Number Bases

Number vs Representation

Example: 27

Tally: ~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ ||

Roman Numerals: XXVII

Positional Notation: 27 (base-10) 11011 (base-2) 1B (base-16)

Bases

decimal : base-10 most people know this

binary : base-2 computers use this

hexadecimal (hex) : base-16 compress binary for readability

Base	Digits
2	0, 1
10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
16	0-9, A (10), B (11), C (12), D (13), E (14), F (15)

Decimal

Decimal Expansion

$$\overset{10^3}{1},\overset{10^2}{5}\overset{10^1}{9}\overset{10^0}{0} = 1\times 10^3 + 5\times 10^2 + 9\times 10^1 + 0\times 10^0$$

$$1\times 1{,}000 + 5\times 100 + 9\times 10 + 0\times 1$$

$$1{,}000 + 500 + 90 + 0$$

$$1{,}590$$

Addition

$$\begin{array}{r} {}^{1}\;\;\;{}^{1}\;\;\;\;\; \\ 1{,}590 \\ +\,3{,}475 \\ \hline 5{,}065 \end{array}$$

Add one column: 0 – 9 no carry

10 – 19 carry a 1

Binary

Binary Expansion

Binary	Decimal
$2^4\ 2^3\ 2^2\ 2^1\ 2^0$	
$1\ 1\ 0\ 1\ 1$	$= 1\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0$

$$1\times16 + 1\times8 + 0\times4 + 1\times2 + 1\times1$$

$$16 + 8 + 0 + 2 + 1$$

$$24 + 3$$

$$27$$

Addition

$2^6\,2^5\,2^4\,2^3\,2^2\,2^1\,2^0$

$$\begin{array}{r} {}^{1\,1\,\,1} \\ 101100 \\ +\ 111010 \\ \hline 1100110 \end{array}$$

Decimal

$= 2^5 + 2^3 + 2^2$

$32 + 8 + 4 = 44_{10}$

$= 2^5 + 2^4 + 2^3 + 2^1$

$32 + 16 + 8 + 2 = 58_{10}$

$$\begin{array}{r} {}^{1} \\ 44 \\ +58 \\ \hline 102 \end{array}$$

$2^6 + 2^5 + 2^2 + 2^1 =$

$64 + 32 + 4 + 2 = 102 \longleftrightarrow 102$

~~76~~ 102 6

Addition Algorithm

add 1 colum

0, 1 → no carry

10, 11 → carry 1

$0_2 = 0_{10}$

$1_2 = 1_{10}$

$10_2 = 2_{10}$

$11_2 = 3_{10}$

Hexadecimal

Hex Expansion

Decimal

16^1 16^0

$1\ B = 1 \times 16^1 + 11 \times 16^0$

$1 \times 16 + 11 \times 1$

$16 + 11$

27

$3E8 = 3 \times 16^2 + 14 \times 16^1 + 8 \times 16^0$

$3 \times 256 + 14 \times 16 + 8 \times 1$

768   160 64 224

$768 + 224 + 8$

$768 + 232$

$3E8_{16} = 1000_{10}$

$398_{16} = 920_{10}$

Hex	Dec	Bin
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

$$\begin{array}{r} {}^{1} \\ 398 \\ +\ \ 1B \\ \hline 3B3 \end{array}$$

$= 920_{10}$

$= 27_{10}$

$+$

947_{10}

$8 + 11 = 19$

13_{16}

B

$3 \times 256 + 11 \times 16 + 3$

$768 + 176 + 3$

$$\begin{array}{r} {}^{1\,1} \\ 179 \\ +768 \\ \hline 947_{10} \end{array}$$

Conversions

How do we convert from base X to base Y?

- Digit expansion is good for converting to base-10 if you know arithmetic in base-10
- Division is good for converting from base-10 if you know how to divide in base-10

Division: divide by target base until you reach zero
remainders are the digits in target base

1,590 / 10 = 159 r 0
159 / 10 = 15 r 9
15 / 10 = 1 r 5
1 / 10 = 0 r 1

1590

Division: $1590_{10} \rightarrow$ binary

$1590 / 2 = 795 \; r\,0$

$795 / 2 = 397 \; r\,1$

$397 / 2 = 198 \; r\,1$

$198 / 2 = 99 \; r\,0$

$99 / 2 = 49 \; r\,1$

$49 / 2 = 24 \; r\,1$

$24 / 2 = 12 \; r\,0$

$12 / 2 = 6 \; r\,0$

$6 / 2 = 3 \; r\,0$

$3 / 2 = 1 \; r\,1$

$1 / 2 = 0 \; r\,1$

$2^{10}\, 2^{9} \;-\; 2^{5}\, 2^{4} \;\; 2^{2}\, 2^{1}$

$\boxed{11000110110}$

2

$$
\begin{array}{r}
1024 \\
512 \\
32 \\
16 \\
4 \\
+ \quad 2 \\
\hline
1590
\end{array}
$$