Exam 3 Review

1) See unit circle

2) a) $360° = 2\pi$ rad

b) $180° = \pi$ rad

c) $90° = \pi/2$ rad

d) $45° = \pi/4$ rad

e) $22°30' = 22.5° = \pi/8$ rad

f) $11°15' = 11.25° = \pi/16$ rad

g) $5°37'30" = 5° + \frac{37.5'}{60} = \frac{1}{2}\left(10° + \frac{37.5'}{30}\right)$

$= \frac{1}{2}(11.25°)$

$= \frac{\pi}{32}$ rad

$$30 \overline{)37.5} \quad 1.25$$
$$-30$$
$$75$$
$$-60$$
$$15$$

h) $1° \cdot \frac{\pi}{180°} = \boxed{\frac{\pi}{180} \text{ rad}}$

i) $1' = \frac{1°}{60'} \cdot \frac{\pi}{180°} = \frac{\pi}{60 \cdot (6 \cdot 30)} = \boxed{\frac{\pi}{1080} \text{ rad}}$

$$\begin{array}{r} 1 \\ 36 \\ \times 3 \\ \hline 108 \end{array}$$

j) $1'' = \frac{1}{60} \cdot \frac{\pi}{1080}$ rad $= \boxed{\frac{\pi}{64800} \text{ rad}}$

$$\begin{array}{r} 4 \\ 108 \\ \times 6 \\ \hline 648 \end{array}$$

3) a) $30.2° = 30° + 0.2° = 30° + \frac{0.2 \cdot 60'}{60} = 30° + \frac{12'}{60} = \boxed{30°12'}$

b) $15.3° = 15° + \frac{3 \cdot 6'}{60} = 15° + 18' = \boxed{15°18'}$

c) $12.05° = 12° + \frac{0.05 \cdot 60'}{60} = 12° + \frac{3'}{60} = \boxed{12°3'}$

d) $12.01° = 12° + \frac{(0.01)(60)'}{60} = 12° + \frac{0.6'}{60} = 12° + 0.6'$

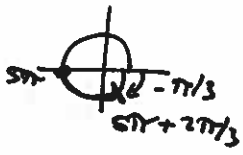
$= 12° + \frac{(0.6)(60)'}{60} = 12° + \frac{36'}{60} = \boxed{12°0'36"}$

4) a) $\sin \pi/3 = \boxed{\sqrt{3}/2}$

b) $\cos \pi/3 = \boxed{1/2}$

c) $\tan \pi/3 = \frac{\sin \pi/3}{\cos \pi/3} = \frac{\sqrt{3}/2}{1/2} = \boxed{\sqrt{3}}$

d) $17/3 = 5 + 2/3 \rightarrow \sin\left(\frac{17\pi}{3}\right) = \sin\left(5\pi + \frac{2\pi}{3}\right)$



$= \sin(-\pi/3)$

$= \boxed{-\sqrt{3}/2}$

e) $\cos\left(\frac{-15\pi}{3}\right) = \cos\left(\frac{15\pi}{3}\right) = \cos(5\pi) = \boxed{-1}$

cos is even

f) $\tan(123456789\pi) = \boxed{0}$

odd multiple of π

g) $\cos\left(\frac{100\pi}{6}\right) = \cos\left(\frac{50\pi}{3}\right)$

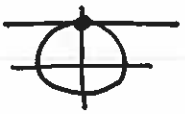
2π-periodic $= \cos\left(16\pi + \frac{2}{3}\pi\right)$

$= \cos\left(\frac{2}{3}\pi\right)$

$= \boxed{-1/2}$

$$3\overline{)50}\ \begin{array}{r} 16 \\ -3 \\ \hline 20 \\ -18 \\ \hline 2 \end{array}$$

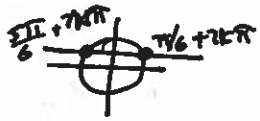
5) a) $\sin\theta = 1$



$\theta = \frac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z}$

b) $\sin\theta = 0$

$\theta = k\pi,\ k \in \mathbb{Z}$

c) $\sin\theta = 1/2$ 

$\theta = \frac{\pi}{6} + 2k\pi$ or $\theta = \frac{5\pi}{6} + 2k\pi$, $k \in \mathbb{Z}$

$\theta \in \{\theta \in \mathbb{R} / \theta = \pi/6 + 2k\pi,\ \theta = \frac{5\pi}{6} + 2k\pi\}$, $k \in \mathbb{Z}$

d) $\sin(3\theta + \pi) = 1 \rightarrow 3\theta + \pi = \frac{\pi}{2} + 2k\pi$ (see a))

$\rightarrow 3\theta = \frac{\pi}{2} + (2k-1)\pi$

$\rightarrow \theta = \frac{\pi}{6} + (2k-1)\frac{\pi}{3},\ k \in \mathbb{Z}$

e) $\sin(3\theta + \pi) = 1/2 \rightarrow 3\theta + \pi = \frac{\pi}{6} + 2k\pi$ or $3\theta + \pi = \frac{5\pi}{6} + 2k\pi$

$\theta = \frac{\pi}{18} + (2k-1)\pi/3$ or $\theta = \frac{5\pi}{18} + (2k-1)\pi$

f) $\tan\theta = 1$

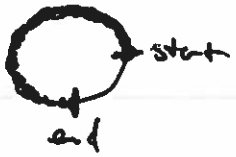
$\theta = \frac{\pi}{4} + k\pi$, $k \in \mathbb{Z}$

g) $\tan 2\theta = -1$ 

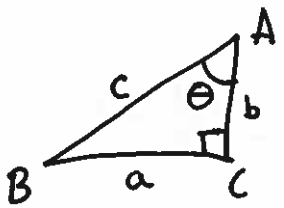
alternate: $2\theta = \frac{3\pi}{4} + k\pi \rightarrow \theta = \frac{3\pi}{8} + k\pi/2,\ k \in \mathbb{Z}$

$2\theta = -\frac{\pi}{4} + k\pi \rightarrow \theta = -\frac{\pi}{8} + k\pi/2,\ k \in \mathbb{Z}$

6) $d = (0.75)(12\,[m]) = \boxed{9\,[m]}$



7)



a) $\boxed{\begin{aligned} \sin\theta &= \frac{a}{c} \\ \cos\theta &= \frac{b}{c} \\ \tan\theta &= \frac{a}{b} \end{aligned}}$

b) $a = 3,\ \theta = 30° \rightarrow \sin 30° = \frac{3}{c} \rightarrow \frac{1}{2} = \frac{3}{c} \rightarrow \boxed{c = 6}$

$\rightarrow \tan 30° = \frac{3}{b} \rightarrow \frac{1}{\sqrt{3}} = \frac{3}{b} \rightarrow \boxed{b = 3\sqrt{3}}$

Double check: Pythagorean thm

$a^2 + b^2 = 3^2 + (3\sqrt{3})^2 = 9 + 9 \cdot 3 = 9 \cdot 4 = 36$ ✓

$c^2 = 6^2 = 36$ ✓

8)

$\sin\theta = \frac{e}{100\,[ft]} \rightarrow e = 100 \sin 1'\ [ft]$

$= 1200 \sin\left(\frac{1}{60}°\right)\,[in] \approx \boxed{0.3491\,[in]}$

$= 1200 \sin\left(\frac{\pi}{60 \cdot 180}\right)\,[in]$

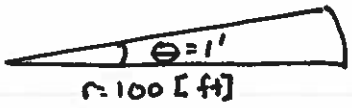
$\approx \frac{1200\pi}{60 \cdot 180} = \frac{\pi}{9} \approx \boxed{0.3491\,[in]}$

more decimals

0.3490658455 [in]

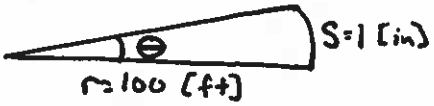
− 0.3490658504 [in]

difference of about $5 \cdot 10^{-9}$

9) 

$$S = \underbrace{\left(\frac{1}{60}\cdot\frac{\pi}{180}\right)}_{\theta \text{ in rad}}\underbrace{(100\cdot 12)}_{r \text{ in [in]}} \text{ [in]}$$

$$= \frac{\pi \cdot 12}{6\cdot 18} = \boxed{\frac{\pi}{9}\text{ [in]}}$$

10) 

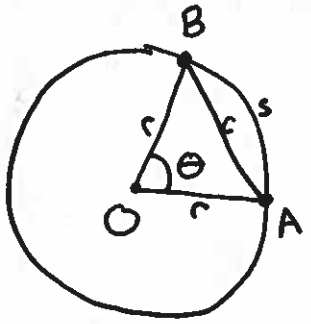
$$S=r\theta \rightarrow \theta = \frac{S}{r} = \frac{1\text{ [in]}}{1200\text{ [in]}} = \frac{1}{1200}\text{ rad}$$

~~$\theta = \frac{1}{1200}\cdot\frac{180^\circ}{\pi}\cdot 60' =$~~

$$\theta = \frac{1\text{ rad}}{1200}\cdot\frac{180^\circ}{\pi\text{ rad}}\cdot\frac{60'}{1^\circ} = \boxed{\frac{9'}{\pi}} = 2+\frac{9}{\pi}\cdot -2$$

$$= 2' + \frac{(9-2\pi)60''}{\pi}$$

11)

a) 

b) $\boxed{S=r\theta}$

c) 

$\sin\frac{1}{2}\theta = \frac{\frac{1}{2}c}{r}$

$\Rightarrow \boxed{c = 2r\sin\left(\frac{1}{2}\theta\right)}$

d) $\theta=0 \rightarrow S=r(0)=0$

$c=2r\sin(0)=0$ ✓

$0<\theta<<1 \rightarrow S=r\theta$ ✓

$c=2r\sin\left(\frac{1}{2}\theta\right) \approx 2r\cdot\frac{1}{2}\theta = r\theta$ ✓

12. Let $f(x) = -2\cos\left(\frac{\pi}{3}x + \frac{\pi}{4}\right) + 1$.

(a) What is the frequency ω? $\pi/3$

(b) What is the period T? $T = 2\pi/\omega = 2\pi/\pi/3 = \boxed{6}$

(c) What is the phase shift ψ? $\psi = \phi/\omega = \frac{-\pi/4}{\pi/3} = \boxed{-3/4}$

(d) What is the amplitude A? $A = 2$

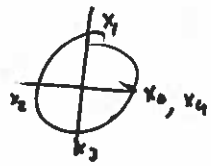
(e) What is the mean value m? $m = 1$

(f) Draw a rectangle in $\mathbb{R}^2$ with lower-left corner at $(\psi, A-m)$ and upper-right corner $(\psi+T, A+m)$. Split the rectangle into four sub-rectangles by partitioning the X-interval $[\psi, \psi+T]$ into for equal sub-intervals.

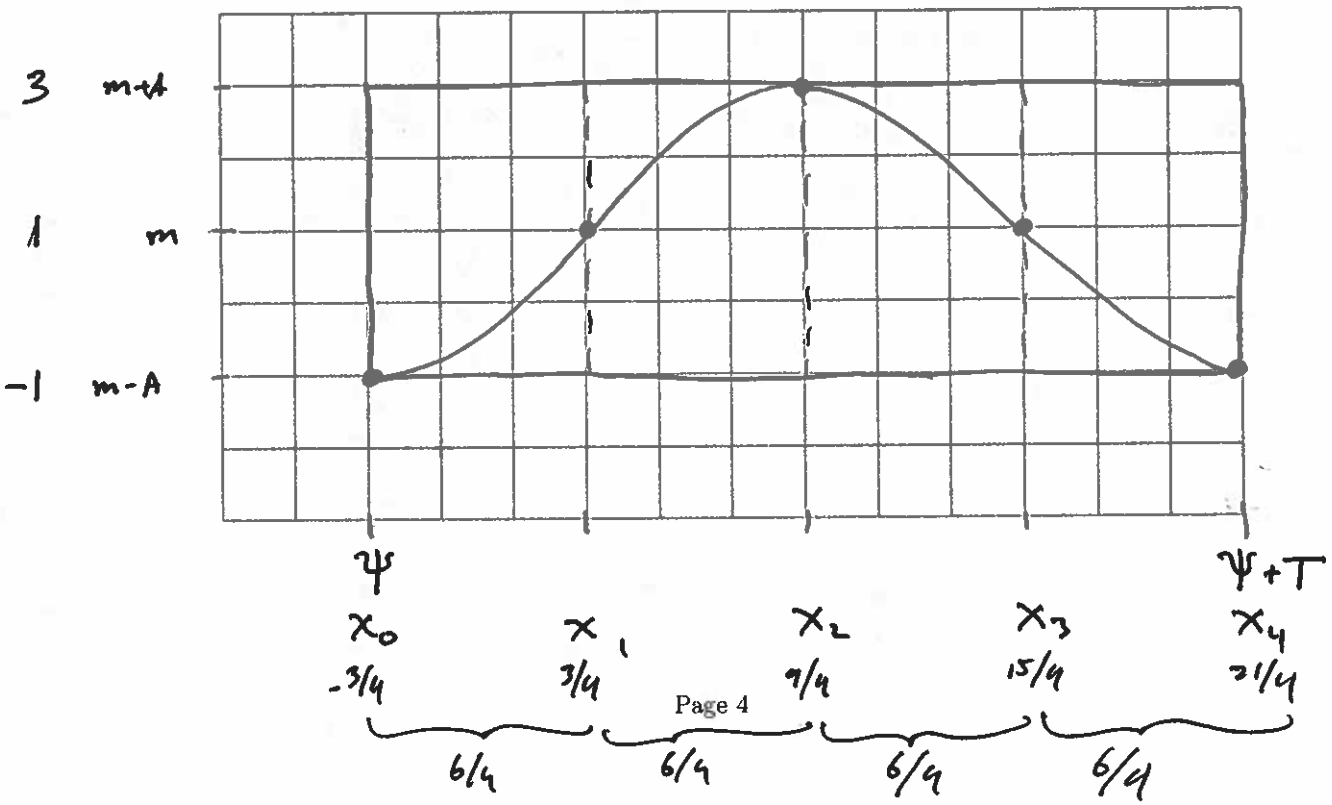
(g) Compute $x_0 = \psi$, $x_1 = x_0 + T/4$, $x_2 = x_1 + T/4$, $x_3 = x_2 + T/4$, $x_4 = x_3 + T/4$. $\frac{T}{4} = \frac{6}{4} = \frac{3}{2}$

(h) Fill out the table below for $x \in \{x_0, x_1, x_2, x_3, x_4\}$. $\pi\left(\frac{x}{3} + \frac{1}{4}\right)$

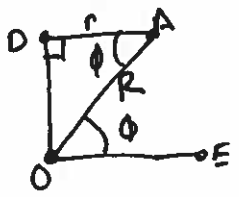
	x	$\frac{\pi}{3}x + \frac{\pi}{4}$	$f(x)$
x_0	$-3/4$	0	$-2\cos(0)+1 = -1$
x_1	$3/4$	$\frac{2\pi}{4} = \frac{\pi}{2}$	$-2\cos\left(\frac{\pi}{2}\right)+1 = 1$
x_2	$9/4$	$\pi\left(\frac{3}{4}+\frac{1}{4}\right) = \pi$	$-2\cos(\pi)+1 = 3$
x_3	$15/4$	$\pi\left(\frac{5}{4}+\frac{1}{4}\right) = \frac{3\pi}{2}$	$-2\cos\left(\frac{3\pi}{2}\right)+1 = 1$
x_4	$21/4$	$\pi\left(\frac{7}{4}+\frac{1}{4}\right) = 2\pi$	-1



(i) Plot $y = f(x)$ over the interval $[\psi, \psi+T]$.



13) a) $s = r\theta \rightarrow \boxed{R \cdot \psi \cdot \frac{\pi}{180} = 1620}$ R in [mi], ψ in deg (7)

b) 

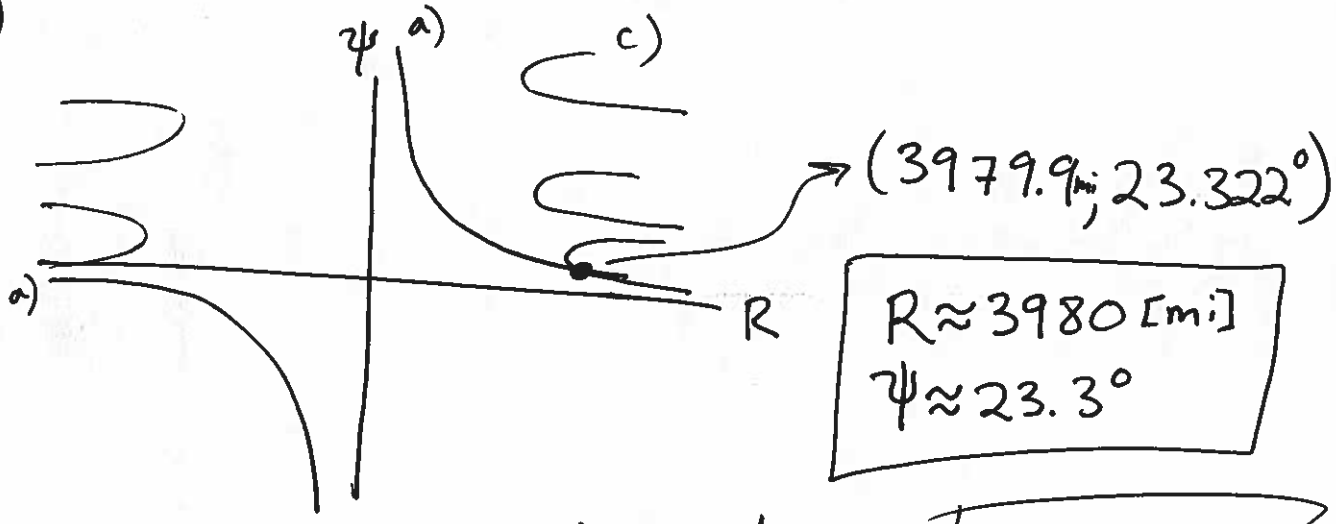
$\cos\phi = r/R \rightarrow \boxed{r = R\cos\phi}$

c) $2\pi r = 2\pi R\cos\phi = 2\pi R\cos(90-\psi) = \boxed{2\pi R\sin\psi = 9900}$

$\cos(90-\psi) = \sin(\psi)$

or $2\pi R\cos(90-\psi) = 9900$

d)



$R \approx 3980$ [mi]
$\psi \approx 23.3°$

Accepted / google: $R \approx 3963$ [mi], $\psi \approx 23.44°$

e) $\phi \approx 90° - 23.3°$
$= 66.7°$
$= 66°42'$ N

f) dist_equator $= 2\pi R \approx 2\pi(3980)$(mi) $\approx$ 25007 [mi]

dist_capricorn $= 2\pi R\cos(23.3°) \approx$ 22967.7 [mi]