MATH 2418: Linear Algebra

Assignment# 1

Due : 08/30/2022, Tuesday, before 11:59pm

Term :Fall 2022

[Last Name]	[First Name]	[Net ID]	[Lab Section]

Recommended Problems:(Do not turn in)
Sec 1.1: 1, 2, 3, 8, 9, 10, 11, 12, 13, 14, 15, 18, 26, 27, 28, 31.

1. Let $\mathbf{v} = (2, 3, 1)$, $\mathbf{w} = (1, -1, -1)$, and $3\mathbf{u} + 2\mathbf{v} - 4\mathbf{w} = (1, -2, 3)$. Find

   (a) the vector $\mathbf{u}$

   (b) the linear combination: $2\mathbf{u} - 3\mathbf{v} + 5\mathbf{w}$.

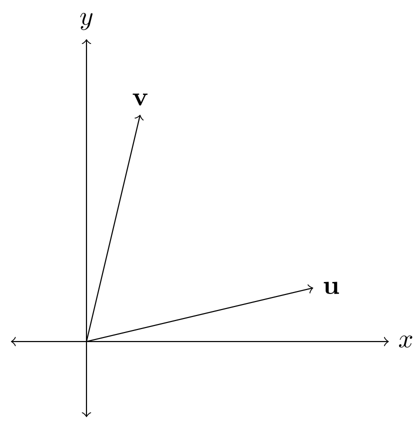
2. Given vectors $\mathbf{u}$ and $\mathbf{v}$ in diagram below, shade in all linear combinations $c\mathbf{u} + d\mathbf{v}$ for

   (a) $0 \leq c \leq 1$ and $0 \leq d \leq 1$

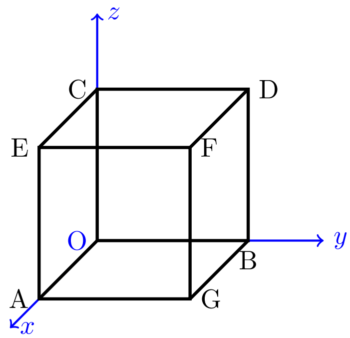
   (b) $0 \leq c \leq 1$ and $d > 1$

   (c) $0 \leq d \leq 1$ and $c > 1$.

   (You can use different shading styles in same picture for all three parts or can graph them separately in three different pictures)



3. Let $\mathbf{0} = (0,0,0)$, $\mathbf{i} = (1,0,0)$, $\mathbf{j} = (0,1,0)$, $\mathbf{k} = (0,0,1)$ be vectors in $\mathbb{R}^3$. Given a cube with an edge 5 inches in the figure below



(a) Write down the vectors $\overrightarrow{OE}$, $\overrightarrow{OD}$, $\overrightarrow{OF}$, $\overrightarrow{OG}$ as linear combinations of $\mathbf{i}, \mathbf{j}, \mathbf{k}$

(b) Let P, Q, R, S, T, U be the centers of the faces $AGFE$, $GBDF$, $DCEF$, $OAEC$, $OAGB$, $OBDC$ respectively, write the vectors: $\overrightarrow{OP}, \overrightarrow{OQ}, \overrightarrow{OR}, \overrightarrow{OS}, \overrightarrow{OT}, \overrightarrow{OU}$ as a linear combinations of $\mathbf{i}, \mathbf{j}, \mathbf{k}$.

4. Let $\mathbf{u} + \mathbf{v} = (3, -3)$ and $\mathbf{u} - \mathbf{v} = (1, 1)$.

   (a) Find $\mathbf{u}$ and $\mathbf{v}$

   (b) Draw the vectors $\mathbf{u},\ \mathbf{v},\ \mathbf{u} + \mathbf{v},\ (\mathbf{u} - \mathbf{v}),\ (-\mathbf{u} + \mathbf{v}),\ (-\mathbf{u} - \mathbf{v})$ in a single $xy-$plane.

5. (a) Determine all real values of p such that the set of all linear combination of $\mathbf{u} = (-3, p)$ and $\mathbf{v} = (2, 3)$ is all of $\mathbb{R}^2$. Justify your answer.

   (b) Determine all real values of p and q such that the set of all linear combinations of $\mathbf{u} = (1, p, -1)$ and $\mathbf{v} = (3, 2, q)$ is a plane in $\mathbb{R}^3$. Justify your answer.

6. Determine whether the set of all linear combinations of the following set of vectors in $\mathbb{R}^3$ is **a line or a plane or all of** $\mathbb{R}^3$. Justify, your answer.

   (a) $\{(-2,5,-3),(6,-15,9),(-10,25,-15)\}$

   (b) $\{(0,0,3),(0,1,2),(1,1,0)\}$

   (c) $\{(1,2,0),(1,1,1),(4,5,3)\}$