Linear Algebra II Quiz 2

Zachary Meyner

18. Do the column of B span $\mathbb{R}^4$? Does the equation $B\mathbf{x} = \mathbf{y}$ have a solution for each $\mathbf{y}$ in $\mathbb{R}^4$?

$$B = \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 4 & -1 & \frac{1}{2} \\ 0 & 1 & 1 & \frac{-1}{13} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

There are only 3 pivots in B so it does not span $\mathbb{R}^4$, thus the equation $B\mathbf{x} = \mathbf{y}$ does not have a solution for each $\mathbf{y}$ in $\mathbb{R}^4$.

43. Suupse A is a 4×3 matrix and $\mathbf{b}$ is a vector in $\mathbb{R}^4$ with the property that $A\mathbf{x} = \mathbf{b}$ has a unique solution. What can you say about the reduced echelon form of A? Justify your answer.
Since $A\mathbf{x} = \mathbf{b}$ has a unique solution we know that the reduced echelon form of $A\mathbf{x} = \mathbf{b}$ augmented matrix is

$$\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where $\mathbf{b} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$. Thus we know that the reduced row echelon form of A is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

49. Determine if the columns of the matrix span $\mathbb{R}^4$.

$$\begin{bmatrix} 12 & 11 & -6 & -7 & 13 \\ -9 & 4 & -8 & 7 & -3 \\ -6 & 11 & -7 & 3 & -9 \\ 4 & -6 & 10 & -5 & 12 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{-7}{12} & \frac{11}{12} & \frac{-3}{4} & \frac{5}{12} \\ 0 & 1 & \frac{-1}{5} & \frac{-1}{5} & \frac{-13}{15} \\ 0 & 0 & 1 & \frac{-41}{84} & \frac{23}{18} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The echelon form of the matrix has 4 pivot positions, thus the matrix span $\mathbb{R}^4$.

51. Find a column of the matrix in exercise 49 that can be deleted and yet have the remaining matrix columns still span $\mathbb{R}^4$.
Further reducing the matrix into reduced echelon form gives

$$\begin{bmatrix} 1 & 0 & 0 & \frac{-10}{21} & 0 \\ 0 & 1 & 0 & \frac{25}{84} & 0 \\ 0 & 0 & 1 & \frac{41}{84} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Since the fourth vector does not give a pivot row it is the only vector in the list that cannot be made with a linear combination of the other 4 vectors, thus you can remove one of the first, second, third, or fifth vector and it will still span $\mathbb{R}^4$.