# Unit 3: Foundations for inference
## 2. Confidence intervals and hypothesis tests

Sta 104 - Summer 2015

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Slides posted at http://bit.ly/sta104su15



- Peer evals - please complete asap after class today
- PS2 feedback:
  - 2.8 (b) - Venn diagrams: intersection is P(A and B), and this value should be subtracted from the values show outside the intersection so that the total probability in the circle for an event adds up to that event's marginal probability.
  - 3.16 -

$$P(X > 2100 | X > 1900) = \frac{P(X > 2100 \text{ and } X > 1900)}{P(X > 1900)} = \frac{P(X > 2100)}{P(X > 1900)}$$

  - Pay attention to which problems are assigned





Hypothesis testing framework:

1. Set the hypotheses.
2. Check assumptions and conditions.
3. Calculate a *test statistic* and a p-value.
4. Make a decision, and interpret it in context of the research question.





1. Set the hypotheses
   - $H_0 : \mu = \textit{null value}$
   - $H_A : \mu <$ or $>$ or $\neq$ *null value*
2. Check assumptions and conditions
   - Independence: random sample/assignment, 10% condition when sampling without replacement
   - Sample size / skew: $n \geq 30$ (or larger if sample is skewed), no extreme skew
3. Calculate a *test statistic* and a p-value (draw a picture!)

$$Z = \frac{\bar{x} - \mu}{SE}, \text{ where } SE = \frac{s}{\sqrt{n}}$$

4. Make a decision, and interpret it in context of the research question
   - If p-value $< \alpha$, reject $H_0$, data provide evidence for $H_A$
   - If p-value $> \alpha$, do not reject $H_0$, data do not provide evidence for $H_A$

**Application exercise: 3.2 Hypothesis testing for a single mean**

See course website for details.



**Clicker question**

Which of the following is the correct interpretation of the p-value from App Ex 3.2?

(a) The probability that average GPA of Duke students has changed since 2001.

(b) The probability that average GPA of Duke students has not changed since 2001.

(c) The probability that average GPA of Duke students has not changed since 2001, if in fact a random sample of 63 Duke students this year have an average GPA of 3.58 or higher.

(d) The probability that a random sample of 63 Duke students have an average GPA of 3.58 or higher, if in fact the average GPA has not changed since 2001.

(e) The probability that a random sample of 63 Duke students have an average GPA of 3.58 or higher or 3.16 or lower, if in fact the average GPA has not changed since 2001.



## Common misconceptions about hypothesis testing

1. P-value is the probability that the null hypothesis is true
   *A p-value is the probability of getting a sample that results in a test statistic as or more extreme than what you actually observed (in the direction of $H_A$, if in fact $H_0$ is correct. It is a conditional probability, conditioned on $H_0$ being correct.*

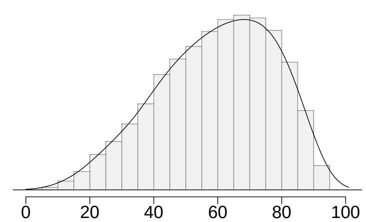
2. A high p-value confirms the null hypothesis.
   *A high p-value means the data do not provide convincing evidence for $H_A$ and hence that $H_0$ can't be rejected.*

3. A low p-value confirms the alternative hypothesis.
   *A low p-value means the data provide convincing evidence for $H_A$, but not necessarily that it is confirmed.*



**Clicker question**

Four plots: Determine which plot (A, B, or C) is which.
(1) At top: distribution for a population ($\mu = 60, \sigma = 18$),
(2) a single random sample of 500 observations from this population,
(3) a distribution of 500 sample means from random samples with size 18,
(4) a distribution of 500 sample means from random samples with size 81.

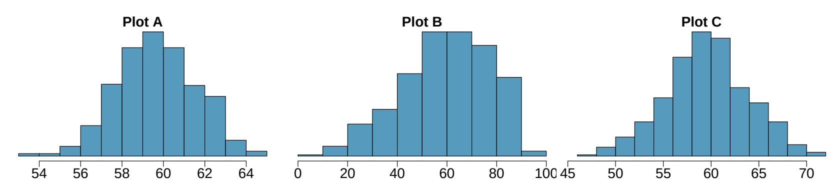



(a) (2) - B; (3) - A; (4) - C

(b) (2) - A; (3) - B; (4) - C

(c) (2) - C; (3) - A; (4) - D

(d) (2) - B; (3) - C; (4) - A



A housing survey was conducted to determine the price of a typical home in Topanga, CA. The mean price of a house was roughly \$1.3 million with a standard deviation of \$300,000. There were no houses listed below \$600,000 but a few houses above \$3 million.

Would you expect most houses in Topanga to cost more or less than \$1.3 million? Hint: What is most likely the shape of this distribution?



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Clicker question

Can we estimate the probability that a randomly chosen house in Topanga costs more than \$1.4 million using the normal distribution?

(a) yes
(b) no



A housing survey was conducted to determine the price of a typical home in Topanga, CA. The mean price of a house was roughly \$1.3 million with a standard deviation of \$300,000. There were no houses listed below \$600,000 but a few houses above \$3 million.

Clicker question

Can we estimate the probability that the mean of 60 randomly chosen houses in Topanga is more than \$1.4 million?

(a) yes
(b) no



A housing survey was conducted to determine the price of a typical home in Topanga, CA. The mean price of a house was roughly \$1.3 million with a standard deviation of \$300,000. There were no houses listed below \$600,000 but a few houses above \$3 million.

What is the probability that the mean of 60 randomly chosen houses in Topanga is more than \$1.4 million?

In order to calculate $P(\bar{X} > 1.4\ mil)$, we need to first determine the distribution of $\bar{X}$. According to the CLT,

$$\bar{X} \sim N\left(mean = 1.3, SE = \frac{0.3}{\sqrt{60}} = 0.0387\right)$$

$$\begin{aligned} P(\bar{X} > 1.4) &= P\left(Z > \frac{1.4 - 1.3}{0.0387}\right) \\ &= P(Z > 2.58) \\ &= 1 - 0.9951 = 0.0049 \end{aligned}$$

**Application exercise: 3.3 Inference for a mean - mechanics**

See course website for details.



**Application exercise: 3.4 Inference for a mean - interpretations**

See course website for details.



## Summary of main ideas

1. Use hypothesis tests to make decisions about population parameters