Assignment 2

1.1 Show that $k \lg k = \Theta(n)$ implies $k = \Theta\left(\frac{n}{\ln n}\right)$.

If $f_n = \Theta(g_n)$, then $\Theta(f_n) = g_n$ (by symmetric property of big-Θ). Note the symmetric property of big-Θ, that if

$$k \ln k = \Theta(n) \Longrightarrow \Theta(k \ln k) = n$$

$$\ln[n] = \Theta(\ln[k \ln k])$$

$$= \Theta(\ln k + \ln \ln k)$$

$$= \Theta(\ln k)$$

$$n = \Theta(k \ln k)$$

$$\frac{n}{\ln n} = \frac{\Theta(k \ln k)}{\Theta(\ln k)} = \Theta\left(\frac{k \ln k}{\ln k}\right) = \Theta(k)$$

$$\Theta(k) = \frac{n}{\ln n}$$

$$k = \Theta\left(\frac{n}{\ln n}\right)$$