#2

ECE-S 681:

Fundamentals of Computer Vision

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1. a) Assume these three textural patterns are 16 × 16 images. Based on previous calculation the co-occurrence matrices are the following:

Image 1: $$[P_1^{(1,0)}] = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 78/240 & 40/240 \\ 40/240 & 82/240 \end{bmatrix} \end{array}$$

$$[P_1^{(0,1)}] = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 78/240 & 40/240 \\ 40/240 & 82/240 \end{bmatrix} \end{array}$$

Image 2: $$[P_2^{(1,0)}] = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 96/240 & 48/240 \\ 48/240 & 48/240 \end{bmatrix} \end{array}$$

$$[P_2^{(0,1)}] = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 120/240 & 24/240 \\ 32/240 & 64/240 \end{bmatrix} \end{array}$$

Image 3: $$[P_3^{(1,0)}] = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 152/240 & 32/240 \\ 32/240 & 24/240 \end{bmatrix} \end{array}$$

$$[P_3^{(0,1)}] = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 152/240 & 32/240 \\ 32/240 & 24/240 \end{bmatrix} \end{array}$$

$P_1^{(1,0)}$, $P_2^{(1,0)}$, and $P_3^{(1,0)}$ are different from each other

also $P_1^{(0,1)}$, $P_2^{(0,1)}$, and $P_3^{(0,1)}$ are different from each other.

these co-occurrence matrices can be used to differentiate between the 3 classes of patterns.

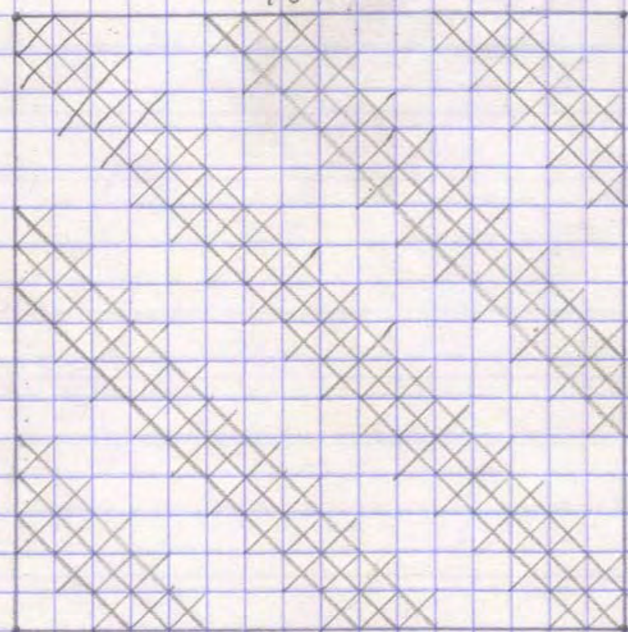
(See page 2-4 for detailed calculation)

□ - White Pixel '0'

╳ - Black Pixel '1'

16

16



0→0	0→1	1→0	1→1
5	2	3	5
4	2	3	6
4	3	2	6
5	3	2	5
6	3	2	4
6	2	3	4
5	2	3	5
4	2	3	6
4	3	2	6
5	3	2	5
6	3	2	4
6	2	3	4
5	2	3	5
4	2	3	6
4	3	2	6
5	3	2	5

0→0	5	4	4	5	6	6	5	4	4	5	6	6	5	4	4	5
0→1	2	2	3	3	2	2	2	3	3	3	3	2	2	2	3	3
1→0	3	3	2	2	3	3	3	2	2	2	2	3	3	3	2	2
1→1	5	6	6	5	4	4	5	6	6	5	4	4	5	6	6	5

Co-occurrence matrix (horizontal)

$$[P^{(1,0)}] = \begin{array}{cc} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 78/240 & 40/240 \\ 40/240 & 82/240 \end{bmatrix} \end{array}$$

Co-occurrence matrix (vertical)

$$[P^{(0,1)}] = \begin{array}{cc} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 78/240 & 40/240 \\ 40/240 & 82/240 \end{bmatrix} \end{array}$$

IMAGE 1.

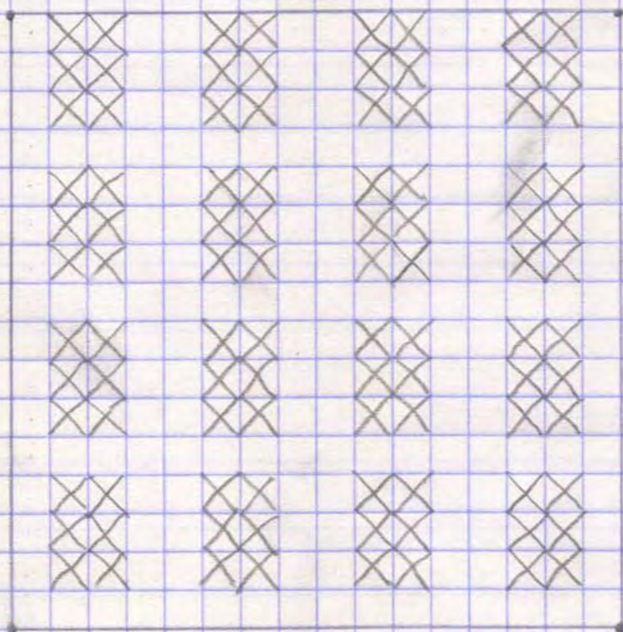
(problem I(a))

□ - White Pixel '0'

X - Black Pixel '1'

16

16



0→0	0→1	1→0	1→1
3	4	4	4
3	4	4	4
3	4	4	4
15	0	0	0
3	4	4	4
3	4	4	4
3	4	4	4
15	0	0	0
3	4	4	4
3	4	4	4
3	4	4	4
15	0	0	0
3	4	4	4
3	4	4	4
3	4	4	4
15	0	0	0

0→0	15	0	0	15	15	0	0	15	15	0	0	15	15	0	0	15
0→1	0	3	3	0	0	3	3	0	0	3	3	0	0	3	3	0
1→0	0	4	4	0	0	4	4	0	0	4	4	0	0	4	4	0
1→1	0	8	8	0	0	8	8	0	0	8	8	0	0	8	8	0

Co-occurence matrix (horizontal)

$$\left[P^{(1,0)}\right] = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 96/240 & 48/240 \\ 48/240 & 48/240 \end{bmatrix} \end{array}$$

Co-occurence matrix (vertical)

$$\left[P^{(0,1)}\right] = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 120/240 & 24/240 \\ 32/240 & 64/240 \end{bmatrix} \end{array}$$

IMAGE 2

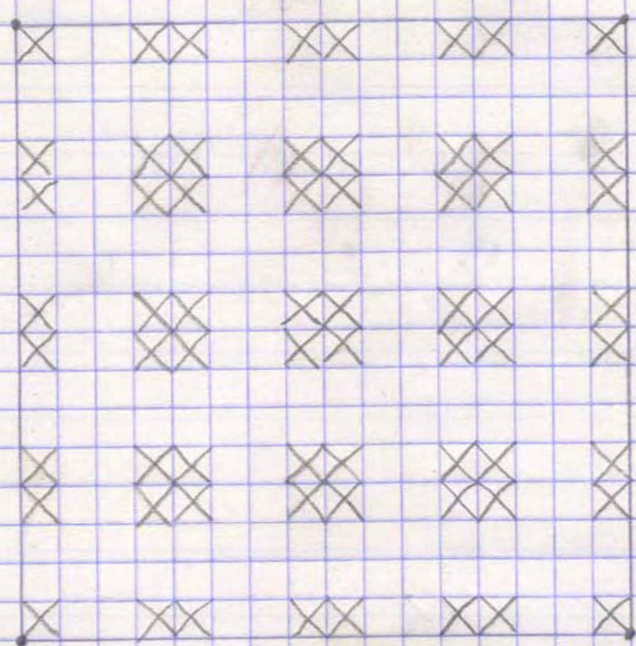
(problem 1(a))

□ - White Pixel. '0'

X - Black Pixel. '1'

16

16



0→0	0→1	1→0	1→1
4	4	4	3
15	0	0	0
15	0	0	0
4	4	4	3
4	4	4	3
15	0	0	0
15	0	0	0
4	4	4	3
4	4	4	3
15	0	0	0
15	0	0	0
4	4	4	3
4	4	4	3
15	0	0	0
15	0	0	0
4	4	4	3

0→0	4	15	15	4	4	15	15	4	4	15	15	4	4	15	15	4
0→1	4	0	0	4	4	0	0	4	4	0	0	4	4	0	0	4
1→0	4	0	0	4	4	0	0	4	4	0	0	4	4	0	0	4
1→1	3	0	0	3	3	0	0	3	3	0	0	3	3	0	0	3

Co-occurrence matrix (horizontal)

$$\left[P^{(1,0)}\right] = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} 152/240 & 32/240 \\ 32/240 & 24/240 \end{pmatrix} \end{array}$$

Co-occurrence matrix (vertical)

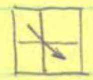
$$\left[P^{(0,1)}\right] = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} 152/240 & 32/240 \\ 32/240 & 24/240 \end{pmatrix} \end{array}$$

IMAGE 3

(problem 1(a))

1. b). For Image 1. co-occurrence matrix $P^{(1,1)}$ 

$$\left[P^{(1,1)}\right] = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 110/225 & 0 \\ 0 & 115/225 \end{bmatrix} \end{array}$$

For Image 2. co-occurrence matrix $P^{(1,1)}$ 

$$\left[P^{(1,1)}\right] = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 69/225 & 56/225 \\ 64/225 & 36/225 \end{bmatrix} \end{array}$$

For Image 3. co-occurrence matrix $P^{(1,1)}$ 

$$\left[P^{(1,1)}\right] = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 136/225 & 40/225 \\ 40/225 & 9/225 \end{bmatrix} \end{array}$$

By using $\{P^{(1,1)}\}$ listed above along with $\{P^{(0,1)}\}$, $\{P^{(1,0)}\}$ in (a), we can have independence to integer multiple of 45° rotation in the pattern.

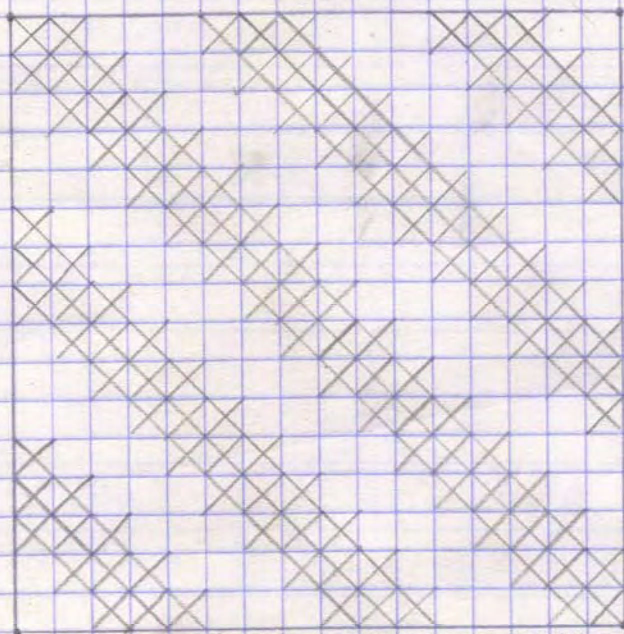
(see page 6-8 for detailed calculation)

□ - White Pixel '0'

X - Black Pixel '1'

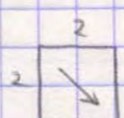
16

16



	0→0	0→1	1→0	1→1
1	0	0	0	0
2	1	0	0	0
3	0	0	0	2
4	0	0	0	3
5	0	0	0	4
6	5	0	0	0
7	6	0	0	0
8	7	0	0	0
9	0	0	0	8
10	0	0	0	9
11	0	0	0	10
12	11	0	0	0
13	12	0	0	0
14	13	0	0	0
15	0	0	0	14
16	0	0	0	15

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	1	0	0	0	5	6	7	0	0	0	11	12	13	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	2	3	4	0	0	0	8	9	10	0	0	0	14



Co-occurrence matrix (45° diagonal)

$$\left\{ P^{(1,1)} \right\} = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 110/225 & 0 \\ 0 & 115/225 \end{bmatrix} \end{array}$$

IMAGE 1

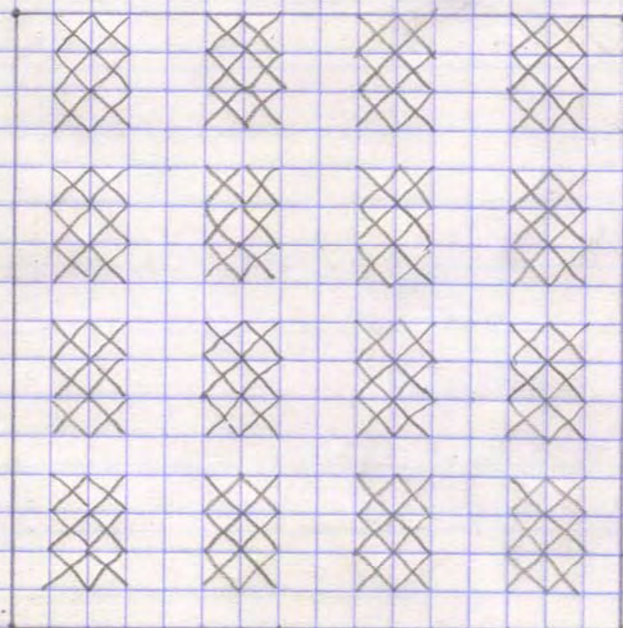
(problem 1(b))

□ - White Pixel '0'

X - Black Pixel '1'

16

16

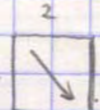


0→0	0→1	1→0	1→1
0	0	0	0
0	0	1	0
0	0	1	1
0	1	1	1
2	1	1	0
1	1	2	0
1	1	2	2
1	2	2	2
4	2	2	0
4	2	3	0
2	2	3	3
2	3	3	3
4	3	3	0
6	3	4	0
3	3	4	4
3	4	4	4

0	1	0	0	1	3	2	2	2	5	3	2	3	7	6
0	0	1	1	1	1	2	2	2	2	3	3	3	3	4
0	0	1	1	1	1	2	2	2	2	3	3	3	3	4
0	0	0	1	1	0	0	2	2	0	0	3	3	0	4

2

2



Co-occurrence matrix (45° diagonal)

$$\left\{P^{(1,1)}\right\} = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 69/225 & 56/225 \\ 64/225 & 36/225 \end{bmatrix} \end{array}$$

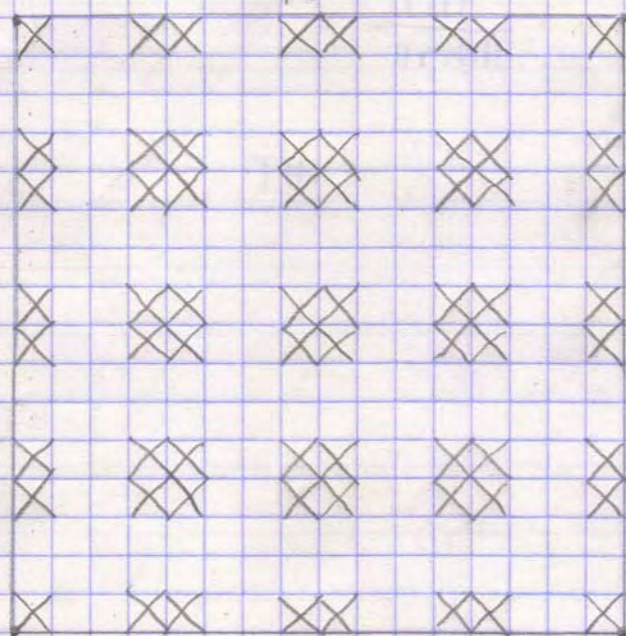
IMAGE 2

(problem 1(b))

□ - White Pixel '0'

X - Black Pixel '1'

16

16 

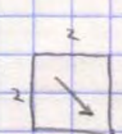
0→0	0→1	1→0	1→1
0	0	0	0
1	0	0	0
2	0	0	0
1	1	1	0
2	1	1	0
5	0	0	0
4	1	1	0
2	2	2	1
4	2	2	0
9	0	0	0
6	2	2	0
3	3	3	2
6	3	3	0
13	0	0	0
8	3	3	0
4	4	4	3

0 1 2 1 2 5 4 2 4 9 6 3 6 13 8

0 0 0 1 1 0 1 2 2 0 2 3 3 0 3

0 0 0 1 1 0 1 2 2 0 2 3 3 0 3

0 0 0 0 0 0 0 1 0 0 0 2 0 0 0



Co-occurrence matrix (45° diagonal)

$$\left[P^{(1,1)}\right] = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 136/225 & 40/225 \\ 40/225 & 9/225 \end{bmatrix} \end{array}$$

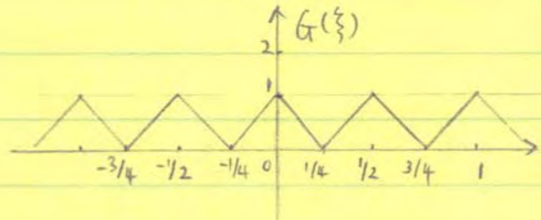
IMAGE 3

(problem 1(b))

2. (a)

$$G(\xi) = f_s \sum_{k=-\infty}^{+\infty} G(\xi - kf_s)$$

$f_s = 0.5$ Hz.



(b) $h(x) = g(2x)$.

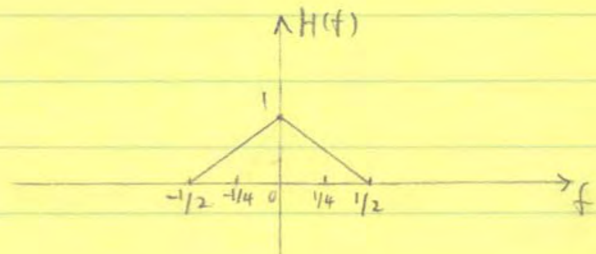
$$H(f) = \int_{-\infty}^{+\infty} g(2x) e^{-j2\pi f x} dx$$

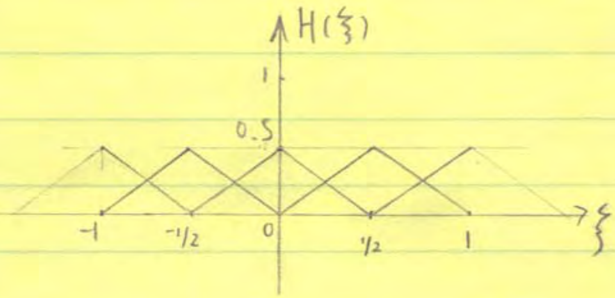
let $u = 2x$.

$$H(f) = \int_{-\infty}^{+\infty} g(u) e^{-j2\pi f \frac{1}{2} u} d\tfrac{1}{2}u$$

$$= \frac{1}{2} \int_{-\infty}^{+\infty} g(u) e^{-j2\pi \frac{1}{2} f u} du$$

$$= \frac{1}{2} G(\tfrac{1}{2} f)$$





(c) $H(\xi) = f_s \sum_{k=-\infty}^{+\infty} H(\xi - kf_s)$, $f_s = 0.5$ Hz

$$= f_s \sum_{k=-\infty}^{+\infty} \frac{1}{2} G(\tfrac{1}{2}\xi - kf_s)$$

(d) If the scaling factor is an unknown constant a

then, $H(\xi) = f_s \sum_{k=-\infty}^{+\infty} \frac{1}{|a|} G(\frac{1}{a}\xi - kf_s)$

$$H(\xi) = f_s \frac{1}{|a|} \sum_{k=-\infty}^{+\infty} G(\tfrac{1}{a}\xi - kf_s) \qquad ①$$

In equation ①, f_s is known (A/D converter sampling rate)

$G(\xi)$ is known, and also $H(\xi)$ is known,

so a can be estimated by evaluating the characteristics of $H(\xi)$ and $G(\xi)$

for example in the last plot the amplitude of $H(\xi)$ is 0.5. and the amplitude (max) of $G(f)$ is 2. so $a = 2$.

3 (a) Find the eigenvalues and eigenvectors of $[R_r]$

$$\det[R_r - \lambda I] = 0$$

$$\det\begin{pmatrix} 2-\lambda & 1 \\ 1 & 1-\lambda \end{pmatrix} = 0.$$

$$(2-\lambda)(1-\lambda) - 1 = 0.$$

$$2 - \lambda - 2\lambda + \lambda^2 - 1 = 0.$$

$$\lambda^2 - 3\lambda + 1 = 0.$$

$$\begin{cases} \lambda_1 = \dfrac{3+\sqrt{9-4}}{2} = \dfrac{3+\sqrt{5}}{2} = 2.618. \\ \lambda_2 = \dfrac{3-\sqrt{9-4}}{2} = \dfrac{3-\sqrt{5}}{2} = 0.382 \end{cases}$$

$$[R_r]\cdot\begin{pmatrix} a \\ a' \end{pmatrix} = \lambda_1\begin{pmatrix} a \\ a' \end{pmatrix} \quad \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} a \\ a' \end{pmatrix} = \lambda_1\begin{pmatrix} a \\ a' \end{pmatrix} \quad \begin{cases} a = 0.5257 \\ a' = -0.8507 \end{cases}$$

$$[R_r]\cdot\begin{pmatrix} b \\ b' \end{pmatrix} = \lambda_2\begin{pmatrix} b \\ b' \end{pmatrix} \quad \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} b \\ b' \end{pmatrix} = \lambda_2\begin{pmatrix} b \\ b' \end{pmatrix} \quad \begin{cases} b = -0.8507 \\ b' = -0.5257 \end{cases}$$

$\begin{pmatrix} a \\ a' \end{pmatrix}$ and $\begin{pmatrix} b \\ b' \end{pmatrix}$ are eigenvectors associated with λ_1 and λ_2

$$\therefore\ [\Phi] = \begin{pmatrix} 0.5257 & -0.8507 \\ -0.8507 & -0.5257 \end{pmatrix}$$

(b) $$[R_u] = \Phi \cdot R_r \cdot \Phi^T = \begin{pmatrix} 0.3820 & 0 \\ 0 & 2.6180 \end{pmatrix}$$

(c) let $W = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ be the matrix for 2-point DFT.

Before KL:

$$S_r = W \cdot R_r \cdot W^T = \begin{pmatrix} 2.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$$

After KL:

$$S_u = W \cdot R_u \cdot W^T = \begin{pmatrix} 1.5 & -1.1180 \\ -1.1180 & 1.5 \end{pmatrix}$$

(d) $\{R_v\} = A \cdot R_r A^T = \begin{bmatrix} 2.6160 & 0.0670 \\ 0.0670 & 0.3840 \end{bmatrix}$

(e) Before A:

$$S_r = W \cdot R_r \cdot W^T = \begin{bmatrix} 2.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$$

After A:

$$S_v = W \cdot R_v W^T = \begin{bmatrix} 1.5670 & 1.1160 \\ 1.1160 & 1.4330 \end{bmatrix}$$

(f) The PSD matrix S_r, S_u, and S_v were calculated in (c), (d), and (e)

$$S_r = \begin{bmatrix} 2.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$$
$$S_u = \begin{bmatrix} 1.5 & -1.1180 \\ -1.1180 & 1.5 \end{bmatrix}$$
$$S_v = \begin{bmatrix} 1.5670 & 1.1160 \\ 1.1160 & 1.4330 \end{bmatrix}$$