Historical Roots of Mathematics Homework 2

Zachary Meyner

1. In problem 30 of the Rhind Papyrus the area of a circle is eqated to the square of $\frac{8}{9}$ of the diameter. Prove that this leads to the approximation $\pi \approx \frac{256}{81}$

Proof. Consider a circle with radius r and diameter $d = 2r$. Consider the approximation of the area

$$A \approx \left(= \frac{8}{9} \right)^2 d$$

Then we know

$$\begin{aligned} A = \pi r^2 &\approx \left(\frac{8}{9} \cdot d \right)^2 \\ \pi r^2 &\approx \left(\frac{8}{9} \cdot 2r \right)^2 && (d = 2r) \\ \pi r^2 &\approx \left(\frac{16}{9} r \right)^2 \\ \pi r^2 &\approx \frac{256}{81} r^2 \\ \pi &\approx \frac{256}{81} \end{aligned}$$

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2. Calcualte using the Egyptian method:

(a) $12 \div 23$

1	23
$\checkmark\overline{2}$	$11 + \overline{2}$
$\checkmark\overline{46}$	$\overline{2}$

$1 \div 2$

$\checkmark\overline{2}$	1

$$12 \div 23 = \overline{2} + \overline{46}$$

(b) $11 \div 13$

1	13
$\checkmark\overline{3}$	$4+\overline{3}$
$\checkmark\overline{2}$	$6+\overline{2}$
$\checkmark\overline{78}$	$\overline{6}$

Complete $10+\overline{3}+\overline{2}$ to 11
Complete $\overline{3}+\overline{2}$ to 1
Sum: 5 Remainder: 1
$1 \div 6$

$\checkmark\overline{6}$	1

$1 \div 6 = \overline{78}+\overline{3}+\overline{2}$

(c) $15 \div 19$

1	19
$\checkmark\overline{2}$	$9+\overline{2}$
$\checkmark\overline{4}$	$4+\overline{2}+\overline{4}$
$\checkmark\overline{38}$	$\overline{2}$
$\checkmark\overline{76}$	$\overline{4}$

Complete $14+\overline{4}$ to 15
Complete $\overline{4}$ to 1
Sum:1 Remainder 3
$3 \div 4$

1	4
$\checkmark\overline{2}$	2
$\checkmark\overline{4}$	1

$15 \div 19 = \overline{76}+\overline{38}+\overline{4}+\overline{2}$

(d) $33 \div 7$

1	7
2	14
$\checkmark 4$	28
$\checkmark\overline{\overline{3}}$	$4+\overline{\overline{3}}$
$\checkmark\overline{21}$	$\overline{3}$

Complete $32+\overline{\overline{3}}$ to 33
Complete $\overline{\overline{3}}$ to 1
Sum: 2 Remainder 1
$1 \div 3$

$\checkmark\overline{3}$	1

$33 \div 7 = 4+\overline{21}+\overline{\overline{3}}$

(e) $11 \div 65$

1	65
$\overline{3}$	$21+\overline{\overline{3}}$
$\checkmark\overline{6}$	$10+\overline{2}+\overline{3}$
$\checkmark\overline{390}$	$\overline{6}$

Complete $10+\overline{2}+\overline{3}$ to 11
Complete $\overline{2}+\overline{3}$ to 1
Sum: 5 Remainder: 1 $1 \div 6$

$\checkmark\overline{6}$	1

$11 \div 65 = \overline{390} + \overline{6}$

(f)

$9 \div 23$

1	23
$\overline{2}$	$11 + \overline{2}$
$\checkmark\overline{3}$	$7 + \overline{\overline{3}}$
$\checkmark\overline{23}$	1
$\checkmark\overline{69}$	$\overline{3}$

Complete $8 + \overline{\overline{3}}$ to 9.

Complete $\overline{\overline{3}}$ to 1

Sum: 2 Remainder: 1 $1 \div 3$

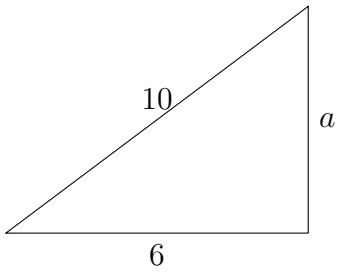
$\checkmark\overline{3}$ 1

$9 \div 23 = \overline{69} + \overline{23} + \overline{3}$

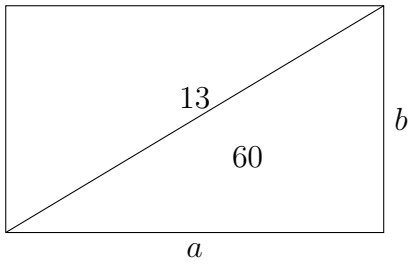
3. Solve the following problems from the Cairo Papyrus (300 BC):

(a) A ladder of 10 cubuts has its foot 6 cubits from a wall. To what height does the ladder reach?

$$a^2 + 6^2 = 10^2$$
$$a^2 + 36 = 100$$
$$a^2 = 64$$
$$a = 8$$

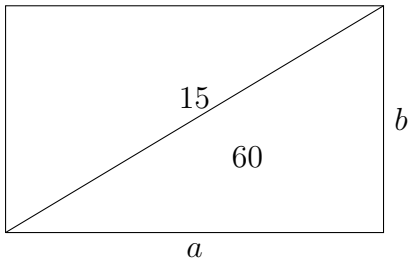


(b) A rectangle with an aera of 60 square cubits has a diagonal of 13 cubits. Find the sides of the rectangle.



$$a \cdot b = 60$$
$$a^2 + b^2 = 169$$
$$5 \cdot 12 = 60$$
$$5^2 + 12^2 = 25 + 144 = 169$$

(c) A rectangle with an aera of 60 square cubits has a diagonal of 15 cubits. Find the sides of the rectangle.



$$\begin{aligned}
a \cdot b &= 60 \\
a^2 + b^2 &= 225 \\
a^2 + 2ab + b^2 &= 225 + 2ab \\
a^2 - 2ab + b^2 &= 225 - 2ab \\
\begin{cases} a + b = \sqrt{345} \\ a - b = \sqrt{105} \end{cases} & \\
2a &= \sqrt{345} - \sqrt{105} \\
\begin{cases} a = \frac{\sqrt{345} - \sqrt{105}}{2} \\ b = \frac{60}{\frac{\sqrt{345} - \sqrt{105}}{2}} \end{cases} &
\end{aligned}$$

4. Use the method of false position to solve the following problems from the Rhind Papyrus. Express your answers in unit fractions.

(a) A quantity and its fourth, added together, give 15. What is the quantity?

$$x + \frac{x}{4} = 15$$

Guess 8:

$$8 + \frac{8}{4} = 10$$

$15 \div 10$

✓1	10
✓$\overline{2}$	5

$15 \div 10 = 1 + \overline{2}$

$$8(1 + \overline{2}) = 12$$

$$x = 12$$

(b) A quantity and its fifth, added together, give 21. What is the quantity?

$$x + \frac{x}{5} = 21$$

Guess 15:

$$15 + \frac{15}{5} = 18$$

$21 \div 18$

$$21 - 18 = 3$$

$$1 + \frac{3}{18} = 1 + \frac{1}{6}$$

$$15(1 + \frac{1}{6}) = 15 + \frac{15}{6} = 17 + \frac{3}{6} = 17 + \frac{1}{2}$$

$$x = 17 + \frac{1}{2}$$