## **Assignment 2**

## 1.1 Use induction to prove $F_i = \frac{\phi^i - \hat{\phi}^i}{\sqrt{5}}$ ; where $F_i = F_{i-2} + F_{i-1}$ , and $\phi$ is the golden ratio $\frac{1+\sqrt{5}}{2}$ .

To prove by induction, write out the expressions  $f_n$  and  $f_{n+1}$  (note:  $f_{n+1}$  is the same as  $f_n$ , but with (n+1) substituted everywhere in place of n). Next, if applicable, re-write the expression  $f_{n+1}$  in terms of  $f_n$  then perform algebraic manipulations on the expression until you reach some variation of  $f_{n+1} = f_{n+1}$ . Lastly, show that the expression  $f_c$  also holds for some constant  $f_c$ . The algebra is called "the inductive step", and the calculation for on the constant is called "the base case".

In this problem, the expression to prove is  $F_i = \frac{\phi^i - \hat{\phi}^i}{\sqrt{5}}$ , where  $\phi = \frac{1+\sqrt{5}}{\sqrt{5}}$ . Start by demonstrating the expression holds for constants c=0, c=1 (e.g., the "base case").

$$F_0 = \frac{\phi^0 - \hat{\phi}^0}{\sqrt{5}} = \frac{1 - 1}{\sqrt{5}} \tag{1.1}$$

$$=0 (1.2)$$

After showing the expression holds for some base cases  $F_0$  and  $F_1$ , the next step is algebra. Setup the expression  $F_{n+1}$  in terms of  $F_n$ , then solve (see below).

$$F_i = \frac{\phi^i - \hat{\phi}^i}{\sqrt{5}} = F_{i-1} + F_{i-2}$$
  $F_{i+1} = \frac{\phi^{i+1} - \hat{\phi}^{i+1}}{\sqrt{5}} = F_i + F_{i-1}$ 

$$F_{i+1} = F_i + F_{i-1} \tag{1.1}$$

$$\frac{\phi^{i+1} - \hat{\phi}^{i+1}}{\sqrt{5}} = \frac{\phi^{i} - \hat{\phi}^{i}}{\sqrt{5}} + \frac{\phi^{i-1} - \hat{\phi}^{i-1}}{\sqrt{5}}$$
(1.2)