Thm 3.3.9 (Ray Theorem). Let $l \in \mathcal{L}, A \in l, B \notin l$ If $C \in \overrightarrow{AB}$ and $C \neq A$, then $C \in H_B(l)$.

Thm 3.3.12 (Pasch's Axiom). $\triangle ABC, l \in \mathcal{L}, A, B, C \notin l$. If $l \cap \overline{AB} \neq \emptyset$ then $l \cap \overline{AC} \neq \emptyset$ or $l \cap \overline{BC} \neq \emptyset$.

Lemma 3.5.0. Let $A, B \in \mathbb{P}$ distinct. Then $\exists C, D \in \mathbb{P}$ s.t. $A * C * B$ and $A * B * D$.

Thm 3.5.3. $D \in \text{int}\angle BAC$ iff $\overrightarrow{AD} \cap \text{int}\overline{BC} \neq \emptyset$.

Thm 4.3.4. Let $l \in \mathcal{L}, P \in \mathbb{P}$ wiht $P \notin \mathcal{L}$. Let F be the foot of the $\perp$ from P to l. If $R \in l, R \neq F$ then $PR > PF$.

Thm 4.6.4. If $\square ABCD$ is a convex quadrilateral then $\sigma(\square ABCD) \leq 360$.

Thm 4.6.6. Every parallelogram is a convex quadrilateral.

Thm 4.6.8. $\square ABCD$ is convex iff $\overline{AB} \cap \overline{BD} \neq \emptyset$.

Def EPP. Let $l \in \mathcal{L}, P \in \mathbb{P} \backslash l$. $\exists! m \in \mathcal{L}$ s.t $P \in m, m \parallel l$.

Thm 4.7.3. The following are equivalent to the EPP

1. (Proclus' Axion) If $l \parallel l'$ and $t \neq l$ with $t \cap l \neq \emptyset$ then $t \cap l' \neq \emptyset$.
2. If $l, m \in \mathcal{L}$ s.t. $l \parallel m$ and $n \perp l$ then $n \perp m$.
3. If $l, m, n, k \in \mathcal{L}$ s.t. $k \parallel l, m \perp k, n \perp l$ then $m = n$ or $m \perp n$.
4. (Transitivity) If $l \parallel m, m \parallel n$ then $l \parallel n$ or $l = n$.

Thm 4.8.10 (Properties of Sacherri quadrilaterals). Let $\square ABCD$ be a Sacherri quadrilateral

1. $AC = BD$.
2. $\angle BCD \cong \angle ACD$
3. If E mid $\overline{AB}$ and F mid $\overline{CD}$ then $\overline{EF} \perp \overline{AB}, \overline{CD}$
4. $\square ABCD$ is a parallelogram
5. $\square ABCD$ is convex
6. $\mu(\angle BCD), \mu(\angle ADC) \leq 90$

Thm 4.8.11 (Properties of Lambert quadrilaterals). Let $\square ABCD$ be a lambert quadrilateral with right angles at $\angle A, \angle B, \angle C$.

1. $\square ABCD$ is a parallelogram.
2. $\square ABCD$ is a convex quadrilateral.
3. $\mu(\angle D) \leq 90$.
4. $BC \leq AD$.

Thm 5.1.10 (Properties of Euclid Geometry). Let $\square ABCD$ be a parallelogram.

1. $\triangle ABC \cong \triangle CDA$ and $\triangle ABD \cong CBD$
2. $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$
3. $\angle DAB \cong \angle BCD$ and $\angle ABC \cong \angle CDA$
4. $\overline{AC} \cap \overline{BD} = \{E\}$ where E is the midpoint of $\overline{AC}, \overline{BD}$

Thm 5.2.1 (Parallel Projection Theorem). Let $l, m \in \mathcal{L}$ be distinct mutually parallel lines. Let $a, b \in \mathcal{L}$ be transversals that cut these lines at A, B, C and D, E, F with $A * B * C$ and $D * E * F$. Then $\frac{AB}{AC} = \frac{DE}{DF}$.

Lemma 5.3.0. Let $A, B, C, D \in \mathbb{P}$ be distinct. If $AB > CD$ and $E \in \overrightarrow{AB}$ s.t. $AE = CD$ then $A * E * B$.

Thm 5.3.1 (Fundamental Theorem of Similar Triangles). If $\triangle ABC \sim \triangle DEF$ then $\frac{AB}{AC} = \frac{DE}{DF}$.

Thm 5.4.3. The height of a right triangle is the geomtric mean of the lengths of the projections of the legs. $h = \sqrt{(AB)(DB)}$

Thm 5.4.4. The length of one leg of a right traingle is the geometric mean of the length of the hypotenuse and the projection of that leg onto the hypotenuse. $b = \sqrt{C(AD)}$ $a = \sqrt{C(BD)}$

Thm 8.1.7 (Tangent Line Theorem). Let $C(O, r) \in \mathfrak{C}, l \in \mathcal{L}, P \in l \cap C(O, r)$. Then $l \cap C(O, r) = \{P\}$ iff $\overleftrightarrow{OP} \perp l$.

Thm 8.1.9 (Secant Line Theorem). Let $C(O, r) \in \mathfrak{C}, l \in \mathcal{L}$ be a second line at $\{P, Q\}$. If m is the $\perp$-bisector of $\overline{PQ}$ then $O \in m$.

Thm 8.1.11 (Elementary Circular Continuity). A line cannot get from the inside to the outside of a circle without crossing the circle.

Thm 8.1.16. Let $C(O,r) \in \mathcal{C}, l, m \in \mathcal{L}$ be nonparallel and tangent to the circle at P, Q. Let $A \in l \cap m$. Then

1. If $\overrightarrow{AB}$ is the angle bisector of $\angle PAQ$ then $O \in \overrightarrow{AB}$
2. $PA = QA$
3. $PQ \perp OA$.

Thm 10.1.6. The composition of two isometries is an isometry and the inverse of an isometry is an isometry.

Thm 10.1.7 (Properties of Isometries). Let T be an isometry then T preserves the following

1. Colinearity
2. Betweenness of Points
3. Segments
4. Lines
5. Betweenness of Rays
6. Angles
7. TrianglesCircles

Thm 10.2.2 ($\frac{1}{2}$-turn theorem). Let $l, m \in \mathcal{L}, l \perp m, O \in l \cap m$ and $h_O = \rho_l \circ \rho_m$. If $P \in \mathbb{P} \backslash \{O\}$ then O is the midpoint of $\overline{Ph_O(P)}$.

Thm 10.2.5 (The Rotation Theorem). Let R_{AOB} be a rotation with center O and angle $\angle AOB$ where $R_{AOB} = \rho_m \circ \rho_l$ where $l = \overleftrightarrow{OA}$ and m containing the angle bisector of $\angle AOB$.

1. If $P = \mathbb{P} \backslash O$ and $P' = R_{AOB}(P)$ then $\mu(\angle AOB) = \mu(\angle POP')$
2. If $n \in \mathcal{L}$ with $O \in n$ then $\exists r, t \in \mathcal{L}$ s.t. $R_{AOB} = \rho_r \circ \rho_n = \rho_n \circ \rho_t$.

Thm 10.2.8 (Translation Theorem).

1. An isometry T is a translation iff $\exists k, l, m \in \mathcal{L}$ s.t. $l, m \perp k$ and $T = \rho_l \circ \rho_m$.
2. Let $T_{AB} = \rho_m \circ \rho_l$ be a translation where $A \neq B, k = \overleftrightarrow{AB}$ If $n \in \mathcal{L}, n \perp k$ Then $\exists r, t \in \mathcal{L}$ s.t. $T_{AB} = \rho_r \circ \rho_n = \rho_n \circ \rho_t$.

Thm 10.3.2 (Glide Reflection Theorem). Let T be an isometry. Then $T = G_{AB}$ iff $\exists l, m, n \in \mathcal{L}$ distinct s.t. $T = \rho_l \circ \rho_m \circ \rho_n$.