Historical Roots of Mathematics Homework 1

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1. Write the follwing problems in hieroglyphics and then perform the addition:

   a. $46 + 23$

   b. $64 + 28$

   c. $4297 + 1351$

2. a. Show that

$$\frac{2}{n} = \frac{1}{3n} + \frac{5}{3n}$$

hence that $2/n$ can be expressed as a sum of unit fractions whenever n is divisible by 5.

That is If $n = 5k$ $k \in \mathbb{Z}$, then $\frac{2}{n} = \frac{1}{3n} + \frac{1}{3k}$.

Proof. Let $n, k \in \mathbb{Z}$ s.t. $n = 5k$. Then

$$\begin{aligned}\frac{2}{n} &= \frac{6}{3n} \\ &= \frac{1}{3n} + \frac{5}{3n} \\ &= \frac{1}{3n} + \frac{5}{3(5k)} \\ &= \frac{1}{3n} + \frac{1}{3k}\end{aligned}$$

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b. Note: $25 = 5(5)$

$$\frac{2}{25} = \frac{1}{75} + \frac{1}{15}$$

Note: $65 = 5(13)$

$$\frac{2}{65} = \frac{1}{195} + \frac{1}{39}$$

Note: $85 = 5(17)$

$$\frac{2}{85} = \frac{1}{255} + \frac{1}{51}$$

3. Use the $2 \div n$ table to write the following as sums of unit fractions without repetition:

a.

$$\begin{aligned}\frac{13}{15} &= \frac{3}{15} + \frac{2}{3} \\ &= \frac{1}{5} + \frac{1}{2} + \frac{1}{6}\end{aligned}$$

b.

$$\begin{aligned}\frac{9}{49} &= \frac{7}{49} + \frac{2}{49} \\ &= \frac{1}{7} + \frac{1}{29} + \frac{1}{196}\end{aligned}$$

c.

$$\frac{19}{35} = \frac{5}{35} + \frac{2}{5}$$
$$= \frac{1}{7} + \frac{1}{3} + \frac{1}{15}$$

4. Compute in the Anceint Egyptian way:

a. $3 \div 4$

1	4
$\checkmark\overline{2}$	2
$\checkmark\overline{4}$	1

$3 \div 4 = \overline{2} + \overline{4}$

b. $5 \div 8$

1	8
$\checkmark\overline{2}$	4
$\overline{4}$	2
$\checkmark\overline{8}$	1

$5 \div 8 = \overline{2} + \overline{8}$

c. $14 \div 24$

1	24
$\checkmark\overline{3}$	8
$\checkmark\overline{2}$	12
$\checkmark\overline{4}$	6

$14 \div 24 = \overline{3} + \overline{4}$

d. $35 \div 32$

$\checkmark 1$	32
$\overline{2}$	16
$\overline{4}$	8
$\overline{8}$	4
$\checkmark\overline{16}$	2
$\checkmark\overline{32}$	1

$35 \div 32 = 1 + \overline{16} + \overline{32}$

e. $5 \div 6$

1	6
$\checkmark\overline{\overline{3}}$	4
$\checkmark\overline{6}$	1

$5 \div 6 = \overline{\overline{3}} + \overline{6}$

f. $17 \div 12$

$\checkmark 1$	12
$\checkmark\overline{3}$	4
$\checkmark\overline{12}$	1

$17 \div 12 = 1 + \overline{3} + \overline{12}$

g. $11 \div 16$

1	16
$\checkmark\overline{2}$	8
$\overline{4}$	4
$\checkmark\overline{8}$	2
$\checkmark\overline{16}$	1

$11 \div 16 = \overline{2} + \overline{8} + \overline{16}$

h. $51 \div 18$

1	18
$\checkmark 2$	36
$\checkmark\overline{\overline{3}}$	12
$\overline{3}$	6
$\checkmark\overline{6}$	3

$51 \div 18 = 2 + \overline{\overline{3}} + \overline{6}$

5. Find a Sylveester-type representation (as a sum of unit fractions) for each of the following:

a.

$$\frac{13}{36} = \frac{1}{36} + \frac{1}{3}$$

b.

$$\begin{aligned}\frac{9}{20} &= \frac{4}{20} + \frac{1}{4} \\ &= \frac{1}{5} + \frac{1}{4}\end{aligned}$$

c.

$$\frac{4}{15} = \frac{1}{15} + \frac{1}{5}$$

d.

$$\begin{aligned}\frac{335}{336} &= \frac{167}{336} + \frac{1}{2} \\ &= \frac{55}{336} + \frac{1}{2} + \frac{1}{3} \\ &= \frac{7}{336} + \frac{1}{2} + \frac{1}{3} + \frac{1}{7} \\ &= \frac{1}{48} + \frac{1}{2} + \frac{1}{3} + \frac{1}{7}\end{aligned}$$