Hist Roots Homework 3

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1. Give three numbers that might be represented by the symbols I don't know how to type

   (a) 2, 120, 7200

   (b) 12, 720, 2,592,000

2. Find a first, second, and third approximation to:

a) $\sqrt{10}$

One:

$$3^2 = 9 < 10, \text{ so}$$
$$3 < \sqrt{10}$$
$$\frac{10}{3} > \sqrt{10}, \text{ so}$$
$$4 < \sqrt{10} < \frac{10}{3}, \text{thus}$$
$$\sqrt{10} \approx \frac{4 + \frac{10}{3}}{2} = \frac{22}{6} = 3.\overline{6}$$

Two:

$$\frac{22}{6} \cdot \frac{60}{22} = 10, \text{ so}$$
$$\sqrt{10} \approx \frac{\frac{22}{6} + \frac{60}{22}}{2} = \frac{211}{66} = 3.19\overline{69}$$

Three:

$$\frac{211}{66} \cdot \frac{660}{211} = 10, \text{ so}$$
$$\sqrt{10} \approx \frac{\frac{211}{66} + \frac{660}{211}}{2} = 3.1624\ldots$$

b) $\sqrt{7}$

One:

$$2^2 = 4 < 7, \text{ so}$$
$$2 < \sqrt{7}$$
$$\frac{7}{2} > \sqrt{7}, \text{ so}$$
$$2 < \sqrt{7} < \frac{7}{2}, \text{ thus}$$
$$\sqrt{7} \approx \frac{2 + \frac{7}{2}}{2} = 11/4 = 2.75$$

Two:

$$\frac{11}{4} \cdot \frac{28}{11} = 7, \text{ thus}$$
$$\sqrt{7} \approx \frac{\frac{11}{4} + \frac{28}{11}}{2} = \frac{233}{88} = 2.647\overline{72}$$

Three:

$$\frac{233}{88} \cdot \frac{616}{233} = 7, \text{ thus}$$
$$\sqrt{7} \approx \frac{\frac{233}{88} + \frac{616}{233}}{2} = 2.64575\ldots$$

3. Use the **Binomial Theorem**

$$(1+x)^m = 1 + \sum_{k=1}^{\infty} \binom{m}{k} x^k$$

to prove that

$$\sqrt{a^2+b} \approx a + \frac{b}{2a} - \frac{b^2}{8a^2}$$

Proof. Consider $(a^2+b)^{\frac{1}{2}}$. Factoring out a we have $a(1+\frac{b}{a^2})^{\frac{1}{2}}$. Thus we have

$$\begin{aligned} a\left(1+\frac{b}{a^2}\right)^{\frac{1}{2}} &= a\left(1+\sum_{k=1}^{\infty}\binom{\frac{1}{2}}{k}\left(\frac{b}{a^2}\right)^k\right) \\ &\approx a\left(1+\frac{b}{2a^2}-\frac{b^2}{8a^3}\right) \\ &= a+\frac{b}{2a}-\frac{b^2}{8a^2} \end{aligned}$$

■

4. Let $n = 2^a 3^b 5^c$, where $a, b, c \in \mathbb{N}$. Prove that n is sexagesimally regular.

Proof. Let $n = 2^a 3^b 5^c$ s.t. $a, b, c \in \mathbb{N}$. Then we know that

$$\begin{gathered} n|2^a \\ n|3^b \\ n|5^c \end{gathered}$$

Thus we can also say,

$$n|2^a 3^b 5^c$$

WLOG let $a = 2m, b = c = m,\ m \in \mathbb{N}$. Then we have

$$\begin{gathered} n|2^{2m}3^m5^m \\ n|(2^2\cdot 3\cdot 5)^m \\ n|60^m \end{gathered}$$

Thus $nz = 60^m,\ z \in \mathbb{N}$, so

$$\frac{1}{n} = \frac{a_1 60^m + a_2 60^{m-1} + \cdots + a_m}{60^m} = \frac{a_1}{60} + \frac{a_2}{60^2} \cdots \frac{a_m}{60^m}$$

$\therefore n$ is sexagesimally regular. ■

5. Write in base-60:

(a) $\frac{5}{6}$

$$\frac{5}{6} = \frac{\frac{5}{6}(60)}{60} = \underline{\frac{50}{60}}$$

(b) $1\frac{4}{9}$

$$\frac{4}{9} = \frac{\frac{4}{9}(60)}{60} = \frac{26}{60} + \frac{\frac{2}{3}}{60}$$

$$\frac{26}{60} + \frac{\frac{2}{3}(60)}{60^2} = \frac{26}{60} + \frac{40}{60^2}$$

$$\underline{1 + \frac{26}{60} + \frac{40}{60^2}}$$

(c) $86\frac{1}{90}$

$$86 = 1, 26\frac{1}{90} = \frac{\frac{1}{90}(60)}{60} = \frac{\frac{2}{3}}{60}$$

$$\frac{\frac{2}{3}(60)}{60^2} = \frac{40}{60^2}$$

$$\underline{1, 26 + \frac{40}{60^2}}$$