MATH 2418: Linear Algebra

Assignment# 5

Due: `Tuesday, 09/27/2022, 11:59pm` Term: `Fall 2022`

[First Name] [Last Name] [Net ID]

Recommended Problems (do not turn in): Sec 2.5: 1, 5, 6, 7, 11, 12, 13, 18, 22, 25, 27, 29, 44. Sec 2.6: 1, 3, 4, 6, 8, 9, 10, 17, 22, 23.

1. Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 4 \\ 5 & 5 & 9 \end{bmatrix}.$$

   (a) Use elementary row operations to reduce A into the identity matrix I.

   (b) List all corresponding elementary matrices.

   (c) Write A^{-1} as a product of elementary matrices.

2. Determine whether the following matrices are invertible. If they are, find the inverses. If not, justify your answer.

$$A_1 = \begin{bmatrix} 2 & 5 \\ 3 & 5 \end{bmatrix}, \; A_2 = \begin{bmatrix} 2 & 5 \\ 0 & 5 \end{bmatrix}, \; A_3 = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}, \; A_4 = \begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix}, \; A_5 = \begin{bmatrix} 3 & 5 \\ 3 & 5 \end{bmatrix}.$$

3. Let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Find the inverse matrix A^{-1}.

4. For which value of x, is

$$\begin{bmatrix} 5 & 3 & 0 \\ 1 & 0 & 1 \\ -3 & -3 & x \end{bmatrix}$$

not invertible?

5. (a) Find the LU decomposition of the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 0 & 2 \\ 6 & 3 & 5 \end{bmatrix}.$$

(b) Find the LDU decomposition of the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 0 & 2 \\ 6 & 3 & 5 \end{bmatrix}.$$

6. Let $\mathbf{b} = \begin{bmatrix} 8 \\ 30 \\ -48 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & -3 & -6 \\ 3 & -11 & -22 \\ -1 & 14 & 32 \end{bmatrix}$. Use the following steps to solve the system $A\mathbf{x} = \mathbf{b}$ by using the LU-decomposition of A.

   (a) Find the LU-decomposition of A, $A = LU$.

   (b) Solve $L\mathbf{y} = \mathbf{b}$ by forward substition.

   (c) Solve $U\mathbf{x} = \mathbf{y}$ by back substition.

   (d) Find the solution of $A\mathbf{x} = \mathbf{b}$.