Homework 3

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Exercise 3.3

```
y.a = c(12,9,12,14,13,13,15,8,15,6)
y.b = c(11,11,10,9,9,8,7,10,6,8,8,9,7)

a.a = 120; a.b = 10
b.a = 12 ; b.b = 1
n.y.a = length(y.a)
n.y.b = length(y.b)
sumy.a = sum(y.a)
sumy.b = sum(y.b)
post.a.a = a.a + sumy.a
post.a.b = n.y.a + a.b

post.b.a = b.a + sumy.b
post.b.b = n.y.b + b.b
```

The closed form of posterior distribution of a poisson sampling model with a gamma prior is $\theta|Y \sim Ga(a + \sum Y, b + n)$

The posterior distribution of $\theta_A|Y_A \sim Ga(237, 20)$, and the posterior distribution of $\theta_B|Y_B \sim Ga(125, 14)$

```
post.mean.a = post.a.a / post.a.b
post.var.a = post.a.a / post.a.b ^2
post.mean.a
```

```
## [1] 11.85
```

```
post.var.a
```

```
## [1] 0.5925
```

```
qgamma(c(.025,.975), post.a.a,post.a.b)
```

```
## [1] 10.38924 13.40545
```

```
post.mean.b = post.b.b / post.b.b
post.var.b = post.b.b / post.b.b ^2
post.mean.b
```

```
## [1] 1
```

```
post.var.b
```

```
## [1] 0.07142857
```

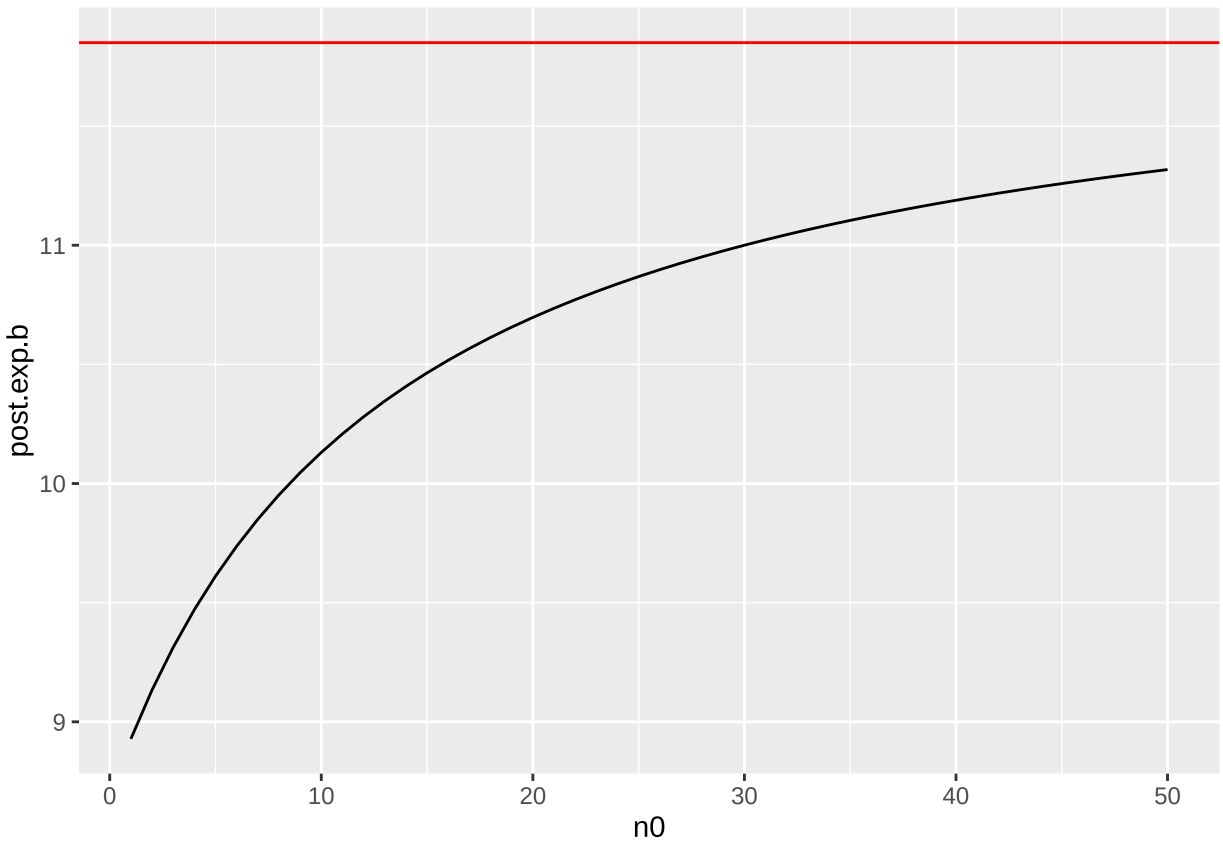
```
qgamma(c(.025,.975), post.b.b,post.b.b)
```

```
## [1] 0.5467093 1.5878854
```

The posterior mean of $\theta = 11.85$

Part B

```
n0 = 1:50
post.b.a.n = 12*n0 + sumy.b
post.b.b.n = n.y.b + n0
post.exp.b = post.b.a.n / post.b.b.n
n0.plot.frame = data.frame(n0,post.exp.b)
plot.exp = ggplot(n0.plot.frame, aes(x= n0, y = post.exp.b))
plot.exp + geom_line() + geom_hline(yintercept = post.mean.a, color = "red")
```



It seems clear that n_0 would need to approach infinity for the posterior expectation of $\theta_B|Y_B$ to approach $E(\theta_A|Y_A)$

Part C

If knowledge about population A informs us about population B, then it doesn't make sense to have a prior $p(\theta_A, \theta_B) = p(\theta_A) \times p(\theta_B)$ because that prior operates under the assumption that they are independent. A informing B makes that assumption invalid.

Exercise 3.9

Exercise 4.1

Exercise 4.2