MATH 2418: Linear Algebra

Assignment# 4

Due: Tuesday, 09/20/2022, 11:59pm Term: `Fall 2022`

[First Name] [Last Name] [Net ID]

Recommended Problems (do not turn in): Sec 2.3: 1, 3, 4, 7, 8, 9, 18, 21, 25, 27, 28.
Sec 2.4: 3, 6, 7, 10, 11, 12, 14, 17, 21, 26, 32.

1. Consider the linear system of equations:

$$\begin{cases} x + 6y + 2z = -13 \\ 12x + 6y + 18z = -6 \\ 6x + 37y + 12z = -80 \end{cases}$$

   (a) Write down its augmented matrix.

   (b) Solve the linear system by reducing the coefficient matrix to an upper triangular matrix followed by back substitution.

   (c) Write down each of the elementary matrices E_{ij} used in step(b).

2. Consider the system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} 1 & -3 & -5 \\ 10 & -2 & -48 \\ -3 & 10 & 15 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -3 \\ 50 \\ 12 \end{bmatrix}.$$

(a) Write down each of the elementary matrices that reduce A to an upper triangular matrix U.

(b) Write down the system $U\mathbf{x} = \mathbf{c}$ which is equivalent to $A\mathbf{x} = \mathbf{b}$.

(c) Solve the system $U\mathbf{x} = \mathbf{c}$ for $\mathbf{x}$.

3. Compute the following products:

(a) $\begin{bmatrix} a & 1 & 2 \\ 0 & b & 3 \\ 0 & 0 & c \end{bmatrix} \begin{bmatrix} d & 2 & 3 \\ 0 & e & 1 \\ 0 & 0 & f \end{bmatrix}$

(b) $\begin{bmatrix} a & 0 & 0 \\ 1 & b & 0 \\ 3 & 2 & c \end{bmatrix} \begin{bmatrix} d & 0 & 0 \\ 2 & e & 0 \\ 3 & 1 & f \end{bmatrix}$

(c) $\begin{bmatrix} 4 & 3 & -1 \\ 1 & 3 & 2 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

(d) $\begin{bmatrix} d & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & f \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

4. Let $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 5 \\ 6 & 9 \\ 4 & 3 \end{bmatrix}$. Without calculating the complete matrices AB and BA, compute (if possible) the following:

   (a) The entry $(AB)_{22}$ of AB.

   (b) The entry $(BA)_{22}$ of BA.

   (c) Column 2 of AB.

   (d) Row 3 of BA.

5. Answer the following (you need to show your work).

(a) Give a 2×2 matrix A such that $A^2 = 0$ but $A \neq 0$.

(b) Give a 2×2 matrix B such that $B^2 = I$ but $B \neq \pm I$ (I is the identity matrix).

(c) Write down a pair of 2×2 non-zero matrices A and B such that $AB = 0$ (0 is the 2×2 zero matrix)

(d) Given $A = \begin{bmatrix} 6 & 2 \\ 3 & 4 \end{bmatrix}$ write down the elementary matrices E_1, E_2 and E_3 such that

$$E_1 A = \begin{bmatrix} 3 & 4 \\ 6 & 2 \end{bmatrix}, \; E_2 A = \begin{bmatrix} 6 & 2 \\ -3 & 2 \end{bmatrix} \text{ and } E_3 A = \begin{bmatrix} 3 & 1 \\ 3 & 4 \end{bmatrix}.$$

6. Let $B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 3 & 3 & 0 \end{bmatrix}$.

(a) For any 4×3 matrix A, what is the column 3 of AB? Write down all components in the column 3 of AB, and explain your answer.

(b) For any 3×2 matrix C, what is the row 2 of BC? Write down all components in the row 2 of BC. Explain your answer.

(c) Is it possible to find a 3×3 matrix D such that $DB = I_3$, the 3×3 identity matrix? Explain your answer.

(d) Is it possible to find a 3×3 matrix F such that $BF = I_3$, the 3×3 identity matrix? Explain your answer.