MATH 2418: Linear Algebra

Assignment# 2

Due :09/06, Tuesday, 11:59pm   Term :`Fall 2022`

[Last Name]   [First Name]   [Net ID]   [Lab Section]

Recommended Problems:(Do not turn in) **Sec 1.2**: 1, 2,5, 6, 7, 8, 12, 13, 19, 29, 31;
Sec 1.3: 1, 2, 3,4, 6, 8.

1. Let $\mathbf{u} = (-1, 2, 4)$ and $\mathbf{v} = (-1, 0, -11)$ be two vectors in $\mathbb{R}^3$.

   (a) Calculate the dot product $\mathbf{u} \cdot \mathbf{v}$. What does it say about the angle between $\mathbf{u}$ and $\mathbf{v}$?

   (b) Compute the lengths $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$ of the vectors.

   (c) Let θ be the angle between $\mathbf{u}$ and $\mathbf{v}$. Find $\cos\theta$, where $0 \leq \theta \leq \pi$.

   (d) Find the unit vector $\hat{\mathbf{u}}$ in the opposite direction of $\mathbf{u}$.

   (e) Find the vector $\hat{\mathbf{v}}$ in the opposite direction of $\mathbf{v}$ and of length 5.

   (f) Find a vector $\mathbf{w}$ parallel to $\mathbf{u}$ that has length 2.

   (g) Find a vector $\mathbf{z}$ in the direction of $\mathbf{v}$ and of length 3.

2. For any geometric vectors $\mathbf{u}$ and $\mathbf{v}$, prove the following using triangle inequality.

   (a) $\|\mathbf{u} - \mathbf{v}\| \leq \|\mathbf{u}\| + \|\mathbf{v}\|$

   (b) $\Big| \|\mathbf{u}\| - \|\mathbf{v}\| \Big| \leq \|\mathbf{u} - \mathbf{v}\|$

3. (a) Let $\mathbf{u}$ and $\mathbf{v}$ be two vectors in $\mathbb{R}^3$ such that $\|\mathbf{u}\| = 3$ and $\|\mathbf{v}\| = 9$.

   (i) Find the maximum and minimum possible values of $\mathbf{u} \cdot \mathbf{v}$.

   (ii) Find the maximum and minimum possible values of $\|\mathbf{u} - \mathbf{v}\|$.

   (b) Let $\mathbf{u} = (1, -3), \mathbf{v} = (2, 4)$ and $\mathbf{w} = (c, d),\ c, d \in \mathbb{R}$, be three vectors in $\mathbb{R}^2$. Find all real values c, d, such that $\mathbf{u}$ and $\mathbf{w}$ are orthogonal, and $\mathbf{v} \cdot \mathbf{w} = 3$.

4. Given a matrix $A = \begin{bmatrix} 3 & 4 & 9 \\ 2 & 1 & 2 \\ 0 & 3 & -1 \\ 5 & -9 & -7 \end{bmatrix}$ and a vector $\mathbf{x} = \begin{bmatrix} -2 \\ 5 \\ 3 \end{bmatrix} \in \mathbb{R}^3$, calculate $A\mathbf{x}$

   (a) as a linear combination of columns of A.

   (b) with entries as dot products of rows of A and $\mathbf{x}$.

5. Let $A = \begin{bmatrix} 2 & -3 & 1 \\ 2 & 7 & -4 \\ 3 & 9 & 9 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$,

(a) write the linear system corresponding to the matrix equation $A\mathbf{x} = \mathbf{b}$.

(b) solve the linear system.

(c) write your answer in the form $\mathbf{x} = A^{-1}\mathbf{b}$. What is A^{-1}?

6. (a) Prove that the vectors $\mathbf{u} = (2, 1, 0)$, $\mathbf{v} = (1, 0, 3)$, $\mathbf{w} = (0, 1, 1)$ are linearly independent.

   (b) Prove that the vectors $\mathbf{u} = (2, 1, 0)$, $\mathbf{v} = (4, 2, 0)$, $\mathbf{w} = (0, 1, 1)$ are linearly dependent.

   (c) Let $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ be three linearly independent vectors, and a, b, c be any three nonzero real numbers. Prove that the vectors $a\mathbf{u}$, $b\mathbf{v}$, $c\mathbf{w}$ are also linearly independent.