Trivial FUNC Lower Bound

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0.1 Main

Lemma 0.1. *Let Y be a set and $x \in Y$ some point in Y. Let $C \subseteq Y$ be a non-empty set satisfying $C \neq x$. Then there is some $y \in Y$ such that $y \neq x$ and $y \in C$.*

Proof. We split into cases depending on whether $x \in C$.

Case 1: $x \notin C$. Since C is non-empty, there is some $y \in C$. We claim such a y satisfies $y \neq x$ and $y \in C$. The latter is by definition. For the former, if $y = x$, then that $y \in C$ implies $x \in C$, contradicting $x \notin C$.

Case 2: $x \in C$. We show that, assuming the conclusion is false, it holds that $C = \{x\}$. To show $\{x\} \subseteq C$, it suffices to show $x \in C$, but we know this already. To show $C \subseteq \{x\}$, we take a $y \in C$, and by assumption, since $y \in C$, we know $y = x$ and hence $y \in \{x\}$. □