Chapter 1

1 What is an algorithm

An algorithm is a sequence of computational steps that transforms an input into an output, generally to solve a well-defined computational problem.

What is a data structure

Data structures are a way to store and organize data in memory to facilitate efficient access and modification (e.g., to enhance the speed of an algorithm).

3 How to quantitatively measure algorithm efficiency

Intuitively, it takes c units of time to perform a given computational operation. Typically the number of operations required by an algorithm corresponds to the size of the input n; therefore, algorithmic efficiency is expressed as a function of input size.

For instance, to sort n integers in increasing order, the *insertion sort* algorithm takes $c \cdot n^2$ units time, whereas the *merge sort* takes $c \cdot n \lg n$. Comparably speaking then, the $n \lg n$ algorithm will outperform the n^2 algorithm for large input sizes n.

Below is a table of input sizes n which could be completed in time t for each efficiency function f_n , assuming each operation takes 1 ms.

| | 1 second | 1 minute | 1 hour | 1 day | 1 month | 1 year | 1 century |
|-----------------------|------------|------------|-----------|---------------|---------------|---------------|---------------|
| $\lg n$ | 2^{10^6} | 2^{10^7} | _ | $2^{10^{10}}$ | $2^{10^{12}}$ | $2^{10^{13}}$ | $2^{10^{15}}$ |
| $\overline{\sqrt{n}}$ | 10^{12} | 10^{15} | 10^{19} | 10^{21} | 10^{24} | 10^{26} | 10^{30} |
| \overline{n} | 10^{6} | 10^{7} | 10^{9} | 10^{10} | 10^{12} | 10^{13} | 10^{15} |
| $n \lg n$ | 10^{4} | 10^{6} | 10^{8} | 10^{9} | 10^{10} | 10^{11} | 10^{13} |
| n^2 | 1000 | 7000 | 10^{4} | 10^{5} | 10^{6} | 10^{6} | 10^{7} |
| n^3 | 100 | 400 | 1500 | 4000 | 10^{4} | 10^{4} | 10^{5} |
| 2^n | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| n! | 9 | 11 | 12 | 13 | 15 | 16 | 17 |