Abstract Algebra Homework 7

Zachary Meyner

2. Find all of the abelian groups of order 200 up to isomorphism.

$$|G| = 200 = 5 \cdot 40 = 5^2 \cdot 8 = 5^2 \cdot 2^3$$
$$\mathbb{Z}_{5^2} \times \mathbb{Z}_{2^3}$$
$$\mathbb{Z}_5 \times \mathbb{Z}_5 \times \mathbb{Z}_{2^3}$$
$$\mathbb{Z}_{5^2} \times \mathbb{Z}_{2^2} \times \mathbb{Z}_2$$
$$\mathbb{Z}_5 \times \mathbb{Z}_5 \times \mathbb{Z}_{2^2} \times \mathbb{Z}_2$$
$$\mathbb{Z}_{5^2} \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$$
$$\mathbb{Z}_5 \times \mathbb{Z}_5 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$$

5. Show that the infinite direct product $G = \mathbb{Z}_2 \times \mathbb{Z}_2 \times \cdots$ is not finitely generated.

Proof. Every element in G has an order of 2, and G is abelian. The group generated by this must have an order of at most 2^n, but G has infinite order. Thus G has to be infinitely generated. ■