Project Report

ECE-S 681:

Fundamentals of Computer Vision

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1. Introduction

9 prototype shapes and 4 samples shapes are give below. (Fig. 1 shows the prototypes. Fig. 2 shows the test objects.) The goal of the project is to come up with a robust shape classifier that will successfully classify the sample objects.

In this project, a classification metric criterion is established. It will be tested against the following assumed transformations:

(i) Similarity transformation;
(ii) Affine transformation;
(iii) Cubic polynomial transformation;

The class alignment will be tested against the prototype for each of the transformation cases. Specifically, test object 1 is prototype 6 after a similarity transformation (rotation, scale, and translation), test object 2 is prototype 9 after an affine transformation (rotation, scale, translation, and shear), test object 3 is prototype 7 after a cubic polynomial transformation and test object 4 is prototype 9 after an affine transformation with occlusion.

The classifier proposed in this project has four parts.

(i) The B-Spline curves were fitted on both prototypes and test objects.

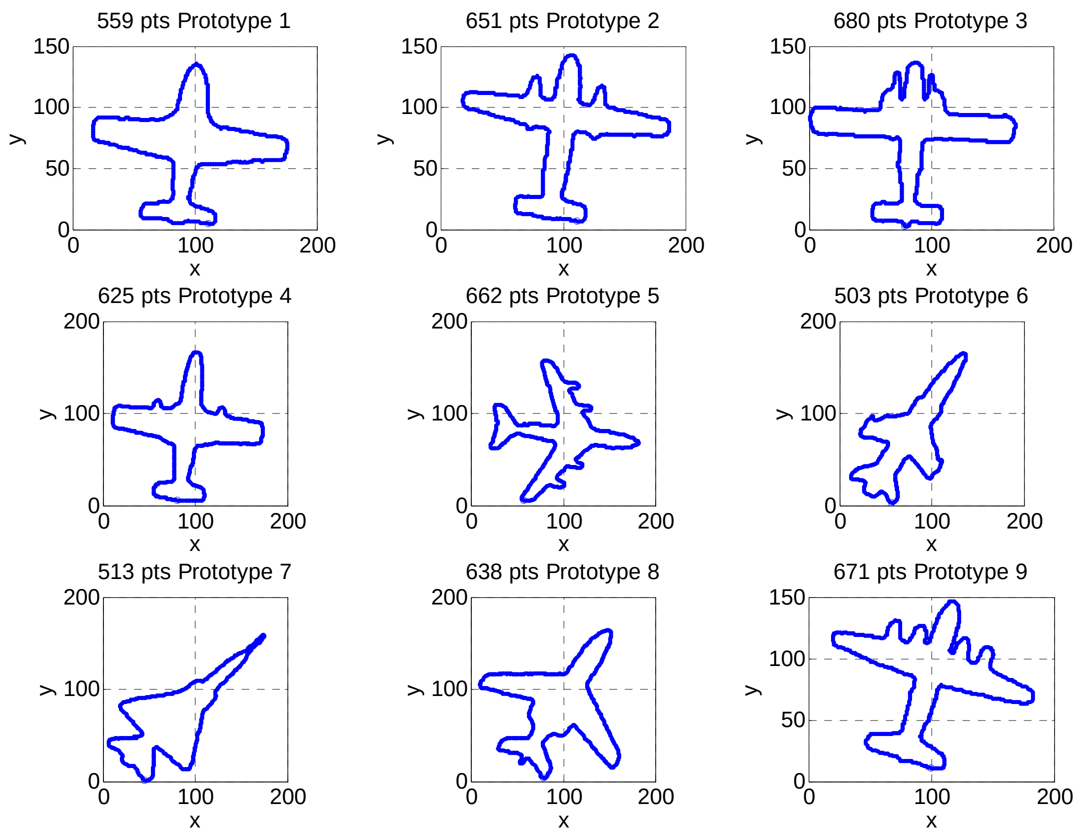

Fig. 1. Prototypes

(ii) Infection points are calculated on these fitted curves.
(iii) The absolute invariants are discovered.
(iv) The transformations are recovered based on these absolute invariants.

In the end, the test objects are aligned with each of the prototypes and the errors were calculated based on ‘closest-point’ manner.

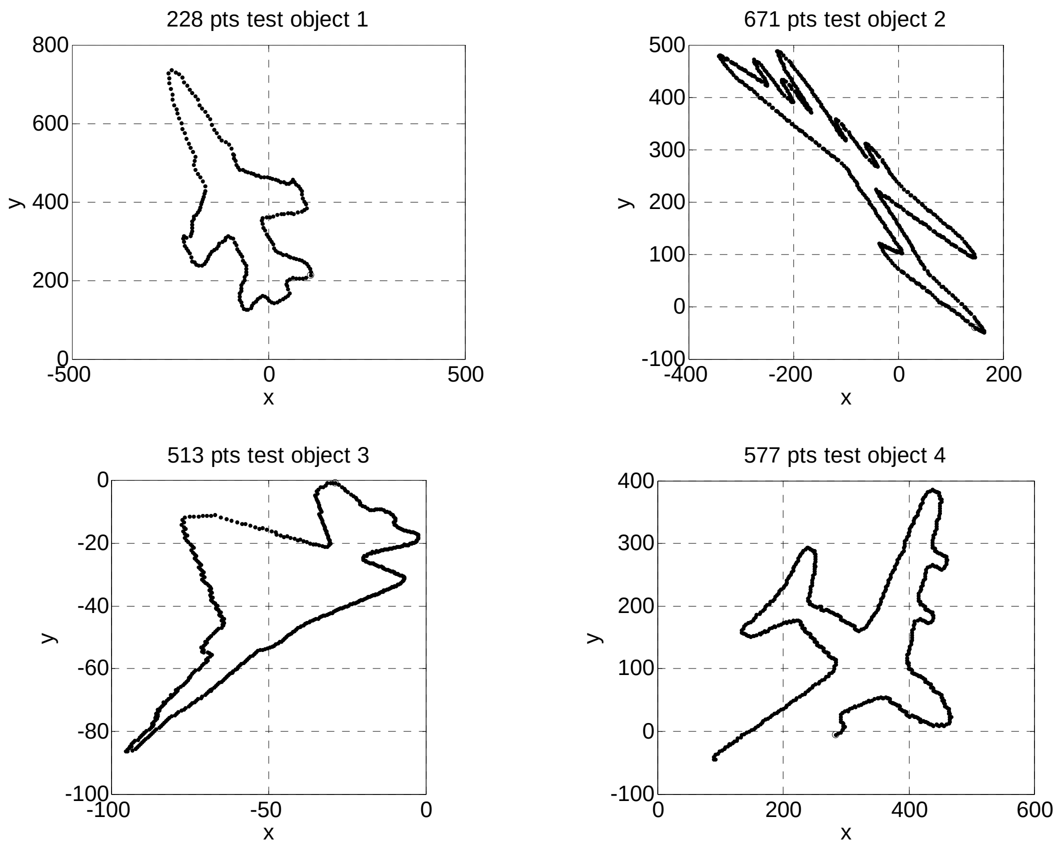

Fig. 2. Test Objects

Fig. 1 shows the prototypes. Fig. 2 shows the test objects. Each plot also has its number of data points indicated in its title. Generally speaking, the points in each object are approximately 500 to 600.

2. B-Spline Curve Fitting

2.1 Introduction

The B-splines are piecewise polynomial functions that provide local approximations to contours/surfaces using a small number of parameters (control points). A k^{th} order B-spline is C^{k-1} continuous, i.e., is continuous and has its (k - 1) derivatives continuous. A k^{th} order B-spline with n+1 parameters $C_0, C_1, \ldots, C_n$, (control points) consists of n-k+1 connected curve segments $r_i(t) = [x_i(t), y_i(t)]$, each of which is a linear combination of (k+1) polynomials of order k in the parameter t, where t is normalized for each such segment between 0 and 1 ($0 \leq t \leq 1$). The parameter t could be thought of as time and the curve could be thought of as the trajectory of a particle moving in two-dimensional (2-D) or three-dimensional (3-D) space with a speed of $\|dr_i(t)/dt\|$. For example, a cubic B-spline with n + 1 control points $C_0, C_1, \ldots, C_n$, consists of n – 2 connected curve segments $r_i(t)= x_i(t), y_i(t)]$, i = 1, . . . , n – 2, each of which is a linear combination of four cubic polynomials in the parameter t. The whole curve $r(t')(0 \leq t \leq 1)$ consists of a concatenation of curve segments $r_i(t)$ that are continuously joint (the joining points are called the knot points) and is expressed as

$$r(t') = \sum_{i=1}^{n-2} r_i(t' - i + 1) = \sum_{N=0}^{n} C_N Q_{N,4}(t') \qquad (2.1)$$

$r_i(t' - i + 1)$ is only nonzero for $i - 1 \leq t' \leq i$. $Q_{N,4}(t')$ ($Q_{N,k+1}(t')$ for a k^{th} order B-spline) are called normalized cubic B-spline bases, and are related to each others by horizontal translation. The knot points are points on the curve that assume the value t' = 0, 1, 2 .., n - 2 (the B-spline knots). There exists a one-to-one relationship between the control points and the knot points.

2.2 Parameterization

Chord Length (CL) and Inverse Chord Length (ICL) methods

CL

$$t'_j = t'_{j-1} + \|r_j - r_{j-1}\| \cdot \frac{t'_{max}}{\sum_{j=2}^{m} \|r_j - r_{j-1}\|} \qquad (2.2)$$

$$j = 2, 3, \ldots, m$$

ICL

$$t'_j = t'_{j-1} + \frac{t'_{max}}{\sqrt[\alpha]{\|r_j - r_{j-1}\|}} \cdot \frac{1}{\sum_{j=2}^{m} \frac{1}{\sqrt[\alpha]{\|r_j - r_{j-1}\|}}} \qquad (2.3)$$

$$j = 2, 3, \ldots, m$$

In this project, ICL is used on test object 1 since it was non-uniform sampling. All the other test objects and prototypes are parameterized by CL.

2.3 Control Points Estimation (Globule Approximation)

Least square approximation (matrix inverse)

$$C_r^* = \left[[N]^T \cdot [N]\right]^{-1} \cdot [N]^T \cdot r \qquad (2.4)$$

r is the curve points

N is the basis function matrix.

$$[N] = \begin{bmatrix} N_{0,4}(t'_1) + N_{n+1,4}(t'_1) & N_{0,4}(t'_2) + N_{n+1,4}(t'_2) & \cdots & N_{0,4}(t'_m) + N_{n+1,4}(t'_m) \\ N_{1,4}(t'_1) + N_{n+2,4}(t'_1) & N_{1,4}(t'_2) + N_{n+2,4}(t'_2) & \cdots & N_{1,4}(t'_m) + N_{n+2,4}(t'_m) \\ N_{2,4}(t'_1) + N_{n+3,4}(t'_1) & N_{2,4}(t'_2) + N_{n+3,4}(t'_2) & \ddots & N_{2,4}(t'_m) + N_{n+3,4}(t'_m) \\ \vdots & \vdots & \cdots & \vdots \\ N_{n+3,4}(t'_1) & N_{n+3,4}(t'_2) & \cdots & N_{n+3,4}(t'_m) \end{bmatrix}$$

m is the number of data points, n+1 is the number of control points.

2.4 Synthesized (Fitted) Curve

$$r' = N' \cdot C_r^* \qquad (2.5)$$

N' is the basis function matrix based on a new set of parameter t.

r is the synthesized curve points

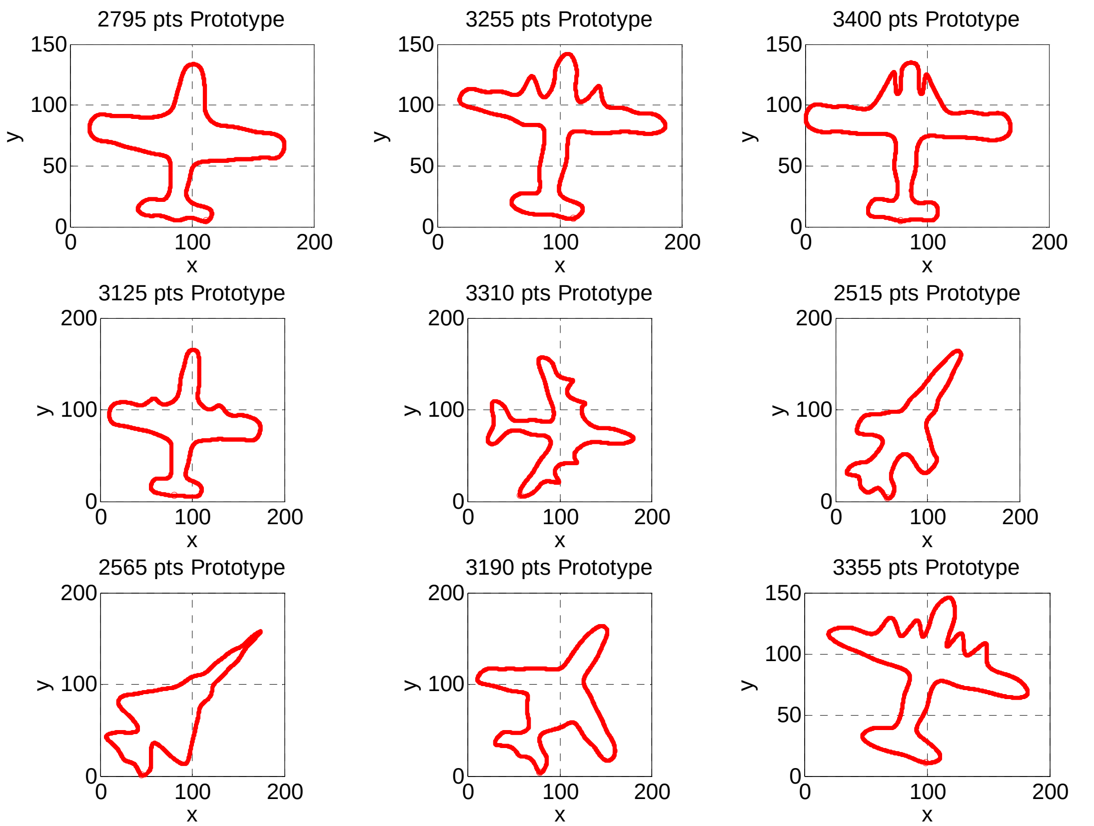

Fig. 3. Synthesized (Fitted) B-Spline Curve for Prototypes

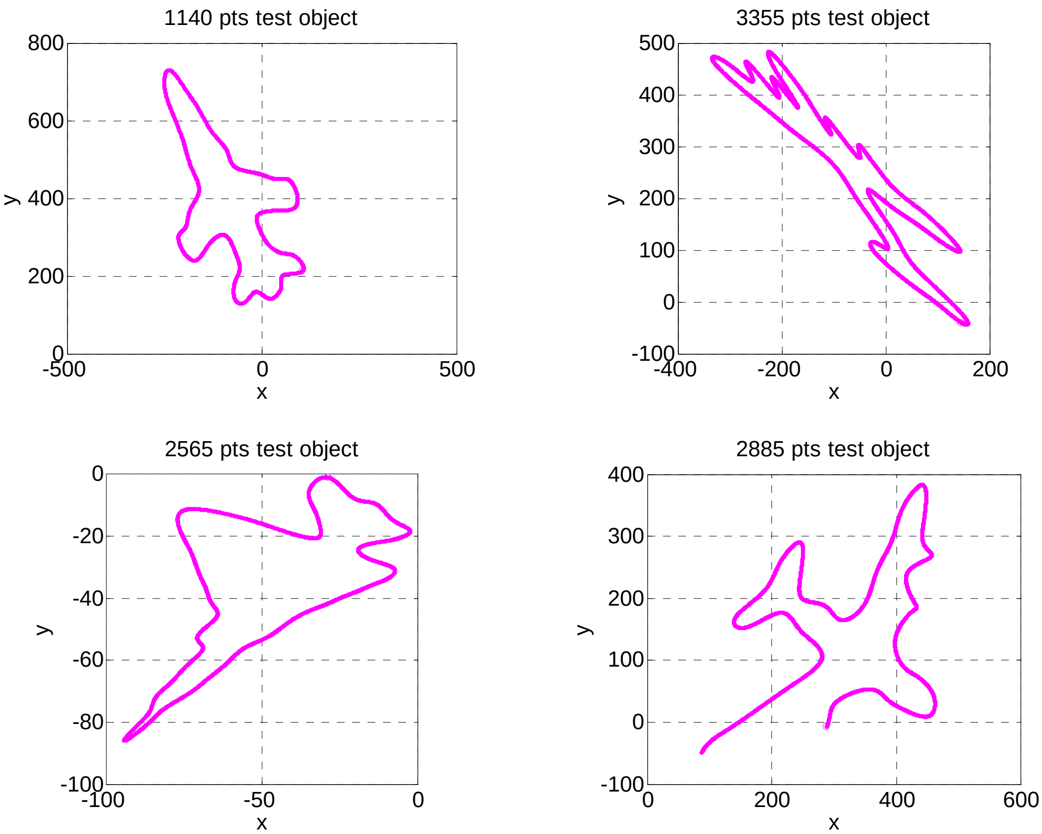

Fig. 4. Synthesized (Fitted) B-Spline Curve for Test Objects

Fig. 3 shows the synthesized B-Spline curve for prototypes. Fig. 4 shows the synthesized B-Spline curve for test objects. All the curves are created in much higher resolution comparing to their raw data (Fig. 1 and Fig. 2). Also, the parameter t for each curve is uniformly spaced, which brings all the curves on a common ground. This makes the following match and alignment processes more accurate.

3. Inflection Points

3.1 Definition

Inflection points are points on the curve at which the curvature k(t) is zero.

The curvature at a given point on curve is

$$k(t) = \frac{\left|r^{(1)}(t) \times r^{(2)}(t)\right|}{\left|r^{(1)}(t)\right|^3} = \frac{\dot{x}(t)\ddot{y}(t) - \ddot{x}(t)\dot{y}(t)}{(\dot{x}(t)^2 - \dot{y}(t)^2)^{3/2}} \qquad (3.1)$$

the inflection points are points which satisfy

$$\left|r^{(1)}(t) \times \dot{r}^{(2)}(t)\right| = x(t)\ddot{y}(t) - \ddot{x}(t)\dot{y}(t) = 0 \qquad (3.2)$$

The inflection points are relative invariants. Therefore, the inflection points of the affine transformed curve are the transformed inflection points of the original curve.

3.2 Inflection Points on Prototype Curves

Inflection points are points on the curve at which the curvature k(t) is zero.

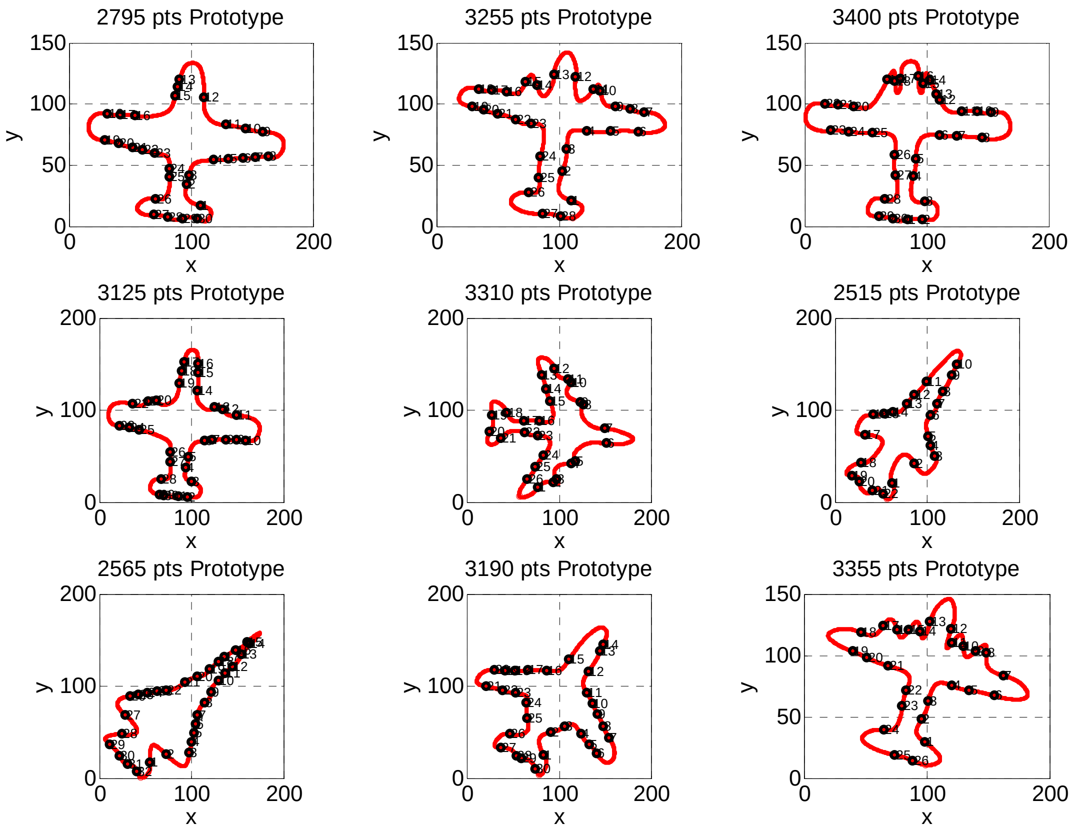

Fig. 5. Inflection Points on Prototypes

3.3 Inflection Points on Test Objects

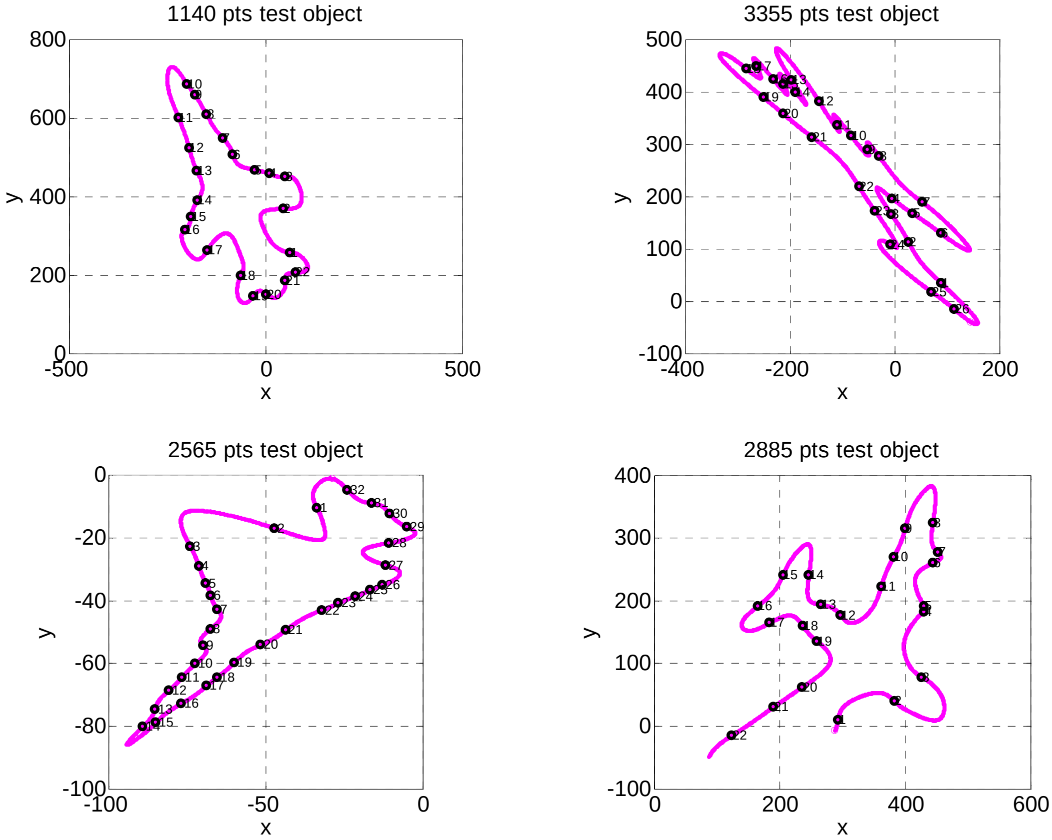

Fig. 6. Inflection Points on Test Objects

Fig. 5 shows the inflection points on prototypes. Fig. 6 shows the inflection points on the test objects. All these inflection points are calculated based on the synthesized B-Spline Curve. As stated in sections 3.2, the inflection points are relative invariants. In the next step, the absolute invariants will be established based on these points.

4. Absolute Invariants

Given a set of fiducial points, from a set of triangle areas by picking up the consecutive three fiducial points, the absolute invariants are the ratio of the neighboring triangles.

$$I = \left\{ I(k) = \frac{\pm A(k)}{\left|A\big((k+1)\ mod\ n\big)\right|}, \qquad k = 1, 2, 3, \ldots, n \right\} \tag{4.1}$$

For transformed curve

$$I_a = \left\{ I_a(k) = \frac{\pm A_a(k)}{\left|A_a\big((k+1)\ mod\ n\big)\right|}, \qquad k = 1, 2, 3, \ldots, n \right\} \tag{4.2}$$

A_a and A satisfy

$$A_a(i) = det\{[L]\}A(k), \qquad k = 1, 2, 3, \ldots, n \tag{4.3}$$

From these three equations, we have

$$I_a = I \tag{4.4}$$

The absolute invariants above can be used to compare how similar or dissimilar two curves are in the presence of an affine transformation, noise, and deformation. In the absence of occlusion or overlap, local deformation or noise, the number of absolute invariants associated with the affine transformed curve will be same as the original curve. Each invariant will have a counterpart with, with that counterpart easily determined through a circular shift involving comparisons, where is the number of invariants.

5. Recovering the Transformations

5.1 Similarity and Affine Transformation

The affine transformation T = {[A], b} can be recovered from a pair of matched triplet, or estimated from more matched vertices by using a least square error (LSE) estimation method. If there are h correspondent vertex, say $\{(x_1, y_1), \ldots, (x_h, y_h)\}$ and $\{(x_{a1}, y_{a1}), \ldots, (x_{ah}, y_{ah})\}$, where $h > 3$, the affine transformation can be obtained as the solution of the LSE estimation which minimized the LSE measure

$$\sum_{i=1}^{h} \|x_{ai} - [A]x_i - b\|^2 \qquad (5.1)$$

with respect to the unknown parameters (a_{11}, a_{12}, a_{21}, a_{22}, b_1, b_2). The LSE solution is given by

$$\begin{bmatrix} a_{11} \\ a_{12} \\ b_1 \end{bmatrix} = \begin{bmatrix} xx^t & xy^t & \bar{x} \\ xy^t & yy^t & \bar{y} \\ \bar{x} & \bar{y} & h \end{bmatrix}^{-1} \cdot \begin{bmatrix} xx_a^t \\ yx_a^t \\ \bar{x}_a \end{bmatrix} \qquad (5.2)$$

$$\begin{bmatrix} a_{21} \\ a_{22} \\ b_2 \end{bmatrix} = \begin{bmatrix} xx^t & xy^t & \bar{x} \\ xy^t & yy^t & \bar{y} \\ \bar{x} & \bar{y} & h \end{bmatrix}^{-1} \cdot \begin{bmatrix} xy_a^t \\ yy_a^t \\ \bar{y}_a \end{bmatrix} \qquad (5.3)$$

$$x = (x_1, \ldots, x_h), \qquad y = (y_1, \ldots, y_h), \qquad \bar{x} = \sum_{i=1}^{h} x_i, \qquad \bar{y} = \sum_{i=1}^{h} y_i$$

$$x_a = (x_{a1}, \ldots, x_{ah}), \qquad y_a = (y_{a1}, \ldots, y_{ah}), \qquad \overline{x_a} = \sum_{i=1}^{h} x_{ai}, \qquad \overline{y_a} = \sum_{i=1}^{h} y_{ai}$$

5.2 Cubic Polynomial Transformation

A cubic polynomial transformation for N surface data (2D) points is defined as

$$S_a = \mathrm{S} \cdot \mathrm{A} \qquad (5.4)$$

where

$$S_a = \begin{bmatrix} \mathrm{x_{a1}} & \mathrm{y_{a1}} \\ \vdots & \vdots \\ \mathrm{x_{aN}} & \mathrm{y_{aN}} \end{bmatrix} \qquad (5.5)$$

$$S = \begin{bmatrix} \mathrm{y_1^3} & \mathrm{x_1^3} & \mathrm{y_1^2x_1} & \mathrm{x_1^2y_1} & \mathrm{y_1^2} & \mathrm{x_1^2} & \mathrm{x_1y_1} & \mathrm{y_1} & \mathrm{x_1} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathrm{x_{aN}} & \mathrm{y_{aN}} & \mathrm{y_N^2x_N} & \mathrm{x_N^2y_N} & \mathrm{y_N^2} & \mathrm{x_N^2} & \mathrm{x_Ny_N} & \mathrm{y_N} & \mathrm{x_N} & 1 \end{bmatrix} \qquad (5.6)$$

and

$$A = \begin{bmatrix} \mathrm{a_{1,1}} & \mathrm{a_{1,2}} \\ \mathrm{a_{2,1}} & \mathrm{a_{2,2}} \\ \vdots & \vdots \\ \mathrm{a_{9,1}} & \mathrm{a_{9,1}} \end{bmatrix} \qquad (5.7)$$

The transformation matrix A can be recovered from 2 pair of 10 matched landmarks, or estimated from using LSE sense if the number of matched landmarks is greater than 20. The LSE solution of A is given as.

$$A = (S^T S)^{-1} S^T S_a \qquad (5.8)$$

The above equation is the well-known normal equation. Note that if all of the components from row 1 to row 5 in the transformation matrix A are zero, the polynomial transformation becomes an affine transformation.

6. Classifier Test Results

The test procedures are as following:

Given a prototype and a test object,

(1) Fit B-Spline curves to both the prototype (output: r) and the test object (output: r').
(2) Find the inflection points on r and r' using (3.1) or (3.2).
(3) Establish the absolute invariant on both curves using (4.1) ~ (4.4).
(4) Find the matching points using circular shift.
(5) Compute A and b using (5.2) and (5.3) or (5.8).
(6) Apply transformation A and b on the B-Spline prototype curve (r). The transformed curve is r''. Calculate the average error between r'' and r'. The error is defined as the distance between corresponding points on r'' and r', if r'' and r' has the same number of points. If r' and r'' has different number of points, the error is defined as the closest point on r'' to every point on r'.
(7) Repeat (1) ~ (6) for each test object. Test it against every prototype. The average error is shown in Table 1.

Table 1 Average Error of Test Planes to Prototype Planes

	Pro 1	Pro 2	Pro 3	Pro 4	Pro 5	Pro 6	Pro 7	Pro 8	Pro 9
Test 1	32.02	29.37	24.00	28.75	26.76	**1.94**	53.27	42.27	23.63
Test 2	17.65	11.15	21.91	24.49	14.44	15.60	28.79	19.38	**0.00**
Test 3	2.90	3.62	3.64	4.44	4.04	4.35	**0.20**	3.51	4.33
Test 4	27.49	56.22	34.79	28.89	**2.51**	31.40	35.08	39.77	16.53

In Table 1, note that the shaded frame indicates the prototype curve that the test curve should belong to, whereas the red number indicates the prototype curve that the test curve is classified to. If the shaded frame and the red number coincide, the classification is correct; otherwise it is wrong.

The classifier correctly classifies all four cases and the error margins between the right and the wrong matches are significant.