Number Bases

Numbers vs Representation

Example: 27

Tally: 𝍸 𝍸 𝍸 𝍸 𝍸 ||

Roman Numerals: XXVII

Positional Notation: 27 (base-10), 11011 (binary)

Bases:		Digits
decimal:	base-10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
binary:	base-2	0, 1
hexadecimal (hex):	base-16	0, 1, ..., 9, A, B, C, D, E, F

Decimal

Decimal Expansion

$1,590 \rightarrow 1\times10^{3} + 5\times10^{2} + 9\times10^{1} + 0\times10^{0}$

$1\times1000 + 5\times100 + 9\times10 + 0\times1$

$1000 + 500 + 90 + 0$

$10^{3}\ 10^{2}\ 10^{1}\ 10^{0}\ 10^{-1}$ 1,590

1 5 9 0.0... ← Each digit represents the base raised to an integer exponent

Addition

```
  1 1
 1,590
+3,475
------
 5,065
```

0-9, 10-19

no carry / carry a 1

Binary

Binary Expansion

$11011 = 1\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0$

$11011_2 = 27_{10}$ $\quad 1\times16 + 1\times8 + 0\times4 + 1\times2 + 1\times1$

$101110_2 = 46_{10}$ $\quad 16 + 8 + 0 + 2 + 1$

$24 + 2 + 1$

$26 + 1$

27

$$\begin{array}{r} {\scriptstyle 1\ 1\ 1} \\ 11011 \\ +101110 \\ \hline 1001001 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 27 \\ +46 \\ \hline 73 \end{array}$$

$2^6 \quad 2^3 \quad 2^0$

$2^6 + 2^3 + 2^0$

$64 + 8 + 1 = 72 + 1 = 73$

0, 1 — no carry

10, 11 — carry a 1

$0_2 = 0_{10}$

$1_2 = 1_{10}$

$10_2 = 2_{10}$

$11_2 = 3_{10}$

$2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

$1\ 1\ 1$

$101001_2 = 41_{10}$

$+111100_2 = 60_{10}$

$1100101 = 101_{10}$

$2^6\ 2^5 \quad 2^2\ 2^0$

$64 + 32 = 96 \quad 4 \quad 1$

$2_{10} = 2$

$2^1\ 2^0$

10

-2

$1\times2 + 0\times1$

$2 + 0$

2

Hexadecimal

Hexadecimal Expansion

Hex | Expansion written in decimal

$3A7_{16} = 3 \times 16^2 + 10 \times 16^1 + 7 \times 16^0$ (A)

$3 \times 256 + 10 \times 16 + 7 \times 1$

$768 + 160 + 7$

$768 + 167$

$935_{10} = 3A7_{16}$

$919_{10} = 397_{16}$

Dec	Hex	Bin
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

Base Conversions

How do we convert from base X to base Y?

Division: divide by base until we reach 0,
remainders are the digits in that base

$1,590_{10}$ = ? in base 2?

To base-10

1590 / 10 = 159 r 0
159 / 10 = 15 r 9
15 / 10 = 1 r 5
1 / 10 = 0 r 1

1590 / 2 = 795 r 0
795 / 2 = 397 r 1
397 / 2 = 198 r 1
198 / 2 = 99 r 0
99 / 2 = 49 r 1
49 / 2 = 24 r 1
24 / 2 = 12 r 0
12 / 2 = 6 r 0
6 / 2 = 3 r 0

11000110110_2
$=$
$1,590_{10}$

3 / 2 = 1 r 1
1 / 2 = 0 r 1