N}$	Natural Numbers	1

References

[1] P. R Halmos. *Naive Set Theory*. eng. Softcover reprint of the original 1st ed. 1974. Undergraduate Texts in Mathematics. New York, NY: Springer, 2013. ISBN: 0387901043.