MATH 164 Exam 1 Review (A)

Name:_____ **SID:**_____

Instructions

Work all problems out on the provided scratch paper, using one side only. Be sure to number each problem and page (if they are not already numbered) and staple them (in order) to this cover sheet at the end of the exam. Be sure that all your work is visible after stapling. In order to receive full credit, you must:

- Fully justify each answer.
- Organize your work so that your line of reasoning is clear.
- Write neatly and legibly.
- Circle your final answer where appropriate.

Unless otherwise specified, calculators are not allowed.

Problems

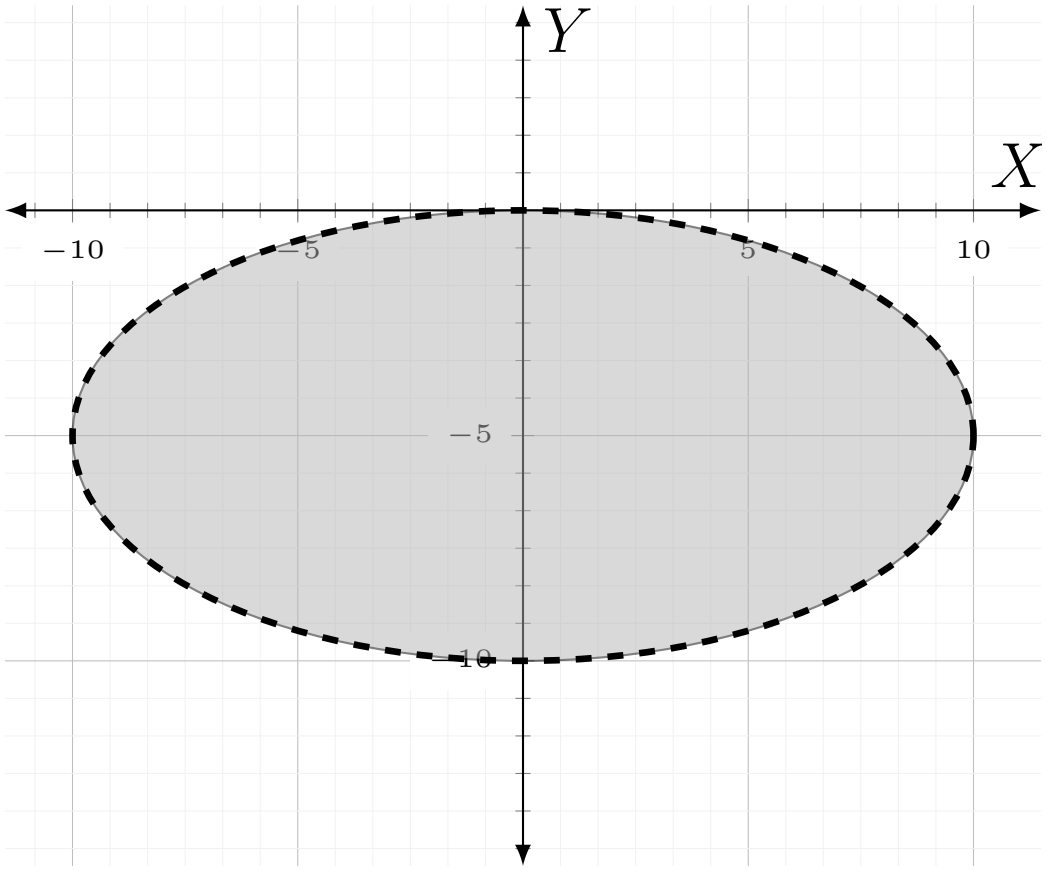
1. Let $X = \{-1, 0, 1\}$ and $Y = \{1, 2, 3\}$. Mark each statement T for *true* or F for *false*.
   (a) $X \cap Y = 1$
   (b) $X \cap Y = \{1\}$
   (c) $(0, 3) \in X \times Y$
   (d) $X \times Y$ has nine elements
   (e) $0 \in X$
   (f) $\{0\} \in X$
   (g) $\{0\} \subset X$
   (h) $\{\} \subset X$
   (i) $\{(-1, 1), (1, 1)\} \subset X \times Y$
   (j) $(3, 0) \in Y \times X$
   (k) $(0, 0) \in X \times X$
   (l) $\{\} \in X$
2. Let $R = \{(x, y) \in \mathbb{R}^2 \mid (x - 2)^2 + (y + 1)^2 = 1\}$.
   (a) Plot the relation R.
   (b) What is the domain of R?
   (c) What is the range of R?
   (d) Is R the graph of a function $y = f(x)$?
   (e) If R is the graph of a function $y = f(x)$, write an expression for $f(x)$.
3. Let $R = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1 \text{ and } y > 0\}$.
   (a) Plot the relation R.
   (b) What is the domain of R?
   (c) What is the range of R?
   (d) Is R the graph of a function $y = f(x)$?
   (e) If R is the graph of a function $y = f(x)$, write an expression for $f(x)$.
4. Let $f\colon X \to Y$ be a function where X and Y are non-empty sets. Answer each question or state that not enough information is provided.
   (a) What is the name of the function?
   (b) What is the domain of the function?
   (c) What is the co-domain of the function?
   (d) What is the range of the function?

5. Let $f(x) = 1 + x^{-1}$ and $g(x) = x^{-1/2}$. Find each of the following sets or evaluate/simplify the expression if possible.

(a) $\mathcal{D}(f)$	(e) $\mathcal{D}(g \circ f)$	(i) $f(9)$	(m) $g(25)$
(b) $\mathcal{R}(f)$	(f) $\mathcal{D}(f \circ g)$	(j) $f(1/9)$	(n) $(g \circ g)(16)$
(c) $\mathcal{D}(g)$	(g) $\mathcal{D}(f \circ f)$	(k) $(f \circ f)(1)$	(o) $g(f(x))$
(d) $\mathcal{R}(g)$	(h) $\mathcal{D}(g \circ g)$	(l) $(f(f(x)))$	(p) $f(g(-1))$

6. Graph the following relations. Label each graph.
   (a) $A = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 25 \text{ and } y > x\}$
   (b) $B = \{(x, y) \in \mathbb{R}^2 \mid -2 < x \leq 2\}$
   (c) $C = \{(x, y) \in \mathbb{R}^2 \mid -2 < x + y \leq 2\}$
   (d) $D = \{(x, y) \in \mathbb{R}^2 \mid xy \geq 25\}$
   (e) $E = [-10, -5] \times (5, 10] \subset \mathbb{R}^2$
   (f) $F = \{(x, y) \in \mathbb{R}^2 \mid y \geq x^2 \text{ or } y < -x^2\}$

7. Provide a **precise** description of the relation drawn below using set builder notation. Here, $X = Y = \mathbb{R}$.



8. For each definition of the function $f\colon X \to \mathbb{R}$ where $X \subset \mathbb{R}$, find the largest possible set X that could be the domain of f.
   (a) $f(x) = 3x + 5$
   (b) $f(x) = x/3 + 5$
   (c) $f(x) = 3/x + 5$
   (d) $f(x) = 3\sqrt{x} + 5$
   (e) $f(x) = 3\sqrt{x+5}$
   (f) $f(x) = 2(x+5)^{-1} + (x-10)^{-2} + (x+3)^{10}$
   (g) $f(x) = x\sqrt{x}$
   (h) $f(x) = \dfrac{x^2(x-1)\sqrt{x}}{x\sqrt{x}}$
   (i) $f(x) = 1/\sqrt{3x^2 - 27}$

9. Let $f(x) = 1/x$ and $g(x) = x + 1/x$. For each $x \in \{-1, 0, 1, 2\}$, evaluate the specified function at x or state that x is not in the domain of the function.
   (a) f
   (b) g
   (c) $f \circ g$
   (d) $g \circ f$
   (e) $f \circ f$

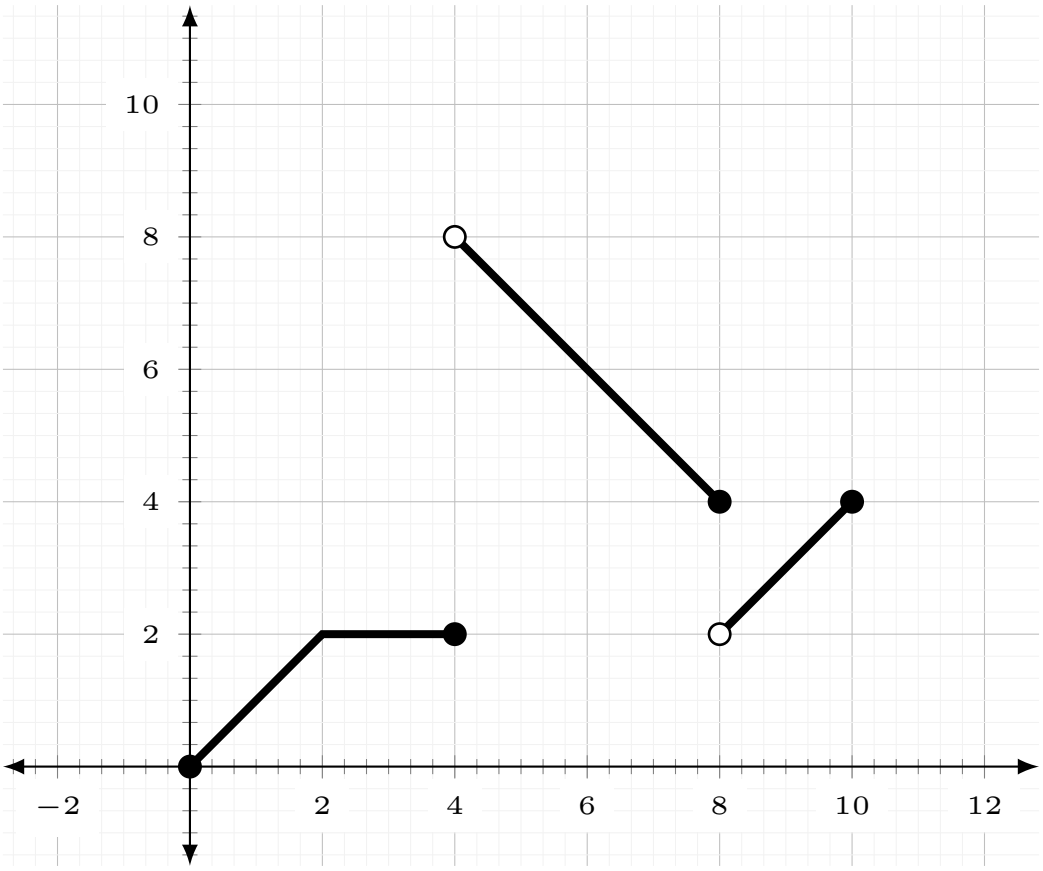
10. For each function h, find functions f and g such that $h = g \circ f$.
   (a) $h(x) = \sqrt{x^2 + 1}$
   (b) $h(x) = (x - 1/x)^2 - 1$
   (c) $h(x) = (x+5)^3 + (x+5)^2 + x + 5$
   (d) $h(x) = (x+5)^3 + (x+5)^2 + x + 6$

11. Provide precise definitions of the followng statements.
   (a) *$B \subset \mathbb{R}$ is an open ball centered at x_0 with radius $\varepsilon > 0$.*
   (b) *$X \subset \mathbb{R}$ is open.*
   (c) *$X \subset \mathbb{R}$ is closed.*

12. Determine whether each subset of $\mathbb{R}$ is open, closed, neither, or both.
   (a) $[0, 1]$
   (b) $[0, \infty)$
   (c) $(-\infty, \infty)$
   (d) $(0, \infty)$
   (e) $\{x \in \mathbb{R} \mid x = 5\}$
   (f) $\{x \in \mathbb{R} \mid x > 5\}$
   (g) $\{\}$
   (h) $\{x \in \mathbb{R} \mid x < 5 \text{ or } x \geq 5\}$

13. Let $f\colon X \to \mathbb{R}$ where $X \subset \mathbb{R}$. Provide precise definitions of the following statements.
   (a) *f is increasing on I where $I \subset X$ is an interval.*
   (b) *f has a local maximum at x_0.*
   (c) *f has a global maximum at x_0.*

14. The function $f\colon \mathcal{D}(f) \to \mathbb{R}$ is graphed below. Hint: use $\{\}$ or $\emptyset$ to denote the empty set.



(a) Find $\mathcal{D}(f)$.
(b) Find $\mathcal{R}(f)$.
(c) Find $A = \{x \in \mathbb{R} \mid (x, f(x)) \text{ is a local minimum of } f\}$
(d) Find $B = \{x \in \mathbb{R} \mid (x, f(x)) \text{ is a local maximum of } f\}$
(e) Find $C = \{x \in \mathbb{R} \mid (x, f(x)) \text{ is a global minimum of } f\}$
(f) Find $D = \{x \in \mathbb{R} \mid (x, f(x)) \text{ is a global maximum of } f\}$
(g) Evaluate $f(10)$.
(h) Evaluate $f(f(10))$.
(i) Evaluate $f(f(f(10)))$.
(j) Evaluate $f(f(f(f(f(6)))))$.
(k) A point $x \in \mathcal{D}(f)$ is called a *fixed point* if $f(x) = x$. Find $E = \{x \in \mathbb{R} \mid f(x) = x\}$.