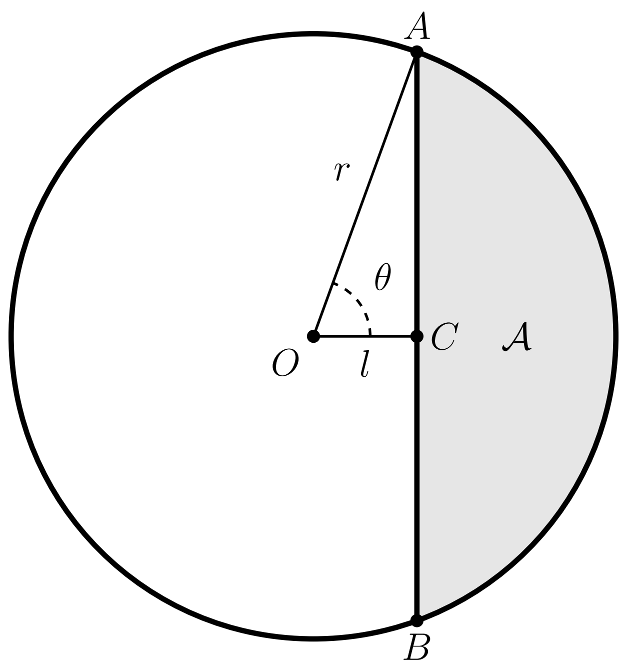
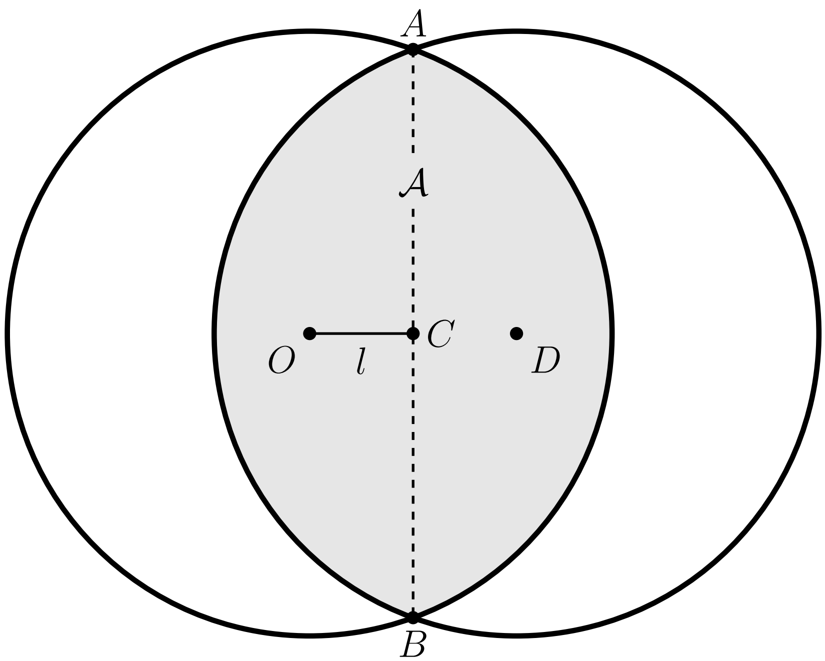
MATH 164 Review (A)

Problems

1. Consider a circle of radius r and center O at the origion. Let C be at the coordinate $(l, 0)$ for some $l \in [-r, r]$ and consider the chord AB perpendicular to OC. Let $\mathcal{A}$ be the area inside the circle and to the right of the chord AB. Find the area of $\mathcal{A}$...
   (a) ...in terms of l and r.
   (b) ...in terms of r and θ.
   (c) Verify that your equations give the same (and correct!) results at $l = r, 0, -r$ and $\theta = 0, \frac{\pi}{2}, \pi$.



2. The Sun and Moon both occupy approximately the same solid angle in the sky (i.e., both are about 0.2 square degrees or $6 \cdot 10^{-5}$ solid radians). This means that during a solar eclipse, it is possible for the Moon to cover a large portion of the Sun. Consider the scenario where O is the center of the outline of the Sun, D is the center of the outline of Moon, and their circular outlines intersect at points A and B. As in Problem 1, let C the midpoint of the chord AB and $l = \overline{OC}$ be the distance from O to C.



(a) Working in units of the apparant common radius of the Sun and Moon (i.e., set $r = 1$), use your result from Problem 1 to compute the total area $\mathcal{A}$ of the eclipse as a function of l. [Hint: AB is a line of symmetry for this problem.]

(b) Suppose the maximum area of the Sun that is covered by the Moon in a particular eclipse is 80%. At what value of l is this maximum achieved?

3. Let $f, g\colon \mathbb{R} \to \mathbb{R}$ be periodic functions. Say $f(t+T_1) = f(t)$ and $g(t+T_2) = g(t)$ for all $t \in \mathbb{R}$ where T_1 and T_2 are positive constants. An important question is that, when viewed as a pair, is the function $(f(t), g(t))$ periodic? That is, is there some (minimal) $T > 0$ such that $f(t+T) = f(t)$ **and** $g(t+T) = g(t)$ for all $t \in \mathbb{R}$.

   (a) Let $T_1 = 1$ and $T_2 = 2$. What is T?

   (b) Let $T_1 = 2$ and $T_2 = 3$. What is T?

   (c) Let $T_1 = 2$ and $T_2 = 4$. What is T?

   (d) Show that if T exists, then there must be integers m and n such that $nT_1 = mT_2 = T$.

   (e) If $T_1 = a/b$ and $T_2 = c/d$, show that $m/n = (bc)/(ad)$.

   (f) If T_1 is rational and T_2 is irrational, show that T does not exist.

   (g) Given parameters A, B, a, and b, a *Lissajous* curve is the set

   $$G = \{(x, y) \in \mathbb{R}^2 \mid x = A\cos(at) \text{ and } y = B\sin(bt), t \in \mathbb{R}\}.$$

   This is the graph of the (parametric) equation $(A\cos(at), B\sin(bt))$. Assume $A, B, a, b > 0$. If $a = b$, what is the shape of G? Plot several examples in Desmos to familiarize yourself with the shape of G. Under what condition will the length of G be finite?

4. Evaluate the following quantities.

   (a) $\arccos(-0.5)$

   (b) $\arcsin(-0.5)$

   (c) $\arctan(1)$

   (d) $\cos(\arcsin(0.25))$

   (e) $\tan(\arccos(a/b))$ where $b > a > 0$

5. Consider the triangle below. Unless otherwise specified, report all angles in degrees. Approximate means use a calculator and round to 5 digits of accuracy.

   (a) Express x and y in terms of r and θ.

   (b) Evaluate $\theta + \psi$ in degrees and radians.

   (c) Suppose $r = 2$ and $\theta = 40°$. Approximate the missing measurements.

   (d) Suppose $x = 3$ and $\psi = 1$ (radian). Approximate the missing measurements.

   (e) Suppose $x = 3$ and $y = 4$. Approximate the missing measurements.

   (f) Suppose $r = 5$ and $\psi = 15°$. Approximate the missing measurements.