# MATH 164 Exam 1 Review (B)

**Name:**________________________ **SID:**________________________

## Instructions

Work all problems out on the provided scratch paper, using one side only. Be sure to number each problem and page (if they are not already numbered) and staple them (in order) to this cover sheet at the end of the exam. Be sure that all your work is visible after stapling. In order to receive full credit, you must:

- Fully justify each answer.
- Organize your work so that your line of reasoning is clear.
- Write neatly and legibly.
- Circle your final answer where appropriate.

**Unless otherwise specified, calculators are not allowed.**

## Problems

1. Let $f\colon X \to \mathbb{R}$ where $X \subset \mathbb{R}$ is symmetric (i.e., $x \in X$ if and only if $-x \in X$).
   (a) Let $h\colon X \to \mathbb{R}$ be defined by $h(x) = \frac{1}{2}\left(f(x) + f(-x)\right)$. Show that $h$ is an even function. $h$ is known as the *even part* of $f$.
   (b) Let $g\colon X \to \mathbb{R}$ be defined by $g(x) = \frac{1}{2}\left(f(x) - f(-x)\right)$. Show that $g$ is an odd function. $g$ is known as the *odd part* of $f$.
   (c) Show that $f = h + g$ for $h$ and $g$ defined above.
   (d) Suppose that $f\colon \mathbb{R} \to \mathbb{R}$ is both even and odd. Find an expression for $f(x)$ and show that it is unique (i.e., there is only one such function). [Hint: can $f(x) = 1$ for any value of $x$?]
   (e) Compute the even and odd part of each function $f$ given below. State if $f$ is even, odd, or neither.
      (i) $f(x) = x$
      (ii) $f(x) = mx$, where $m \in \mathbb{R}$
      (iii) $f(x) = x^2$
      (iv) $f(x) = x + 1$
      (v) $f(x) = 1 + x + x^2$
      (v) $f(x) = 2^x$ [Hint: you may restrict $x$ to $\mathbb{Z}$ if you wish.]
      (vi) $f(x) = x^{1/3}$
      (vii) $f(x) = a|x|$, where $a \in \mathbb{R}$
      (viii) $f(x) = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$

2. Write each quadratic function $f\colon \mathbb{R} \to \mathbb{R}$ in vertex form. Find all of the roots/zeros of $f$. Then plot $y = f(x)$.
   (a) $f(x) = x^2 + 2x + 1$
   (b) $f(x) = x^2 + 2x$
   (c) $f(x) = x^2 + 2x + 2$
   (d) $f(x) = 2x^2 - 4x + 8$
   (e) $f(x) = 4x^2 + 16x + 4$
   (f) $f(x) = -5x^2 + 20x - 14$

3. Given points $A = (x_0, y_0)$ and $B = (x_1, y_1)$ in $\mathbb{R}^2$, write the equation of the line passing through $A$ and $B$ in the forms $y = mx + b$ and $y - y_0 = m(x - x_0)$ where $m, b, x_0, y_0 \in \mathbb{R}$, or state that such representations are impossible.
   (a) $A = (0, 0)$, $B = (1, 1)$
   (b) $A = (0, 0)$, $B = (1, 2)$
   (c) $A = (1, 1)$, $B = (1, 2)$
   (d) $A = (1, 1)$, $B = (2, 1)$
   (e) $A = (1, -3)$, $B = (-2, 3)$

4. Given points $A = (x_0, y_0)$ and $B = (x_1, y_1)$ in $\mathbb{R}^2$, write the equation of the line passing through $A$ and $B$ in the form $ax + by = c$ where $a, b, c \in \mathbb{R}$. In order to make the represenation unique, require $a^2 + b^2 = 1$ and $c \geq 0$.
   (a) $A = (0, 0)$, $B = (1, 1)$
   (b) $A = (0, 0)$, $B = (1, 2)$
   (c) $A = (1, 1)$, $B = (1, 2)$
   (d) $A = (1, 1)$, $B = (2, 1)$
   (e) $A = (1, -3)$, $B = (-2, 3)$

5. For each polynomial function $f : \mathbb{R} \to \mathbb{R}$, answer the followng questions:
   (i) What is the degree of $f$?
   (ii) What is the *leading coefficient* (i.e., the coefficeint of the largest power of $x$ when the polynomial is written as $a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$)?
   (iii) What is the end-behavior of $f$ as $x \to -\infty$? Answer with either $\infty$ or $-\infty$.
   (iv) What is the end-behavior of $f$ as $x \to \infty$? Answer with either $\infty$ or $-\infty$.
   (v) List the zeros of $f$ and their multiplicity.
   (vi) Find the $y$-intercept of the graph of $y = f(x)$.
   (vii) Plot $y = f(x)$, ensuring that the behavior near the zeros and end-behavior are correct. Other than 0, precise values of $y$ are not necessary.
   (a) $f(x) = x$
   (b) $f(x) = -x^2$
   (c) $f(x) = 1 - x^2$
   (d) $f(x) = x(x + 1)^2$
   (e) $f(x) = 2x(x^2 + 1)$
   (f) $f(x) = x(x^2 + 1)(x + 1)^2(3x - 15)^3$

6. Plot the function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} -x, & x < 0 \\ a, & x = 0 \\ x^2, & x > 0 \end{cases}$$

   for the case $a = 1$. To what value could $a$ be changed to to make $f$ continuous?

7. For each function $f$, plot both $y = f(x)$ and $y = \frac{1}{f(x)}$.
   (a) $f(x) = x$
   (b) $f(x) = x^2$

(c) $f(x) = x - 4$

(d) $f(x) = x^2 - 4$

8. Let $f(x) = 1$ and $g(x) = \frac{x}{x}$. Are these the same function? Explain your reasoning.

9. For each pair of integers $a, b \in \mathbb{N}$, find $q, r \in \mathbb{N}$ such that $a = bq + r$ and $0 \leq r < b$. [Hint: use long division.]

   (a) $a = 7$, $b = 5$

   (b) $a = 100$, $b = 33$

   (c) $a = 200$, $b = 60$

   (d) $a = 12321$, $b = 3$

   (e) $a = 12322$, $b = 3$

10. For each pair of polynomials $A, B \in \mathbb{P}$, find $Q, R \in \mathbb{P}$ such that $Q = BQ + R$ and the degree of $R$ is strictly less than the degree of $B$. [Hint: use long division.]

   (a) $A(x) = x^2 + 2x$, $B(x) = x$

   (b) $A(x) = x^2 + 2x + 2$, $B(x) = x + 1$

   (c) $A(x) = x^4 - 1$, $B(x) = x^2 + 1$

   (d) $A(x) = x^3 + 3x + 1$, $B(x) = x$

11. Find the zeros, vertical asymptotes, removable discontinuities, and horizontal/oblique asymptotes of the following rational functions if possible. Plot the graph of each function.

   (a) $f(x) = \dfrac{x^2}{x}$

   (b) $f(x) = \dfrac{x}{x^2}$

   (c) $f(x) = \dfrac{x^3}{x - 1}$

   (d) $f(x) = \dfrac{x^2}{x - 1}$

   (e) $f(x) = \dfrac{x}{x - 1}$

   (f) $f(x) = \dfrac{x}{x^2 - 9}$

   (g) $f(x) = \dfrac{x^2 - 1}{x^2 + x - 5}$

   (h) $f(x) = \dfrac{x^2 - 4}{x^2 + x - 5}$

12. Find the largest set $X \subset \mathbb{R}$ on which the following inequalities are true:

   (a) $x^2 \leq 0$

   (b) $x^2 > 0$

   (c) $x^3 > x^2$

   (d) $x^4 \leq x^2$

   (e) $|x| - x^2 \geq 0$

   (f) $\dfrac{1}{|x|} < 1$