MATH 2418: Linear Algebra

Assignment# 7

Due: Tuesday, 10/18/2022, 11:59pm Term: Fall 2022

[Last Name] [First Name] [Net ID] [Lab Section]

Recommended Problems:(Do not turn in)
Sec 3.3: 1, 2, 3, 4, 5, 7, 16, 17, 25, 27, 29
Sec 3.4: 1, 2, 3, 4, 5, 11, 12, 13, 15, 16.

1. Find the complete solution $\mathbf{x} = \mathbf{x}_p + \mathbf{x}_n$ of the system $A\mathbf{x} = \mathbf{b}$, $A = \begin{bmatrix} 1 & 3 & 2 & 2 \\ 2 & 3 & -2 & 10 \\ 4 & 13 & 7 & -3 \\ 3 & 9 & 5 & 3 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 5 \\ 7 \\ 3 \\ 9 \end{bmatrix}$, where $\mathbf{x}_p$ stands for a particular solution and $\mathbf{x}_n$ the general solution of the associated homogeneous system.

2. (a) Construct a matrix whose null space consists of all linear combinations of vectors $(3, 2, 0, 1)$ and $(-2, 5, 1, 0)$.

   (b) Construct a 2 by 3 system $A\mathbf{x} = \mathbf{b}$ with particular solution $\mathbf{x}_p = (3, 2, 0)$ and special solution $\mathbf{x}_n = (2, 5, 1)$.

3. Consider the system $A\mathbf{x} = \mathbf{b}$, where $A = \begin{bmatrix} 1 & 1 & -1 & 2 \\ 1 & 3 & -5 & 7 \\ 7 & 1 & 5 & -1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$.

(a) Find all possible values of $\mathbf{b}$ so that $\operatorname{rank}(A) = \operatorname{rank}[A \mid \mathbf{b}]$.

(b) Determine the values of k so that the rank of matrix $C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ k & 8 & 9 \end{bmatrix}$ is 2.

4. Consider $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 5 \\ 6 \\ 4 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} -3 \\ -7 \\ -1 \end{bmatrix}$

(a) Show that $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ are linearly independent.

(b) Find the dimension of $\mathrm{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$.

(c) Is any vector $(x, y, z) \in \mathbb{R}^3$ in $\mathrm{Span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$? If so, express the vector as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$.

5. Consider the 4×5 matrix $A = [u_1 \mid u_2 \mid u_3 \mid u_4 \mid u_5]$, where the columns are

$$u_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 5 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \quad u_3 = \begin{bmatrix} 2 \\ -1 \\ -2 \\ 3 \end{bmatrix}, \quad u_4 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \quad u_5 = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 5 \end{bmatrix}$$

a) Find a set of vectors in $\{u_1, u_2, u_3, u_4, u_5\}$ which is a basis of the column space of A.

b) Find the rank of A.

6. For each of the following matrices, find all possible real values of $(c, d) \in \mathbb{R}^2$ such that matrices C and D have same rank and then find the rank.

(a) $C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & c & 0 & 0 \\ c & 0 & 0 & 0 \\ 0 & 0 & 0 & c \end{bmatrix}$, $D = \begin{bmatrix} d \\ d \\ d \\ d \end{bmatrix}$.

(b) $C = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & c & 0 & 0 \end{bmatrix}$, $D = \begin{bmatrix} d & 0 \\ 0 & d \end{bmatrix}$.

(c) $C = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & d & 0 & 0 \\ 0 & 0 & 0 & c \end{bmatrix}$, $D = \begin{bmatrix} d & c \\ c & d \end{bmatrix}$.