## Chapter 1

## 1 What is an algorithm

An algorithm is a sequence of computational steps that transforms an input into an output, generally to solve a well-defined computational problem.

## What is a data structure

Data structures are a way to store and organize data in memory to facilitate efficient access and modification (e.g., to enhance the speed of an algorithm).

## 3 How to quantitatively measure algorithm efficiency

Intuitively, it takes c units of time to perform a given computational operation. Typically the number of operations required by an algorithm corresponds to the size of the input n; therefore, algorithmic efficiency is expressed as a function of input size.

For instance, to sort n integers in increasing order, the *insertion sort* algorithm takes  $c \cdot n^2$  units time, whereas the *merge sort* takes  $c \cdot n \lg n$ . Comparably speaking then, the  $n \lg n$  algorithm will outperform the  $n^2$  algorithm for large input sizes n.

There's an entire mathematical notation for identifying and comparing these input-efficiency functions for algorithms, called *asymptotic notation*; it's discussed at length in chapter 3. Below is a table of input sizes n which could be completed in time t for each efficiency function  $f_n$ , assuming each operation takes 1 ms.

	1 second	1 minute	1 hour	1 day	1 month	1 year	1 century
$\lg n$	$2^{10^6}$	$2^{10^7}$	_	$2^{10^{10}}$	_	$2^{10^{13}}$	$2^{10^{15}}$
$\sqrt{n}$	$10^{12}$	$10^{15}$	$10^{19}$	$10^{21}$	$10^{24}$	$10^{26}$	$10^{30}$
$\overline{n}$	$10^{6}$	$10^{7}$	$10^{9}$	$10^{10}$	$10^{12}$	$10^{13}$	$10^{15}$
$n \lg n$	$10^{4}$	$10^{6}$	$10^{8}$	$10^{9}$	$10^{10}$	$10^{11}$	$10^{13}$
$n^2$	1000	7000	$10^{4}$	$10^{5}$	$10^{6}$	$10^{6}$	$10^{7}$
$n^3$	100	400	1500	4000	$10^{4}$	$10^{4}$	$10^{5}$
$2^n$	20	25	30	35	40	45	50
n!	9	11	12	13	15	16	17