Abstract Algebra Homework 1

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1. Which of the following multiplication tables defined on the set $G = \{a, b, c, d\}$ form a group? Support your answer in each case.

Theorem 1. *Let $a, b, c \in G$. Given a is the Identity element of the set G, then $a \circ (b \circ c) = (a \circ b) \circ c$, $\forall b, c \in G$.*

Proof. Let $d \in G$ with $b \circ c = d$. Then $a \circ (b \circ c) \implies a \circ d = d$.
We also have $(a \circ b) \circ c \implies b \circ c = d$.
$\therefore a \circ (b \circ c) = (a \circ b) \circ c$. ∎

(a)

$\circ$	a	b	c	d
a	a	c	d	a
b	b	b	c	d
c	c	d	a	b
d	d	a	b	c

This Cayley Table does not form a group because it is not Associative:
$a \circ (b \circ c) = d$ and
$(a \circ b) \circ c = a$, so
$a \circ (b \circ c) \neq (a \circ b) \circ c$

(b)

$\circ$	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	b
d	d	c	b	a

Closure: Every element in the Cayley Table is in the set G, so it is closed.
Identity: taking any element and multiplying it by a returns that element. So a is the indentity element.
Inverse: A diagonal is formed in the table with the identity element a, so every element is its own inverse.
Associative: Because a is the identity element it is associative with every