Assignment 3

Written by Zach Leach, February 12, 2024.

1 Give asymptotic upper and lower bounds for $T(n)$ in each recurrence. Assume $T(n)$ is constant for $n \leq 2$. Make bounds as tight as possible

a. $T(n) = 16T(n/4) + n^2$

b. $T(n) = 7T(n/3) + n^2$

c. $T(n) = 7T(n/2) + n^2$

d. $T(n) = 2T(n/4) + n^2$

1.1 What are asymptotic upper and lower bounds?

Asymptotic upper bound refers to $O(g_n)$ notation (e.g., $f_n \leq c \cdot g_n$). Asymptotic lower bound refers to $\Omega(g_n)$ notation (e.g., $f_n \geq c \cdot g_n$).

1.2 How do you find the asymptotic upper and lower bound of a recurrence?

2 Prove the substitution method that the solution to the following recurrence is $S(n) = O(n \lg^2(n))$

$$S(n) = 2S(n/2) + n \lg n$$

$$S(1) = 1$$

2.1 Substitution Method

3 Write the pseudocode for Strassen's Algorithm.

3.1 Strassen's Algorithm

4 Draw the recursion tree and show a good asymptotic upper bound on the recurrence $T(n) = T(n/2) + T(n/3) + T(n/6) + n$.

4.1 Recursion Tree

4.2 Asymptotic Upper Bound

5 Write the pseudocode for an algorithm that finds a local minimum of an array in $O(\lg n)$ time, and explain the algorithm's run time.

5.1 Local Minimum