MATH 2418: Linear Algebra

Assignment# 3

Due :Tuesday, 09/13/2020, 11:59pm Term :Fall 2022

[Last Name]	[First Name]	[Net ID]	[Lab Section]

Recommended Problems:(Do not turn in)
Sec 2.1: 1, 2, 9, 10, 16, 17, 19, 21, 26, 29. Sec 2.2: 5, 6, 7, 8, 12, 13, 19, 23.
Note: The answers to these problems are available at: http://math.mit.edu/~gs/linearalgebra/

1. (a) Find the matrix P that multiplies every vector $(x,\ y,\ z) \in \mathbb{R}^3$ to produce the vector $(3x+2y+z,\ 5y-z,\ 8x)$. Also find P^{-1}.

   (b) Find the matrix P that multiplies every vector $(x,\ y) \in \mathbb{R}^2$ to produce $(5x-4y,\ -2x,\ 3y-2x) \in \mathbb{R}^3$.

2. Given linear system $\begin{cases} x - 5y = 2 \\ -x - 3y = -10 \end{cases}$

(a) Write the corresponding matrix equation $A\mathbf{x} = \mathbf{b}$.

(b) Solve the linear system.

(c) Draw the row picture and the column picture.

3. Consider the function $\mathrm{T} : \mathbb{R}^3 \to \mathbb{R}^2$, defined by $T(x, y, z) = (x + 3y, -2x + 5z)$

   (a) Write the matrix for T.

   (b) For the vectors $\mathbf{u} = (1, 5, -2), \mathbf{v} = (2, 7, 4) \in \mathbb{R}^3$, verify that $\mathrm{T}(2\mathbf{u} + 3\mathbf{v}) = 2\mathrm{T}(\mathbf{u}) + 3\mathrm{T}(\mathbf{v})$.

   (c) For the unit vectors $\mathbf{i} = (1, 0, 0), \mathbf{j} = (0, 1, 0), \mathbf{k} = (0, 0, 1)$, write the matrix $[\mathrm{T}] = \begin{bmatrix} \mathrm{T}(\mathbf{i}) & \mathrm{T}(\mathbf{j}) & \mathrm{T}(\mathbf{k}) \end{bmatrix}$ (i.e. write the matrix [T] whose columns are the vectors $\mathrm{T}(\mathbf{i}), \mathrm{T}(\mathbf{j}), \mathrm{T}(\mathbf{k})$)

4. Solve the system $\begin{cases} 2x + y - 2z = 3 \\ x - y - z = 0 \\ x + y + 3z = 10 \end{cases}$ by reducing into upper triangular form and using back substitution. List all multipliers used and circle all pivots.

5. Given linear system $\begin{cases} (3a+1)x + 3y = -3 \\ 4x - 6y = 6 \end{cases}$

(a) For what value(s) of a does the elimination fail (i) temporarily (ii)permanently?

(b) Solve the system after fixing the temporary failure of the elimination.

(c) Also solve the system in case of permanent failure of elimination.

6. Solve the system $\begin{cases} x + z = 6 \\ -3y + z = 7 \\ 2x + y + 3z = 15 \end{cases}$ by reducing into upper triangular form and using back substitution. List all multipliers used and circle all pivots.