Historical Roots of Mathematics Homework 4

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1. The Pythagorean Theorem states that, if a and b are the sides of a right triangle with hypotenuse c, then $a^2 + b^2 = c^2$:
   Find a if $b = 1,59$ and $c = 2,49$.

$$c_{10} = 169$$
$$b_{10} = 109$$
$$\sqrt{(169)^2 - (109)^2} = \sqrt{16680} = 2\sqrt{4170}$$

2. Use the Babylonian approximation of $\pi = 3$ to calculate

   (a) The circumference of a circle with diameter 20.

$$d = 20$$
$$r = 10$$
$$C = 2\pi r$$
$$C = 2(3)(10) = 60$$

   (b) The areao fa circle with diameter 20.

$$d = 20$$
$$r = 10$$
$$A = \pi r^2$$
$$A = (3)(10)^2 = 300$$

3. Solve the following quadratic equations using the Babylonian method:

   (a) $x^2 + 4x = 21$

$$x(x + 4) = 21$$
$$x = a - 2$$
$$(a - 2)(a + 2) = 21$$
$$a^2 - 4 = 21$$
$$a^2 = 25$$
$$a = 5$$
$$x = 3$$

(b) $x^2 - 2x = 4$

$$x^2 - 2x = 4$$
$$x(x-2) = 4$$
$$x = a + 1$$
$$(a+1)(a-1) = 4$$
$$a^2 - 1 = 4$$
$$a^2 = 5$$
$$a = \sqrt{5}$$
$$x = sqrt5 + 1$$

(c) $x^2 + x = 7$

$$x(x+1) = 7$$
$$x = a - \frac{1}{2}$$
$$(a - \frac{1}{2})(a + \frac{1}{2}) = 7$$
$$a^2 - \frac{1}{4} = 7$$
$$a^2 = \frac{29}{4}$$
$$a = \frac{\sqrt{29}}{2}$$
$$x = \frac{\sqrt{29} - 1}{2}$$

4. Using the Babylonian method, find two numbers whose difference is 3 and whose product is 40.

$$x - y = 3$$
$$xy = 40$$
$$y = 3 + x$$
$$x(x+3) = 40$$
$$x = a - \frac{3}{2}$$
$$(a - \frac{3}{2})(a + \frac{3}{2}) = 40$$
$$a^2 - \frac{9}{4} = 40$$
$$a^2 = \frac{169}{4}$$
$$a = \frac{13}{2}$$
$$x = 5 \quad y = 8$$

5. That system we set up in class, solve

$$\begin{cases} \dfrac{y_2}{2}(x+30) - \dfrac{xy_1}{2} = 420 \\ y_1 - y_2 = 20 \\ \dfrac{y_1}{x} = \dfrac{y_1+y_2}{30} \end{cases}$$

$$y_2 = y_1 - 20, \text{ then}$$

$$\frac{y_1}{x} = \frac{y_1+y_2}{30} = \frac{2y_1-20}{30} = \frac{y_1-10}{15}, \text{ then}$$

$$\frac{y_2(x+30)}{2} - \frac{xy_1}{2} = \frac{(y_1-20)(x-30)}{2} - \frac{xy_1}{2} = \frac{1}{2}(y_1x + 30y_1 - 20x - 600 - y_1x) = 420$$

$$\implies 15y_1 - 10x - 300 = 420 \implies 5(3y_1 - 2x - 60) = 420 \implies 3y_1 - 2x - 60 = 90$$

$$\implies x = \frac{3y_1 - 144}{2}, \text{ finally}$$

$$\frac{y_1}{x} = \frac{y_1-10}{15} \implies \frac{2y_1}{3y_1-144} = \frac{y_1-10}{15} \implies 2y_1 = \frac{3y_1^2 - 174y_1 + 1440}{15}$$

$$\implies 30y_1 = 3y_1^2 - 174y_1 + 1440 \implies 3y_1^2 - 204y_1 + 1440 = 0$$

$$y_1 = \frac{204 + \sqrt{204^2 - 4(3)(1440)}}{6} \implies y_1 = 60$$

$$y_2 = 60 - 20 = 40$$

$$x = \frac{3(60) - 144}{2} = 18$$

Thus, $x - 18, y_1 = 60, y_2 = 40$