Number Bases

Number vs Representation

Example: 27

Tally: 𝍸 𝍸 𝍸 𝍸 𝍸 ||

Roman Numerals: XXVII

Positional Notation: 27 (decimal) 11011 (binary) 1B (hexadecimal)

Bases

decimal : base-10, what we use

binary : base-2, what computers use

hexadecimal (hex) : base-16, compresses binary for readability

Base	Digits	Base written in its base
2	0, 1	10
10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9	10
16	0-9, A_{10}, B_{11}, C_{12}, D_{13}, E_{14}, F_{15}	10

Decimal

Decimal Expansion

$$\overset{10^3}{1},\overset{10^2}{5}\,\overset{10^1}{9}\,\overset{10^0}{0} = 1\times10^3 + 5\times10^2 + 9\times10^1 + 0\times10^0$$

$$1\times1000 + 5\times100 + 9\times10 + 0\times1$$

$$1000 + 500 + 90 + 0$$

$$1590$$

Addition

$$\begin{array}{r} {}^{1}\ {}^{1}\ \ \ \ \\ 1590 \\ +3475 \\ \hline 5{,}065 \end{array}$$

Add column by column

Keep 1's digit 0–9 no carry

Carry 10's digit 10–19 carry 1

Binary

Decimal

Binary Expansion

$2^4\ 2^3\ 2^2\ 2^1\ 2^0$

$1\ 1\ 0\ 1\ 1 = 1\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0$

$1\times16 + 1\times8 + 0\times4 + 1\times2 + 1\times1$

$16 + 8 + 0 + 2 + 1$

27

Addition

$2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

$\ \ \ 1\ 1\ 1$

$\ \ \ \ 101100 = 2^5 + 2^3 + 2^2 = 32+8+4 = 44$

$+\ 111010 = 32+16+8+2 = 58$

1100110

$2^6+2^5+2^2+2^1$

$64+32+4+2$

$70+32 = 102$

102

$0 = 0_{10}$

$1 = 1_{10}$ no carry

$10 = 2_{10}$

$11 = 3_{10}$ carry 1

Hexadecimal

$16^1 \; 16^0$

$$1\underline{B} = \underbrace{1 \times 16^1 + \underline{11} \times 16^0}_{\text{Decimal}}$$

$$1 \times 16 + 11 \times 1$$

$$16 + 11$$

$$27$$

$$3\underline{A}7 = 3 \times 16^2 + \underline{10} \times 16^1 + 7 \times 16^0$$

$$3 \times 256 + 10 \times 16 + 7 \times 1$$

$$768 + 160 + 7$$

$$768 + 167$$

$$935_{10} = 3A7_{16}$$

$$919_{10} = 397_{16}$$

Addition

$$\begin{array}{r} {}^{1} \\ 3A7 \\ +\ 1B \\ \hline 3C2 \end{array} \qquad \begin{array}{r} {}^{1} \\ = 935_{10} \\ = +\ 27_{10} \\ \hline 962_{10} \end{array}$$

$$7 + B$$

$$7 + 11$$

$$18$$

$$12_{16} = 18_{10}$$

$$3 \times 16^2 + 12 \times 16^1 + 2 \times 16^0$$

768   160 2×1

32

770   192   2

962

Conversion

If we have a number in base x,
how do we represent it in base y?

- Base expansion: good for converting to decimal because the arithmetic is in the target base.
- Division: dividing in original base can convert to any target base, so this is good for converting <u>from</u> decimal

Division: divide by target base until you reach 0
the remainders are the digits in target base

$1590 / 10 = 159 \; r \; 0$

$159 / 10 = 15 \; r \; 9$

$15 / 10 = 1 \; r \; 5$

$1 / 10 = 0 \; r \; 1$

Division : 1590 to base - 2

1590 / 2 = 795 r0
795 / 2 = 397 r1
397 / 2 = 198 r1
198 / 2 = 99 r0
99 / 2 = 49 r1
49 / 2 = 24 r1
24 / 2 = 12 r0
12 / 2 = 6 r0
6 / 2 = 3 r0
3 / 2 = 1 r1
1 / 2 = 0 r1

10 9 8 7 6 5 4 3 2 1 0
1 1 0 0 0 1 1 0 1 1 0

$2^{10} + 2^{9} + 2^{5} + 2^{4} + 2^{2} + 2^{1}$

1024
512
32
16
4
+ 2

1590