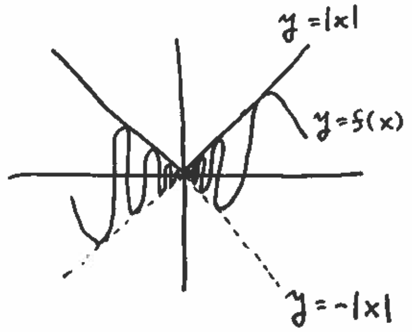
1) $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{(x+h)^2 - x^2}{h} = \lim\limits_{h\to 0} \dfrac{x^2+2xh+h^2-x^2}{h}$

$= \lim\limits_{h\to 0}(2x+h) = \boxed{2x}$

2) $f'(x) = \lim\limits_{h\to 0} \dfrac{(x+h)^3 - x^3}{h} = \lim\limits_{h\to 0} \dfrac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$

$= \lim\limits_{h\to 0}(3x^2+3xh+h^2) = \boxed{3x^2}$

3)



$\lim\limits_{h\to 0} \dfrac{f(h)-f(0)}{h} = \lim\limits_{h\to 0} \dfrac{h\sin(1/h)}{h}$

$= \lim\limits_{h\to 0} \sin(1/h)$

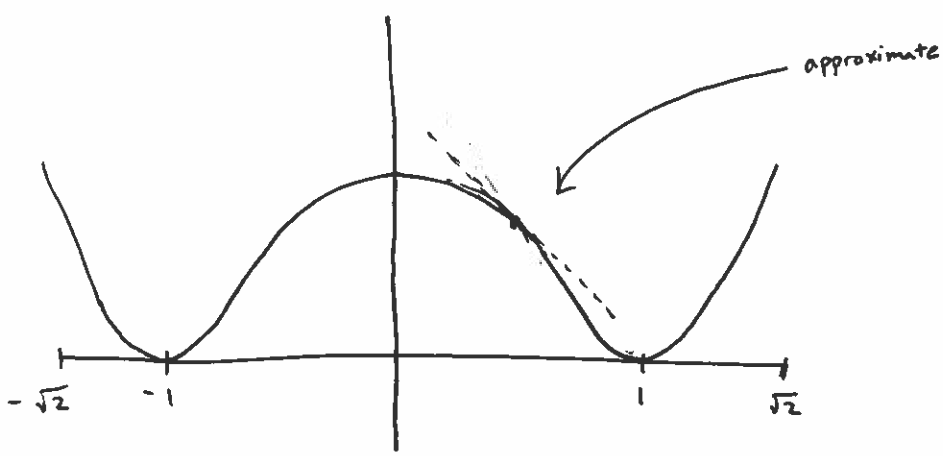
This limit DNE, so $f'(0)$ DNE.

4) $f(x) = \frac{1}{4}(1-x^2)^2 \longrightarrow f'(x) = \frac{1}{4}\cdot 2(1-x^2)^1\cdot(-2x) = -x(1-x^2)$

$y_0 = f(x_0) = \frac{1}{4}\left(1-\frac{1}{4}\right)^2 = \frac{1}{4}\cdot\left(\frac{3}{4}\right)^2 = \frac{9}{64}$

$m = f'(x_0) = -\frac{1}{2}\left(1-\frac{1}{4}\right) = -\frac{1}{2}\cdot\frac{3}{4} = -3/8$

tangent line at $x_0 = 1/2$: $\boxed{y - \frac{9}{64} = -\frac{3}{8}\left(x-\frac{1}{2}\right)}$



5) $f(x) = (1+x+x^2)^2 = (1+x+x^2)(1+x+x^2)$

$= 1+x+x^2 + x+x^2+x^3 + x^2+x^3+x^4$

$= 1+2x+3x^2+2x^3+x^4$

$\rightarrow f'(x) = 2+6x+6x^2+4x^3 = \boxed{2(1+3x+3x^2+2x^3)}$

6) $f(x) = (1+x+x^2)^2 \rightarrow f'(x) = \boxed{2(1+x+x^2)(1+2x)}$

$= 2(1+x+x^2+2x+2x^2+2x^3)$

$= 2(1+3x+3x^2+2x^3)$

7) a) $\frac{d}{dx}\left(\frac{1+x+x^2}{1-x+x^2}\right) = \boxed{\frac{(1+2x)(1-x+x^2) - (1+x+x^2)(-1+2x)}{(1-x+x^2)^2}}$

b) $\frac{d}{dx}\left(10^{10} + \pi x + \frac{1}{x} + \frac{2}{\sqrt{x}}\right) = \pi + \frac{d}{dx}(x^{-1}) + 2\frac{d}{dx}(x^{-1/2})$

$= \pi - x^{-2} + 2\cdot(-1/2)\, x^{-3/2}$

$= \boxed{\pi - \frac{1}{x^2} - \frac{1}{\sqrt{x^3}}}$

c) $\frac{d}{dx}\left(xy^2 - x^2y + y^{-\pi}\right)$ where y is independent of x

$= \boxed{y^2 - 2xy}$

d) $\frac{d}{dx}\left(xy^2 - x^2y + y^{-\pi}\right)$ where y is dependent on x and $y' = \frac{dy}{dx}$.

$= (y^2 + x\cdot 2yy') - (2xy + x^2y') + (-\pi y^{-\pi-1}\, y')$

$= \boxed{y^2 - 2xy + (2xy - x^2 - \pi y^{-\pi-1})\, y'}$

e) $\frac{d}{dy}\left(xy^2 - x^2y + y^{-\pi}\right)$ where x is independent of y

$= \boxed{2xy - x^2 - \pi y^{-\pi-1}}$

f) $\frac{d}{dt}(x^2+y^2)$ where x and y are differentiable functions of t

$= \boxed{2x\dot{x} + 2y\dot{y}}$ where $\dot{x} = \frac{dx}{dt}$ and $\dot{y} = \frac{dy}{dt}$

g) $\frac{d}{dt}(xy) = \boxed{\dot{x}y + x\dot{y}}$ where $\dot{x} = \frac{dx}{dt}$ and $\dot{y} = \frac{dy}{dt}$

h) $\frac{d}{dt}\left(\sqrt{3t} + (t-1)^{-2}\right) = \frac{d}{dt}\left((3t)^{1/2} + (t-1)^{-2}\right)$

$= \frac{1}{2}(3t)^{-1/2}\cdot 3 - 2(t-1)^{-3} = \frac{3}{2\sqrt{3t}} - \frac{2}{(t-1)^3}$

$= \boxed{\frac{\sqrt{3}}{2\sqrt{t}} - \frac{2}{(t-1)^3}}$

i) $\frac{d}{dx}\left(\sin(x)\cos(2x)\right) = \boxed{\cos(x)\cos(2x) - \sin(x)\sin(2x)\cdot 2}$

j) $\frac{d}{dx}\sin(1/x) = \cos(1/x)\cdot\frac{d}{dx}(x^{-1}) = \boxed{-\frac{1}{x^2}\cos(1/x)}$

k) $\frac{d}{dx}x^2\sin(1/x) = 2x\sin(1/x) + x^2\frac{d}{dx}\sin(1/x)$

$= \boxed{2x\sin(1/x) - \cos(1/x)}$

l) $\frac{d}{d\theta}\tan\theta = \frac{d}{d\theta}\frac{\sin\theta}{\cos\theta} = \frac{\cos\theta\cdot\cos\theta - \sin\theta\cdot(-\sin\theta)}{\cos^2\theta} = \frac{1}{\cos^2\theta} = \boxed{\sec^2\theta}$

m) $\frac{d}{d\theta}\sec\theta = \frac{d}{d\theta}\frac{1}{\cos\theta} = \frac{d}{d\theta}(\cos\theta)^{-1} = -(\cos\theta)^{-2}(-\sin\theta) = \frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta}$

$= \boxed{\tan\theta\sec\theta}$

NOT $\cos^{-1}\theta$

7 cont.)

n) $\frac{d}{d\phi}\left(\cos^2\phi + \sin^2\phi\right) = \frac{d}{d\phi}(1) = \boxed{0}$

o) $\frac{d}{d\phi}\left(\cos^2(2\phi+1)\right) = \cancel{2\cos(2\phi+1) \cdot 2 = 4\cos}$

$$= 2\cos(2\phi+1)\cdot\frac{d}{d\phi}\cos(2\phi+1)$$

$$= 2\cos(2\phi+1)\cdot\left(-\sin(2\phi+1)\cdot 2\right)$$

$$= \boxed{-4\cos(2\phi+1)\sin(2\phi+1)}$$

p) $\frac{d}{dx}\sin\left(\sin^2 x\right) = \cos(\sin^2 x)\cdot\frac{d}{dx}\sin^2 x$

$$= \cos(\sin^2 x)\cdot 2\sin x\cdot\frac{d}{dx}\sin x$$

$$= \boxed{\cos(\sin^2 x)\cdot 2\sin x\cos x}$$

8) <u>Claim</u>: $(kf)'(x_0) = k f'(x_0)$

<u>Proof</u>: $(kf)'(x_0) = \lim_{h\to 0}\frac{(kf)(x_0+h) - (kf)(x_0)}{h} = \lim_{h\to 0}\frac{k f(x_0+h) - k f(x_0)}{h}$

$$= k\lim_{h\to 0}\frac{f(x_0+h) - f(x_0)}{h} = k f'(x_0). \blacksquare$$

9) <u>Claim</u>: $(f+g)'(x_0) = f'(x_0) + g'(x_0)$

<u>Proof</u>: $(f+g)'(x_0) = \lim_{h\to 0}\frac{(f+g)(x_0+h) - (f+g)(x_0)}{h}$

$$= \lim_{h\to 0}\frac{f(x_0+h) + g(x_0+h) - f(x_0) - g(x_0)}{h}$$

$$= \lim_{h\to 0}\left(\frac{f(x_0+h) - f(x_0)}{h} + \frac{g(x_0+h) - g(x_0)}{h}\right)$$

$$= f'(x_0) + g'(x_0). \blacksquare$$

10) Let $f(x)=mx$.

Claim: f is linear.

Proof:

1) $f(u_1+u_2)=m(u_1+u_2)=mu_1+mu_2=f(u_1)+f(u_2)$

2) $f(au)=m\cdot(au)=a\cdot(mu)=a\cdot f(u)$. ∎

11) Let $f(x)=mx+b$.

Claim: f is not linear in general.

Proof: $f(u_1+u_2)=m(u_1+u_2)+b=mu_1+mu_2+b=mu_1+f(u_2)$

$=mu_1+b+f(u_2)-b=f(u_1)+f(u_2)-b$

$\neq f(u_1)+f(u_2)$ unless $b=0$. ∎

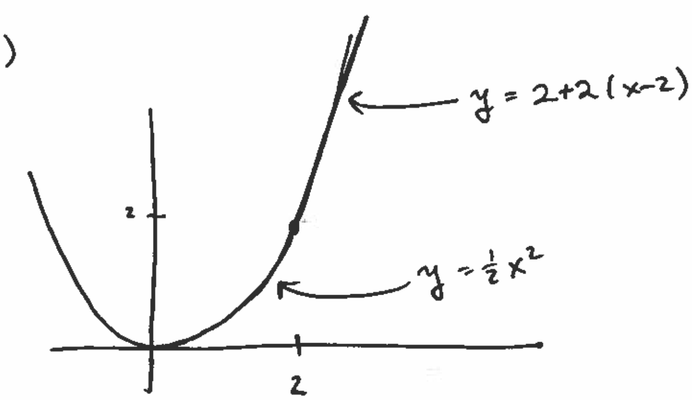
12) Suppose $\lim\limits_{x\to x_0}\frac{f(x)}{x}=L$. Then $\lim\limits_{x\to x_0}\frac{f(3x)}{x}=\lim\limits_{x\to x_0}3\cdot\frac{f(3x)}{3x}=\boxed{3L}$.

13) $f(x)=\begin{cases}\frac{1}{2}x^2 & ,\ x<2\\ y_0+m(x-2) & ,\ x\geq 2\end{cases}$

a) If $y_0=\frac{1}{2}\cdot 2^2=2$, then f is continuous.

b) If $m=\frac{d}{dx}\left(\frac{1}{2}x^2\right)\Big|_{x=2}=2$, then f is differentiable.

c)



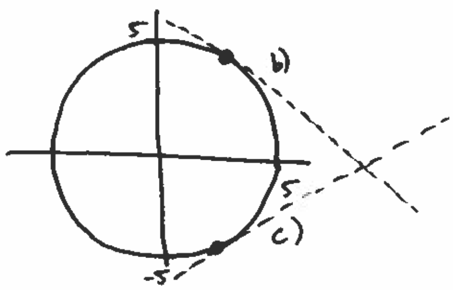
14) Let C be given by $x^2+y^2=25$.

a) $2x+2yy'=0 \to \boxed{y'=-x/y}$

b) $m=-3/4 \to \boxed{y-4=-3/4\,(x-3)}$

c) $m=-3/-4=3/4 \to \boxed{y+4=3/4\,(x-3)}$

d)



15) Let C be given by $\cos(xy)=1/\sqrt{2}$ and set $(x_0,y_0)=(\pi,1/4)$

$$\frac{d}{dx}\cos(xy)=\frac{d}{dx}\frac{1}{\sqrt{2}} \implies -\sin(xy)\left(y+xy'\right)=0$$

$$\implies -\sin(\pi/4)\left(1/4+\pi m\right)=0$$

$$\implies m=-\frac{1}{4\pi}$$

$$\implies \boxed{y-1/4=-\frac{1}{4\pi}(x-\pi)}$$

16) Recall $V = \frac{4\pi}{3} r^3$. Suppose $\dot{r} = 1 \text{ cm/s}$.

a) $\dot{V} = 4\pi r^2 \dot{r} = 4\pi (1\text{ cm})^2 (1 \text{ cm/s}) = \boxed{4\pi \text{ cm}^3/\text{s}}$

b) $\dot{V} = 4\pi r^2 \dot{r} = 4\pi (2\text{cm})^2 (1 \text{ cm/s}) = \boxed{16\pi \text{ cm}^3/\text{s}}$

17) Recall $A = \pi r^2$. Suppose $\dot{A} = 1 \text{ cm}^2/\text{s}$.

Note $\dot{A} = 2\pi r \dot{r} \implies \dot{r} = \dot{A}/2\pi r$.

a) $\dot{r} = \frac{\dot{A}}{2\pi r} = \frac{1 \text{ cm}^2/\text{s}}{2\pi \text{ cm}} = \boxed{\frac{1}{2\pi} \text{ cm/s}}$

b) $\dot{r} = \frac{\dot{A}}{2\pi r} = \frac{1 \text{ cm}^2/\text{s}}{4\pi \text{ cm}} = \boxed{\frac{1}{4\pi} \text{ cm/s}}$