

MATH 2418: Linear Algebra

Assignment# 6

Due: Tuesday, 10/11/2022, 11:59pm

Term: Fall 2022

[Last Name]	[First Name]	[Net ID]	[Lab Section]
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Recommended Problems:(Do not turn in)

Sec 2.7: 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 20, 22, 23.

Sec 3.1: 1, 2, 5, 9, 10, 11, 12, 19, 20, 24, 26.

Sec 3.2: 1, 2, 3, 5, 12, 13, 15, 18, 25, 31, 43, 53.

1. Find the LDL^T decomposition of $A = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 7 & 5 \\ 2 & 5 & 8 \end{bmatrix}$.

2. Find a permutation matrix P so that $PA = LU$.

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 6 & 8 \\ 0 & 9 & 0 \end{bmatrix}$$

3. Determine whether the below statements are True or False. Justify your answer.

(a) **T F**: Let A be any square matrix, then $A + A^{\mathbf{T}}$ is symmetric.

(b) **T F**: If A is invertible, then $A^{\mathbf{T}}$ is not invertible.

(c) **T F**: Let $A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$, then $(AB)^T = (A)^T(B)^T$.

(d) **T F**: All 2×2 symmetric matrices is a subspace of M . (Here M is the vector space of all 2×2 matrices.)

4. Determine whether the given set V is a vector space, justify your answer.

(a) $V = \{\mathbf{x} = (x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1x_2 - 2x_3 = 0\}.$

(b) $V = \{\mathbf{x} = (x_1, x_2, x_3) \in \mathbb{R}^3 \mid 9x_1 + 4x_2 - x_3 = 0\}.$

(c) $V = \{\mathbf{x} = (x_1, x_2, x_3) \in \mathbb{R}^3 \mid 3x_1 - x_2 + 7x_3 = 1\}.$

(d) $V = \{\text{set of all } 3 \times 3 \text{ diagonal matrices}\}.$

5. Let $U = \{(u_1, u_2, u_3) \in \mathbb{R}^3 \mid u_1 + u_2 - u_3 = 0\}$ and $V = \{(v_1, v_2, v_3) \in \mathbb{R}^3 \mid v_1 - 3v_2 + 5v_3 = 0\}$.
- (a) Show that U and V are subspaces of \mathbb{R}^3 .
 - (b) Is the set $U \cup V := \{\mathbf{x} \mid \mathbf{x} \in U \text{ or } \mathbf{x} \in V\}$ a subspace of \mathbb{R}^3 ? Justify your answer.
 - (c) Is the set $U \cap V := \{\mathbf{x} \mid \mathbf{x} \in U \text{ and } \mathbf{x} \in V\}$ a subspace of \mathbb{R}^3 ? Justify your answer.

6. Check whether $Ax = b$ is solvable or not i.e., if the vector $b = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$ is in the column space of the matrix

$$A = \begin{bmatrix} 1 & 0 & 5 & 3 & 1 \\ 1 & 4 & 1 & 2 & 5 \\ 2 & 3 & 0 & 1 & 2 \end{bmatrix} ? \text{ Justify your answer.}$$

7. For which condition on $\mathbf{b} = (b_1, b_2, b_3)$ do there exist solution(s) for the linear system $A\mathbf{x} = \mathbf{b}$?

(a) $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 7 & 5 \\ 3 & 6 & 8 & 0 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 2 & 2 & 3 \end{bmatrix}$

8. Given

$$A = \begin{bmatrix} 1 & 2 & 5 & 3 & 4 \\ 3 & 6 & 15 & 9 & 12 \\ 5 & 10 & 25 & 15 & 20 \end{bmatrix}$$

- (a) Compute the reduced row echelon form $\mathbf{rref}(A)$ of the matrix A . What is the rank of the matrix A ?
- (b) Find vectors $\mathbf{u} \in \mathbb{R}^3$ and $\mathbf{v} \in \mathbb{R}^5$ such that $A = \mathbf{u}\mathbf{v}^T$.

9. Construct a matrix B whose column space contains $\begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \\ 2 \\ 0 \end{bmatrix}$ and whose nullspace contains $\begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}$.