

Assignment 2

1.1 Show that $k \lg k = \Theta(n)$ implies $k = \Theta\left(\frac{n}{\lg n}\right)$.

If $f_n = \Theta(g_n)$, then $\Theta(f_n) = g_n$ by symmetric property of big- Θ . We can use this with some algebra to solve this problem:

$$k \lg k = \Theta(n) \implies \Theta(k \lg k) = n$$

$$\lg[n] = \Theta(\lg[k \lg k])$$

$$= \Theta(\lg k + \lg \lg k)$$

$$= \Theta(\lg k)$$

$$n = \Theta(k \lg k)$$

$$\frac{n}{\lg n} = \frac{\Theta(k \lg k)}{\Theta(\lg k)} = \Theta\left(\frac{k \lg k}{\lg k}\right) = \Theta(k)$$

$$\Theta(k) = \frac{n}{\lg n}$$

$$k = \Theta\left(\frac{n}{\lg n}\right)$$