Lakehead University

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COMPUTATIONAL METHODS AND MODELING IN MECHANICAL ENGINEERING PROJECT 2

As a submission to

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1.0 Introduction

1.1 Project Definition

Students are required to develop a mathematical model to determine the heat distribution in a longitudinal rib on a plate. The temperature distribution in the plate can be determined by assuming two dimensional steady state conditions. The heat distribution will be calculated based on equally spaced nodes on the rib. Students will be required to complete the mathematical model and simulate the results using computational analysis and programming skill, using matlab to plot the temperature distribution of the rib.

1.2 Project Purpose

The purpose of this project is to allow students the opportunity to practice the skills and computational techniques learned in class in order to solve a developed system of equations based on a provided complex heat transfer problem. This will allow the students to obtain a better understanding of the material required to analyze and solve the problem, based on the knowledge acquired during the course.

2.0 Methodology

2.1 Process

The first step in solving this complex heat transfer problem using numerical methods was to identify what was required in order to analyze the problem. In order to solve the problem using numerical methods a system of equations would be required, which means the rib could be considered a plate with nodes spaced at equal distances apart. Each node has an associated temperature, and each node is influenced by the surrounding temperatures. Therefore, a temperature, convection and conduction equation could be developed for each node in the plate based on an energy balance based on the heat equation in figure 1. Since the total sum of energies flowing through the system equals 0, it can be determined that the total sum of energies flowing into each individual node is also 0. Figure 2 shows a 2-D representation of the longitudinal rib in question, by considering the rib as a series of nodes, a system of equations can be determined for the plate. The plate consists of different cases in which different energy balances are required to analyze; such as, node 1 being exposed to heat conduction, and heat conduction while node 11 is only exposed to heat conduction. The energy balance and equation derivations based on finite differences for the exterior nodes and interior nodes are shown in figures 4 and 3, respectively. To keep the derivations short and concise the other alternative cases such as the corners nodes will not be shown, however, figure 5 shows the final equations in which case 1, 3 and 4 were utilized for analysis.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

Figure 1: Formula 1 Heat Equation

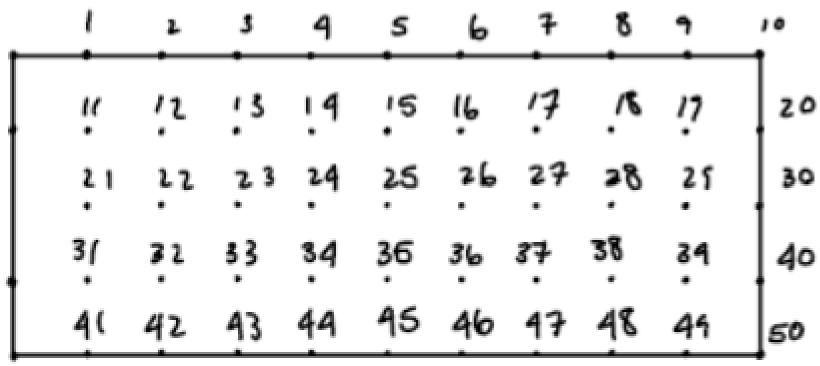


Figure 2: Longitudinal Rib

2.2 Equation Derivations

Energy Balance for Interior Nodes

$$\frac{\Delta E_{element}}{\Delta_t} = 0$$

$$\dot{Q}_{cond,left} + \dot{Q}_{cond,top} + \dot{Q}_{cond,right} + \dot{Q}_{cond,bottom} = 0$$

$$k\Delta y \frac{T_{m-1,n} - T_{m,n}}{\Delta x} + k\Delta x \frac{T_{m,n+1} - T_{m,n}}{\Delta y} + k\Delta y \frac{T_{m+1,n} - T_{m,n}}{\Delta x} + k\Delta x \frac{T_{m,n-1} - T_{m,n}}{\Delta y} = 0$$

Dividing each term by $(\Delta x) * (\Delta y)$ gives,

$$\frac{T_{m-1,n} - 2T_{m,n} + T_{m+1,n}}{\Delta x^2} + \frac{T_{m,n-1} - 2T_{m,n} + T_{m,n+1}}{\Delta y^2} = 0$$

Since $\Delta x = \Delta y$, multiplying both sides by Δx^2 gives,

$$T_{m-1,n} + T_{m+1,n} + T_{m,n-1} + T_{m,n+1} - 4T_{m,n} = 0$$

Figure 3: Formula 2 Interior Node Equation

Energy Balance for Plane Exterior Nodes

$$\frac{\Delta E_{element}}{\Delta_t} = 0$$

$$\dot{Q}_{cond,left} + \dot{Q}_{cond,top} + \dot{Q}_{cond,right} + \dot{Q}_{cond,bottom} = 0$$

$$k\frac{\Delta y}{2}\frac{T_{m-1,n} - T_{m,n}}{\Delta x} + k\frac{\Delta \mathrm{y}}{2}\frac{T_{m+1,n} - T_{m,n}}{\Delta \mathrm{x}} + k\Delta \mathrm{x}\frac{T_{m,n-1} - T_{m,n}}{\Delta \mathrm{y}} + h\Delta \mathrm{x}\left(T_{oo} - T_{m,n}\right) = 0$$

Multiplying each term by $(\Delta \mathrm{x}) * (\Delta \mathrm{y})$ gives,

$$k\frac{\Delta \mathrm{y}^2}{2}\left(T_{m,n-1} - T_{m,n}\right) + k\frac{\Delta y^2}{2}\left(T_{m+1,n} - T_{m,n}\right) + k\Delta x^2\left(T_{m,n-1} - T_{m,n}\right) + (h\Delta x^2\Delta \mathrm{y})\left(T_{oo} - T_{m,n}\right) = 0$$

Divide both sides by $(\Delta x^2) * (k)$ gives,

$$\frac{1}{2}\left(T_{m-1,n} - T_{m,n}\right) + \frac{1}{2}\left(T_{m+1,n} - T_{m,n}\right) + \left(T_{m,n-1} - T_{m,n}\right) + \frac{h\Delta \mathrm{x}}{k}\left(T_{oo} - T_{m,n}\right) = 0$$

Multiplying by 2 and simplifying gives,

$$\left(2T_{m,n-1} + T_{m-1,n} + T_{m+1,n}\right) + \frac{2h\Delta x}{k}T_{oo} - 2T_{m,n}\left(2 + \frac{h\Delta x}{k}\right) = 0$$

Figure 4: Formula 3 Plane exterior node equation

TABLE 4.2 Summary of nodal finite-difference equations

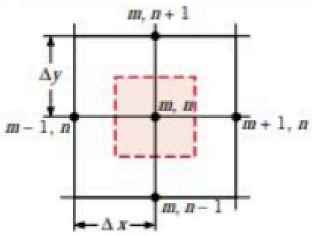
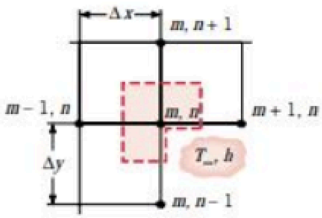
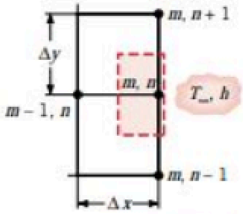
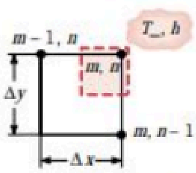
Configuration	Finite-Difference Equation for $\Delta x = \Delta y$	
	$T_{m,n+1} + T_{m,n-1} + T_{m+1,n} + T_{m-1,n} - 4T_{m,n} = 0$ **Case 1.** Interior node	(4.29)
	$2(T_{m-1,n} + T_{m,n+1}) + (T_{m+1,n} + T_{m,n-1}) + 2\frac{h\Delta x}{k}T_\infty - 2\left(3 + \frac{h\Delta x}{k}\right)T_{m,n} = 0$ **Case 2.** Node at an internal corner with convection	(4.41)
	$(2T_{m-1,n} + T_{m,n+1} + T_{m,n-1}) + \frac{2h\Delta x}{k}T_\infty - 2\left(\frac{h\Delta x}{k} + 2\right)T_{m,n} = 0$ **Case 3.** Node at a plane surface with convection	(4.42)[a]
	$(T_{m,n-1} + T_{m-1,n}) + 2\frac{h\Delta x}{k}T_\infty - 2\left(\frac{h\Delta x}{k} + 1\right)T_{m,n} = 0$ **Case 4.** Node at an external corner with convection	(4.43)

Figure 5: Finite-Difference Equations

3.0 Results and Discussion

3.1 Discussion

The initial statements in the matlab code require the user to input a value for dy and dx; for this project dy is equal to dx and they dictate the spacing between the nodes. The size of dy and dx determine the set up of the initial matrices in which they are dependent on the size of the dy and dx. Matrix 'A' in the matlab code is a system of equations automatically generated based on the size of the nodes and the finite difference equations found in figure 5. Based on the system of equations matrix and the jacobi method, a solution can be acquired along with plots and figures required for analysis which can be found in the appendix.

Initially it was required to analyze a system of equations based on a node size (dy and dx) of 1mm. The temperature profiles plotted along vertical lines at x=1mm, 4mm and 6mm can be seen in figures 8, 9 and 10 respectively in the appendix. These plots are expected to be symmetrical since the conditions of the experiment are steady state and no other temperature influences are considered such as radiation. The figures show that the temperature distribution along the vertical lines at x=1mm, 4mm and 6mm are all symmetrical; this makes sense because it should be expected that the exterior temperatures of the plate should be lowest since they are exposed to convection and conduction, while the interior temperatures are the lowest since they are only exposed to conduction.

The temperature contour for an initial node size of 1mm is shown in figure 11, in which observations show that the hottest temperature remains at the base and a gradient pattern cools the plate down to its lowest temperature at the tip. The curves of the contour indicate that the edges of the plate cool quicker than the inner temperatures because of the edges exposure to convection as discussed earlier.

Figure 12 shows the temperature profile plotted with a node size of 0.5mm rather than the initial 1mm. The curve of this plot is much smoother than the plot of the 1mm temperature profile because more nodes are considered. In terms of numerical methods the node size could be considered analogous to step size, in which a smaller step size or a smaller node size will grant more accurate results because a larger number of entities is considered. If a higher accuracy was required then the node size should be decreased further.

Figures 6 and 7 show the temperature distribution matrices for the node sizes of 1 mm and 0.5 mm, respectively. The smaller node size increases the number of nodes considered by a significant amount and thus increases the accuracy of the results. The results show more than 2 degrees of difference in the tip temperatures of the ribs. This basically shows that numerical methods may not show the true results based on a given step size, it could be expected that an even smaller step size could cause the results to converge on an even smaller value and thus contributing to a larger true error in the results of the 1mm node size.

3.2 Summary of Results

```
Dimensions of plate is x=10 mm by y=4 mm
Please enter deltas (mm): 1
The Final matrix is:

50.000  46.704  44.101  41.913  40.052  38.475  37.153  36.065  35.195  34.529  34.056
50.000  47.137  44.589  42.378  40.479  38.866  37.513  36.400  35.509  34.827  34.342
50.000  47.253  44.740  42.529  40.621  38.997  37.634  36.512  35.614  34.926  34.439
50.000  47.137  44.589  42.378  40.479  38.866  37.513  36.400  35.509  34.827  34.342
50.000  46.704  44.101  41.913  40.052  38.475  37.153  36.065  35.195  34.529  34.056
```

Figure 6: Temperature Distribution Matrix for Node Size of 1 mm

```
Dimensions of plate is x=10 mm by y=4 mm
Please enter deltas (mm): 0.5
The Final matrix is:

50.000  47.984  46.293  44.772  43.371  42.069  40.856  39.727  38.678  37.708  36.815  35.997  35.253  34.583  33.984  33.456  32.998  32.609  32.288  32.035  31.847
50.000  48.233  46.593  45.075  43.664  42.347  41.119  39.975  38.912  37.929  37.023  36.194  35.440  34.759  34.152  33.617  33.153  32.758  32.433  32.176  31.985
50.000  48.357  46.777  45.280  43.872  42.550  41.314  40.160  39.087  38.094  37.179  36.342  35.580  34.892  34.279  33.738  33.269  32.871  32.542  32.282  32.090
50.000  48.422  46.883  45.406  44.005  42.683  41.442  40.283  39.204  38.205  37.284  36.440  35.673  34.981  34.364  33.819  33.347  32.946  32.615  32.353  32.159
50.000  48.450  46.931  45.466  44.069  42.748  41.506  40.344  39.262  38.260  37.336  36.490  35.720  35.026  34.406  33.860  33.386  32.983  32.651  32.388  32.194
50.000  48.450  46.931  45.466  44.069  42.748  41.506  40.344  39.262  38.260  37.336  36.490  35.720  35.026  34.406  33.860  33.386  32.983  32.651  32.388  32.194
50.000  48.422  46.883  45.406  44.005  42.683  41.442  40.283  39.204  38.205  37.284  36.440  35.673  34.981  34.364  33.819  33.347  32.946  32.615  32.353  32.159
50.000  48.357  46.777  45.280  43.872  42.550  41.314  40.160  39.087  38.094  37.179  36.342  35.580  34.892  34.279  33.738  33.269  32.871  32.542  32.282  32.090
50.000  48.233  46.593  45.075  43.664  42.347  41.119  39.975  38.912  37.929  37.023  36.194  35.440  34.759  34.152  33.617  33.153  32.758  32.433  32.176  31.985
50.000  47.984  46.293  44.772  43.371  42.069  40.856  39.727  38.678  37.708  36.815  35.997  35.253  34.583  33.984  33.456  32.998  32.609  32.288  32.035  31.847
```

Figure 7: Temperature Distribution Matrix for Node Size of 0.5 mm

4.0 Conclusion

In conclusion the purpose of this project was to compute the heat distribution in rectangular ribs attached to a plate. The cross-section of a rib was parted into even nodes. These individual nodes can have a unique heat distribution equation depending on their orientation. The outside nodes will have different heat distribution from the inside nodes because they are exposed to the atmospheric temperature of 22°C. Thus it is important to specify the location of each node so the temperature can be computed accordingly. After determining mathematical equations for all the nodes, Using a matrix allows us to solve several equations more efficiently, the matrix also allows us to organize according to their specific location. The matrix was put into the matlab software to develop a code, where the temperature distribution of the rib was plotted. The results show the hottest temperature remains at the base and a gradient pattern cools the plate down to its lowest temperature at the tip, which was anticipated. Figure 11 temperature contour shows the temperature variation down the total rib distance of 10mm. The computation shows the range of temperature variation is from 50°C at the base, to 34.056°C at the tip.

5.0 References

Tarokh, A. (2021). *EMEC 3559: Computational Methods and Modeling in Mechanical Engineering Project #2* [Class Handout].

Cengel, Yunus A., and Afshin J. Ghajar. *Heat and Mass Transfer: Fundamentals and Applications*. 6th ed., McGraw-Hill Professional, 2014.

Appendix: Matlab Developed Plots and Figures

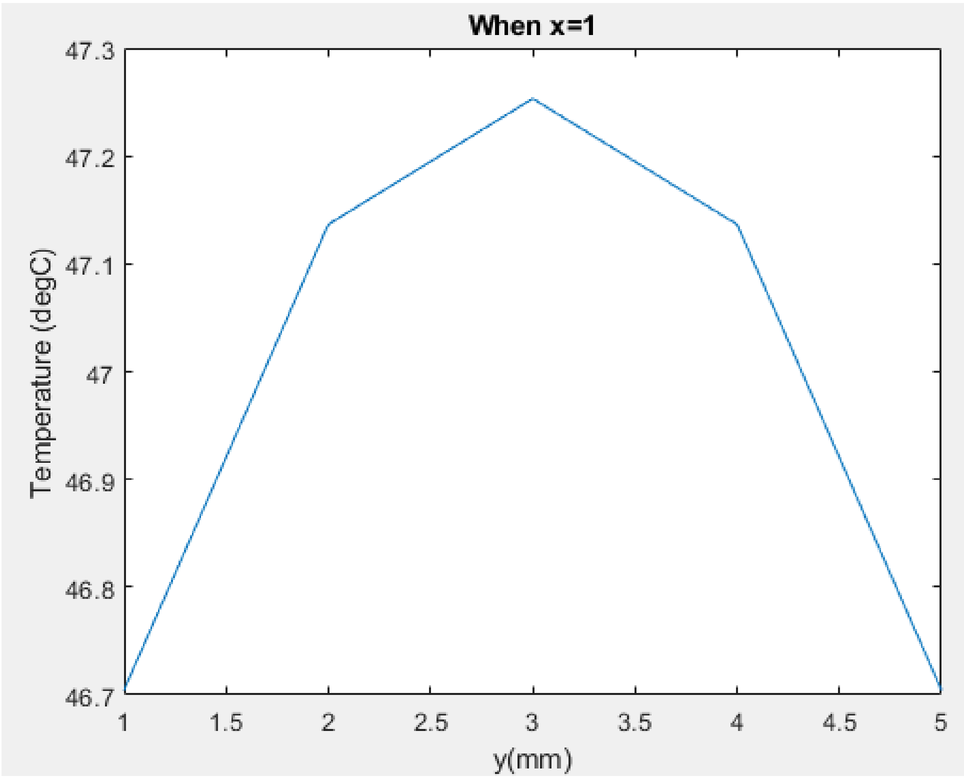


Figure 8: X=1; dy/dx = 1mm

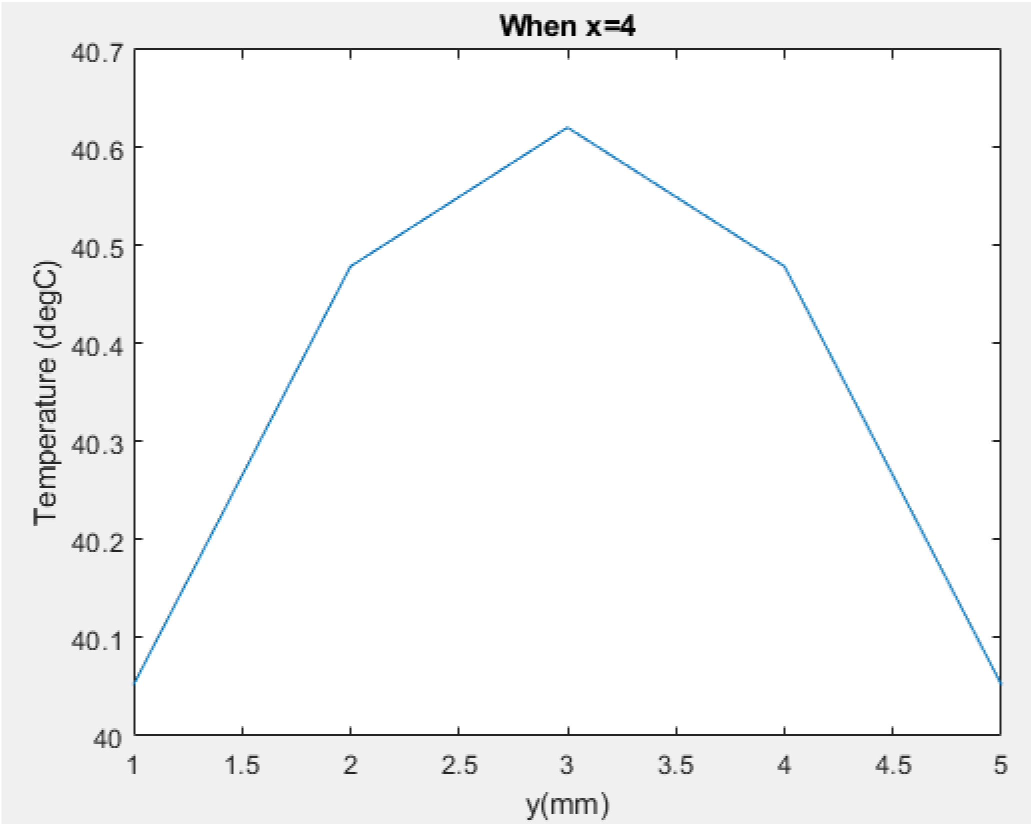


Figure 9: X=4; dy/dx = 1mm

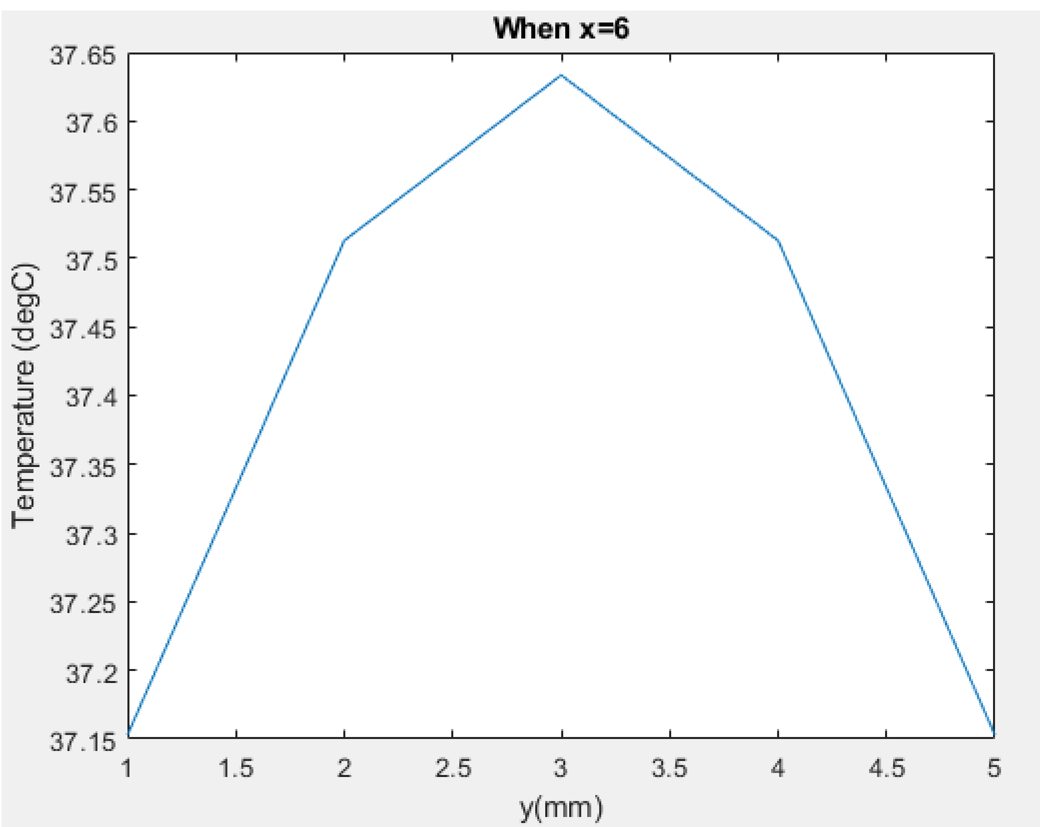


Figure 10: X=6; dy/dx = 1mm

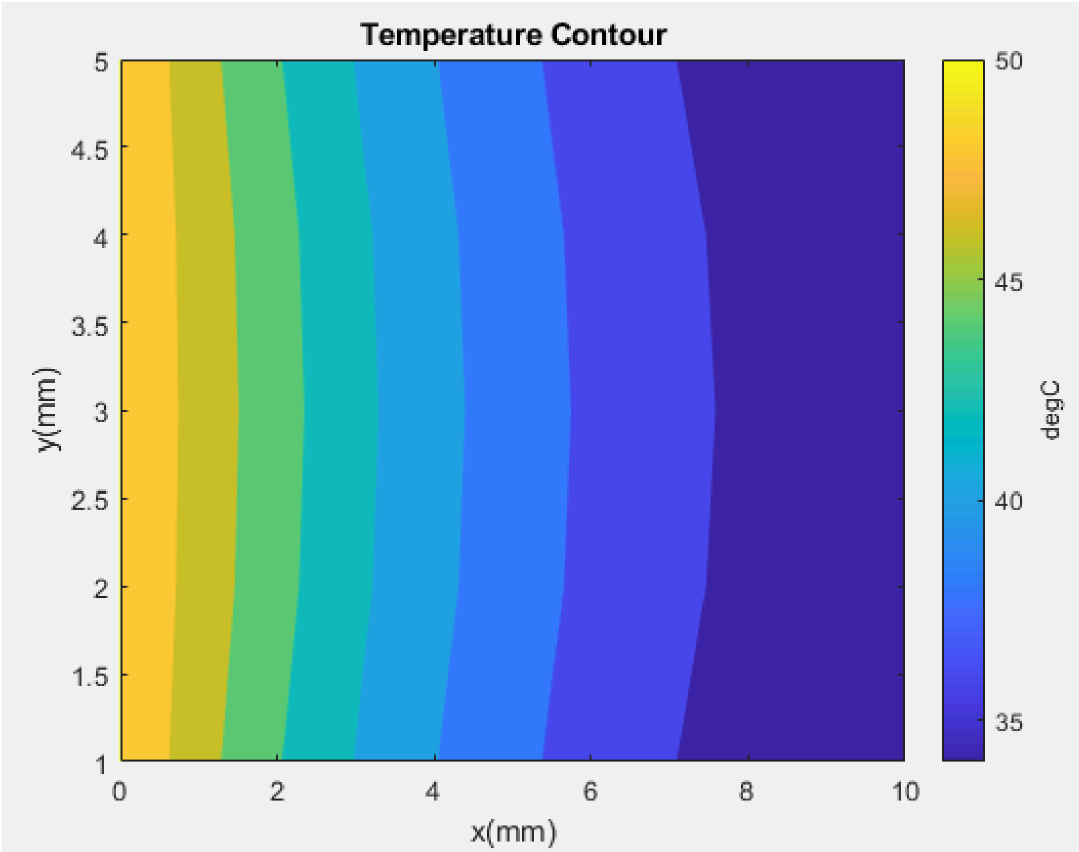


Figure 11: Temperature Contour; dy/dx = 1mm

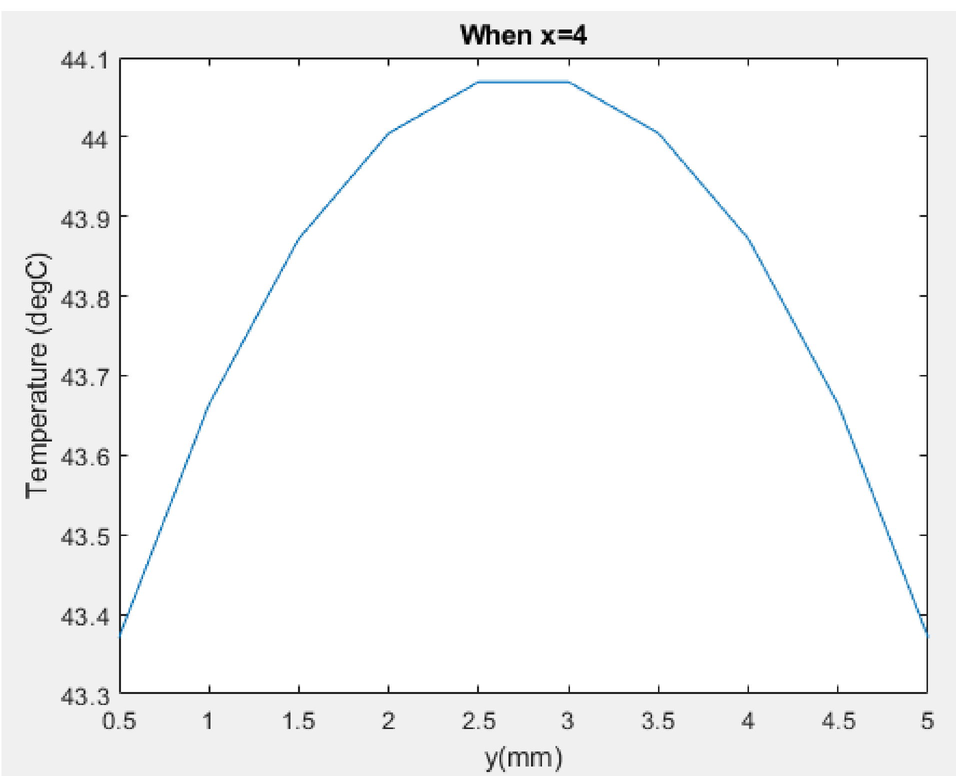


Figure 12: X=4; dy/dx = 0.5mm