# Assignment 2

## 1.1 Use induction to prove $F_i = \frac{\phi^i - \hat{\phi}^i}{\sqrt{5}}$; where $F_i = F_{i-2} + F_{i-1}$, and $\phi$ is the golden ratio $\frac{1+\sqrt{5}}{2}$.

To prove by induction, write out the expressions $f_n$ and $f_{n+1}$ (note: $f_{n+1}$ is the same as $f_n$, but with $(n+1)$ substituted everywhere in place of $n$). Next, if applicable, re-write the expression $f_{n+1}$ in terms of $f_n$ then perform algebraic manipulations on the expression until you reach some variation of $f_{n+1} = f_{n+1}$. Lastly, show that the expression $f_c$ also holds for some constant $c$. The algebra is called "the inductive step", and the calculation for on the constant is called "the base case".

In this problem, the expression to prove is $F_i = \frac{\phi^i - \hat{\phi}^i}{\sqrt{5}}$, where $\phi = \frac{1+\sqrt{5}}{\sqrt{5}}$. Start by demonstrating the expression holds for constants $c = 0, c = 1$.

$$F_0 = \frac{\phi^0 - \hat{\phi}^0}{\sqrt{5}} = \frac{1-1}{\sqrt{5}} \tag{1.1}$$

$$= 0 \tag{1.2}$$

After showing the expression holds for some base cases $F_0$ and $F_1$, the next step is algebra. Setup the expression $F_{n+1}$ in terms of $F_n$, then solve (see below).

$$F_i = \frac{\phi^i - \hat{\phi}^i}{\sqrt{5}} = F_{i-1} + F_{i-2} \qquad F_{i+1} = \frac{\phi^{i+1} - \hat{\phi}^{i+1}}{\sqrt{5}} = F_i + F_{i-1}$$

$$F_{i+1} = F_i + F_{i-1} \tag{1.1}$$

$$\frac{\phi^{i+1} - \hat{\phi}^{i+1}}{\sqrt{5}} = \frac{\phi^{i} - \hat{\phi}^{i}}{\sqrt{5}} + \frac{\phi^{i-1} - \hat{\phi}^{i-1}}{\sqrt{5}} \tag{1.2}$$

$$= \frac{\phi^{i-1} + \hat{\phi}^{i-1} - \phi^{i-2} - \hat{\phi}^{i-2}}{\sqrt{5}} \tag{1.3}$$

$$= \frac{\phi^{i-1} - \phi^{i-2} + \hat{\phi}^{i-1} - \hat{\phi}^{i-2}}{\sqrt{5}} \tag{1.4}$$

$$= \frac{\left[(\phi \cdot \phi^{i-2}) + \phi^{i-2}\right] - \left[(\hat{\phi} \cdot \hat{\phi}^{i-2}) + \hat{\phi}^{i-2}\right]}{\sqrt{5}} \tag{1.5}$$

$$= \frac{\phi^{i-2}\left(\phi^{2}\right) - \hat{\phi}^{i-2}\left(\hat{\phi}^{2}\right)}{\sqrt{5}} \tag{1.6}$$