Abstract Algebra Homework 9

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2. Compute each of the following.

(a) $(5x^2 + 3x - 4) + (4x^2 - x + 9)$ in $\mathbb{Z}_{12}[x]$
$= (9x^2 + 2x + 5) \mod 12$

(b) $(5x^2 + 4x - 4)(4x^2 - x + 9)$ in $\mathbb{Z}_{12}[x]$
$= 20x^4 + 11x^3 + 25x^2 + 40x - 36 = (8x^4 + 11x^3 + x^2 + 4x) \mod 12$

(c) $(7x^3 + 3x^2 - x) + (6x^2 - 8x + 4)$ in $\mathbb{Z}_9[x]$
$= (7x^3 + 4) \mod 9$

(d) $(3x^2 + 2x - 4) + (4x^2 + 2)$ in $\mathbb{Z}_5[x]$
$= (2x^2 + 2x + 3) \mod 5$

(e) $(3x^2 + 2x - 4)(4x^2 + 2)$ in $\mathbb{Z}_5[x]$
$= 12x^4 + 8x^3 - 10x^2 + 4x - 8 = (2x^4 + 3x^3 + 4x + 2) \mod 5$

(f) $(5x^2 + 3x - 2)^2$ in $\mathbb{Z}_{12}[x]$
$= 25x^4 + 30x^3 - 11x^2 - 12x + 4 = (x^4 + 6x^3 + x^2 + 4) \mod 12$

3. Use the division algorithm to find $q(x)$ and $r(x)$ such that $a(x) = q(x)b(x) + r(x)$ with $\deg r(x) < \deg b(x)$ for each of the following pairs of polynomials.

(a) $a(x) = 5x^3 + 6x^2 - 3x + 4$ and $b(x) = x - 2$ in $\mathbb{Z}_7[x]$

$$
\begin{array}{r l}
 & 5x^2 + 16x + 29 \\
x - 2 & \big) \overline{5x^3 + 6x^2 - 3x + 4} \\
 & \underline{-(5x^3 - 10x^2)} \\
 & 0x^3 + 16x^2 - 3x \\
 & \quad \underline{-(16x^2 - 32x)} \\
 & \qquad\qquad 29x + 4 \\
 & \qquad\qquad \underline{-(29x - 58)} \\
 & \qquad\qquad\qquad 62
\end{array}
$$

$5x^3 + 6x^2 - 3x + 4 = (x - 2)(6x^2 + 16x + 29) + 62$
$= [(x - 2)(5x^2 + 2x + 1) + 6] \mod 7$

(b) $a(x) = 6x^4 - 2x^3 + x^2 - 3x + 1$ and $b(x) = x^2 + x - 2$ in $\mathbb{Z}_7[x]$

$$
\begin{array}{r l}
 & 6x^2 - 8x + 21 \\ \cline{2-2}
x^2 + x - 2 \big) & 6x^4 - 2x^3 + \quad x^2 - 3x + 1 \\
 & \underline{-(6x^4 + 6x^3 - 12x^2)} \\
 & \qquad - 8x^3 + 13x^3 - 3x \\
 & \qquad \underline{-(-8x^3 - 8x^2 + 16x)} \\
 & \qquad\qquad 21x^2 - 19x + 1 \\
 & \qquad\qquad \underline{-(21x^2 + 21x - 42)} \\
 & \qquad\qquad\qquad - 40x + 43
\end{array}
$$

$6x^4 - 2x^3 + x^2 - 3x + 1 = (x^2 + x - 2)(6x^2 - 8x + 21) + (-40x + 43)$
$= (x^2 + x - 2)(6x^2 - x) + (2x + 1)$

(c) $a(x) = 4x^5 - x^3 + x^2 + 4$ and $b(x) = x^3 - 2$ in $\mathbb{Z}_5[x]$

$$
\begin{array}{r l}
 & 4x^2 - 1 \\ \cline{2-2}
x^3 - 2 \big) & 4x^5 \;-\; x^3 \;+\; x^2 + 4 \\
 & \underline{-(4x^5 + 0x^3 - 8x^2)} \\
 & \qquad - x^3 + 9x^2 + 4 \\
 & \qquad \underline{-(-x^3 + 0x^2 + 2)} \\
 & \qquad\qquad 9x^2 - 2
\end{array}
$$

$4x^5 - x^3 + x^2 + 4 = (x^3 - 2)(4x^2 - 1) + (9x^2 - 2)$
$= (x^3 - 2)(4x^2 - 1) + (4x^2 - 2) \mod 5$

(d) $a(x) = x^5 + x^3 - x^2 - x$ and $b(x) = x^3 + x$ in $\mathbb{Z}_2[x]$

$$
\begin{array}{r l}
 & x^2 \\ \cline{2-2}
x^3 + x \big) & x^5 + x^3 - x^2 - x \\
 & \underline{-(x^5 + x^3)} \\
 & \qquad 0 - x^2 - x
\end{array}
$$

$x^5 + x^3 - x^2 - x = (x^3 + x)(x^2) + (-x^2 - x) \mod 2$