Abstract Algebra Homework 3

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1. Write the following permutations in cucle notation.

(a)

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 1 & 5 & 3 \end{pmatrix} = (1 \quad 2 \quad 4 \quad 5 \quad 3)$$

(b)

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 1 & 3 \end{pmatrix} = (1 \quad 4)(3 \quad 5)$$

2. Compute each of the following.

(a)

$$(1\ 3\ 4\ 5)(2\ 3\ 4) = (3\ 4\ 5\ 2\ 1)$$

(b)

$$(1\ 2)(1\ 2\ 5\ 3) = (1\ 5\ 2\ 4\ 3)$$

(c)

$$(1\ 4\ 3)(2\ 3)(2\ 4) = (1\ 4\ 3)(4\ 3) = (4\ 3\ 1)$$

3. Express the following permutations as products of transpositions and identify them as even or odd.

(a)

$$(1\ 4\ 3\ 5\ 6) = (1\ 6)(1\ 5)(1\ 3)(1\ 4)$$

Even transposition

(b)

$$(1\ 5\ 6)(2\ 3\ 4) = (1\ 6)(1\ 5)(2\ 4)(2\ 3)$$

Even transposition

(c)

$$(1\ 4\ 2\ 6)(1\ 4\ 2) = (1\ 6)(1\ 2)(1\ 4)(1\ 2)(1\ 4)$$

Odd transposition

4. Find $(a_1, a_2, \ldots, a_n)^{-1}$.

$$(a_1, a_2, \ldots, a_n)^{-1} = (a_n, a_{n-1} \ldots, a_2, a_1) = (a_1, a_n, a_{n-1}, \ldots, a_2)$$

13. Let $\sigma = \sigma_1 \cdots \sigma_m \in S_n$ be the product of disjoint cycles. Prove that the order of σ is the least common multiple of the lengths of the cycles $\sigma_1, \dots \sigma_m$.

Proof. Let $\sigma = \sigma_1 \cdots \sigma_m$ where $\sigma_1, \dots, \sigma_m$ are disjoint. Also let $|\sigma| = k$. We know $\sigma_1, \dots, \sigma_m$ commute with each other so

$$\sigma^k = \sigma_1^k \cdots \sigma_m^k = id \Longleftrightarrow \sigma_1^k = \cdots = \sigma_m^k = id$$

because they are disjoint. This also means that k is a common multiple of $|\sigma_1|, \dots, |\sigma_m|$. Because the order of the cycle is its lenght, the smallest k must then be the least common multiple of the order of the cycles. ■