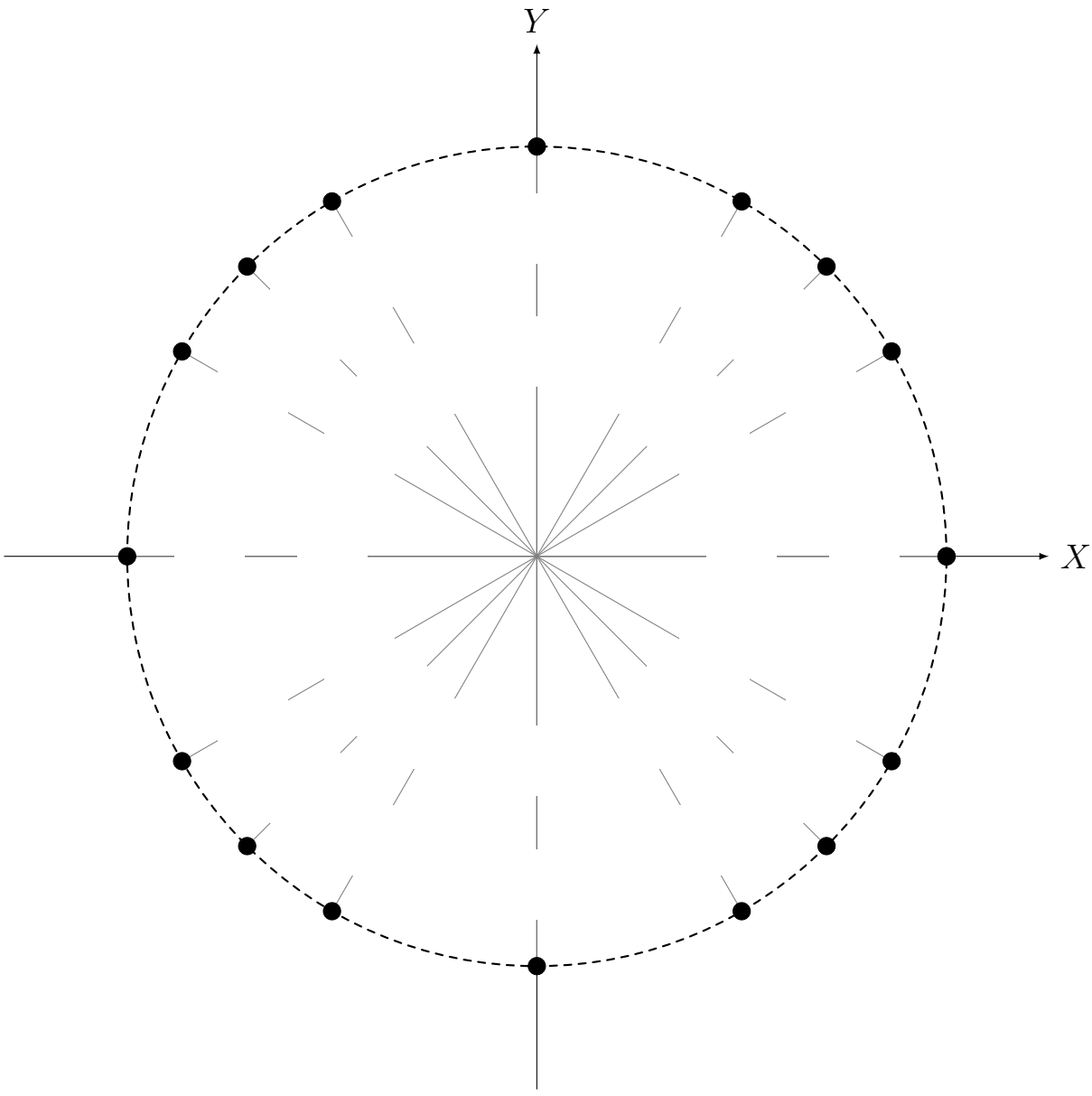
MATH 164 Exam 3 Review

Problems

1. Complete the unit circle.



2. Convert from DMS (*Degree Minute Second*) to radians. Write your answer in exact form and simplified.
   (a) 360°
   (b) 180°
   (c) 90°
   (d) 45°
   (e) $22^\circ 30'$
   (f) $11^\circ 15'$
   (g) $5^\circ 37' 30''$
   (h) 1°
   (i) $1'$
   (j) $1''$
3. Convert from a decimal degree to DMS. For example:

$$5.4^\circ = 5^\circ + 0.4^\circ = 5^\circ + 0.4^\circ \cdot \frac{60'}{1^\circ} = 5^\circ + 24' = 5^\circ 24'$$

   (a) 30.2°
   (b) 15.3°
   (c) 12.05°
   (d) 12.01°
4. Evaluate the following expressions exactly and without a calculator.
   (a) $\sin \dfrac{\pi}{3}$
   (b) $\cos \dfrac{\pi}{3}$
   (c) $\tan \dfrac{\pi}{3}$
   (d) $\sin \dfrac{17\pi}{3}$
   (e) $\cos \dfrac{-15\pi}{3}$
   (f) $\tan 123456789\pi$
   (g) $\cos \dfrac{100\pi}{6}$
5. Solve each equation for *all* solutions exactly and without a calculator.
   (a) $\sin\theta = 1$
   (b) $\sin\theta = 0$
   (c) $\sin\theta = \dfrac{1}{2}$
   (d) $\sin(3\theta + \pi) = 1$
   (e) $\sin(3\theta + \pi) = \dfrac{1}{2}$
   (f) $\tan(\theta) = 1$
   (g) $\tan(2\theta) = -1$

6. Suppose a circle has a circumference of 12 [m]. What is the distance if one were to walk 75% of the way around the circle?

7. Let A, B, and C be distinct points in the plane such that $\triangle ABC$ is a right triangle. Let $\angle ACB$ be the right angle and $\theta = \angle CAB$. Let $a = \overline{BC}$, $b = \overline{AC}$, and $c = \overline{AB}$.
   (a) Write $\sin\theta$, $\cos\theta$, and $\tan\theta$ as ratios of a, b, and c.
   (b) If $a = 3$ and $\theta = 30°$, find the remaining side lengths.

8. Consider a right triangle with $\theta = 1'$ and hypotenuse 100 [ft]. What is the length of the opposite side? Use your calculator to give a decimal answer in units of [in]. Next, use the small angle approximation $\sin\theta \approx \theta$ to approximate the answer in units of [in]. How close are your two answers?

9. What is the length of a $1'$ arc of a circle with a 100 [ft] radius? Express your answer exactly and in units of [in].

10. Suppose a circle has a 100 [ft] radius and 1 [in] arc is drawn on its perimeter. What is the central angle of the arc? Express your answer exactly and in DMS.

11. Let the point O be at the center of a circle of radius r, A and B two distinct points on the perimeter of the circle. Let θ be the angle $\angle AOB$ measured in radians, s the distance along the perimeter of the circle from A to B, and c be the straight-line distance $\overline{AB}$ (i.e., the length of the chord AB).
   (a) Draw a diagram with O, A, B, r, θ, s, and c all labeled.
   (b) Find an expression for s in terms of r and θ.
   (c) Find an expression for c in terms of r and θ. [Hint: let C be the point halfway along the chord AB. Draw the line segment OC. This line segment will be interior to the circle, bisect θ, and be perpendicular to AB.]
   (d) Sanity check your formulas by verifying that $s = c = 0$ when $\theta = 0$ and $s \approx c$ when $0 < \theta << 1$. [Hint: for the second check, use the small angle approximation $\sin x \approx x$ when x is small.]

12. Let $f(x) = -2\cos\left(\frac{\pi}{3}x + \frac{\pi}{4}\right) + 1$.

(a) What is the frequency ω?

(b) What is the period T?

(c) What is the phase shift ψ?

(d) What is the amplitude A?

(e) What is the mean value m?

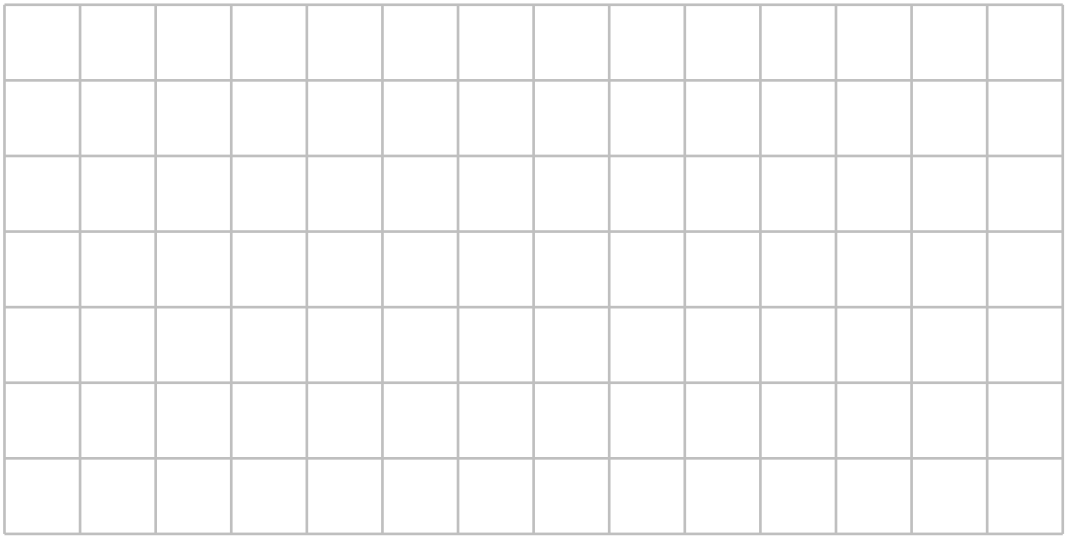
(f) Draw a rectangle in $\mathbb{R}^2$ with lower-left corner at $(\psi, A-m)$ and upper-right corner $(\psi+T, A+m)$. Split the rectangle into four sub-rectangles by partitioning the X-interval $[\psi, \psi+T]$ into for equal sub-intervals.

(g) Compute $x_0 = \psi$, $x_1 = x_0 + T/4$, $x_2 = x_1 + T/4$, $x_3 = x_2 + T/4$, $x_4 = x_3 + T/4$.

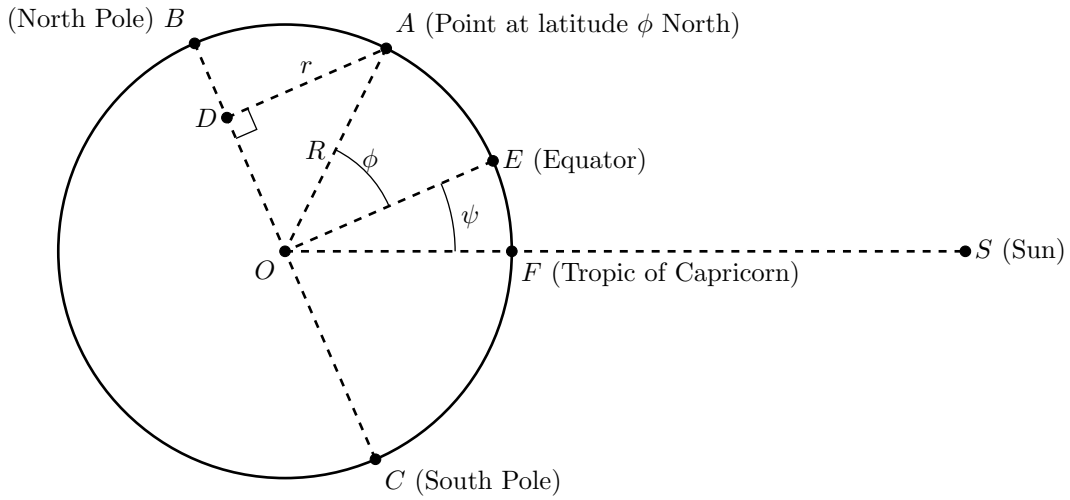
(h) Fill out the table below for $x \in \{x_0, x_1, x_2, x_3, x_4\}$.

x	$\frac{\pi}{3}x + \frac{\pi}{4}$	$f(x)$

(i) Plot $y = f(x)$ over the interval $[\psi, \psi+T]$.



13. Consider the diagram (not drawn to scale) below where O is the center of the Earth and the Sun S is far enough away to be considered a point. E is the point on the equator closest to S, and the axis of rotation of the earth BC is tilted by an angle of ψ from being perpendicular to OS (drawn is the equivalent statement that $\angle EOS = \psi$). The point A is at the latitude of ϕ (North). The lines AD and OE are parallel to each other and are both perpendicular to the line BC. Let $R = \overline{OB} = \overline{OA} = \overline{OE} = \overline{OC}$ [mi] be the radius of the Earth. Let D the point (interior to the Earth) closest to A so that $\angle ODA$ is a right angle.



(a) The Tropic of Capricorn is the circle of latitude at ψ South. It is the farthest South one can go where the sun *can* appear to be directly overhead at noon. Suppose the distance (as measured on the surface of the earth) from the Tropic of Capricorn to the equator is surveyed and found to be 1620 [mi]. Write a relation between the angle ψ in degrees and the radius of the Earth R. [Hint: the *arc length* from E to F was given.]

(b) Find the distance $r = \overline{AD}$ in terms of R and ϕ. Note that the distance around the Earth at latitude ϕ is $2\pi r$.

(c) The Arctic Circle is circle of latitude such that $\phi + \psi = 90°$. Suppose that the distance around the Arctic Circle is surveyed to be approximately 9900 [mi]. Use this information and your solution from part (b) to write another relation between the angle ψ (in degrees) and R.

(d) Use Desmos to plot the relations in parts (a) and (c) and find their intersection. Look up the correct values for R and ψ. How close are your estimates? [Hint: set Desmos to use degrees and use variables x and y instead of R and ψ.]

(e) Use your estimate of ψ to compute the latitude of the Arctic Circle in DMS.

(f) Use your estimate of R to compute the distance around the Earth at the equator and the Tropic of Capricorn.