Assignment 2

1.1 Are either $\lceil \lg n \rceil!$ or $\lceil \lg\lg n \rceil!$ polynomially bounded?

Polynomially bounded means $f_n = O(n^k)$ for some constant k (e.g., whether $f_n \le c \cdot n^k$ for constants c and k as n approaches ∞).

In other words, this problem asks you to verify whether these two inequalities hold: $\lceil \lg n \rceil! \le c \cdot n^k$. $\lceil \lg\lg n \rceil! \le c \cdot n^k$.

1.2 Use induction to prove $F_i = \frac{\phi^i - \hat{\phi}^i}{\sqrt{5}}$; where $F_i = F_{i-2} + F_{i-1}$, and ϕ is the golden ratio $\frac{1+\sqrt{5}}{2}$.

1.3 Show that $k \lg k = \Theta(n)$ implies $k = \Theta\left(\frac{n}{n \ln n}\right)$.

1.4 Are either 2^{n+1} or 2^{2n} big-O of 2^n?

1.5 For each pair of functions (A, B), indicate whether A is O, o, Ω, ω, or Θ of B. Assume $k \ge 1$, $\epsilon > 0$, $c > 1$ are constants.

A	B	O	o	Ω	ω	Θ
$\lg^k n$	n^ϵ	yes	yes	yes	yes	yes
n^k	c^n	yes	yes	yes	yes	yes
$\sqrt{n}$	$n^{\sin n}$	yes	yes	yes	yes	yes
2^n	$2^{n/2}$	yes	yes	yes	yes	yes
$n^{\lg c}$	$c^{\lg n}$	yes	yes	yes	yes	yes
$\lg(n!)$	$\lg(n^n)$	yes	yes	yes	yes	yes
A	B	yes	yes	yes	yes	yes

1.6 Order the following functions such that $f_1 = \Omega(f_2), f_2 = \Omega(f_3), \ldots, f_{29} = \Omega(f_{30})$, and partition them into equivalence classes such that each function is big-Θ of each other.