

# HW07

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## Exercise 7.3

### Part A

```
blue = read.table("/home/grad/zmw5/Fall 2016/STA 601/sta601/HW/bluecrab.dat")
orange = read.table("/home/grad/zmw5/Fall 2016/STA 601/sta601/HW/orangecrab.dat")
L.b.0 = S.b.0 = cov(blue)
L.o.0 = S.o.0 = cov(orange)
mu.b.0 = ybar.b = apply(blue,2,mean)
mu.o.0 = ybar.o = apply(orange,2,mean)

L.b.0.inv = S.b.0.inv = solve(L.b.0)
L.o.0.inv = S.o.0.inv = solve(L.o.0)

ybar.b = apply(blue,2,mean)
ybar.o = apply(orange,2,mean)
Sigma.b = cov(blue)
Sigma.o = cov(orange)
Sigma.b.inv = solve(Sigma.b)
Sigma.o.inv = solve(Sigma.o)

n.b = nrow(blue)
n.o = nrow(orange)
nu.0 = 4

n.iter = 10000

theta.b.post = theta.o.post = NULL
Sigma.b.post = Sigma.o.post = NULL

for(i in 1:n.iter){
  ### Update theta.b, theta.o
  Ln.b = solve(S.b.0.inv + n.b* Sigma.b.inv)
  Ln.o = solve(S.o.0.inv + n.o * Sigma.o.inv)
  mu.n.b = Ln.b %*% (L.b.0.inv %*% mu.b.0 + n.b*Sigma.b.inv %*% ybar.b)
  mu.n.o = Ln.o %*% (L.o.0.inv %*% mu.o.0 + n.o*Sigma.o.inv %*% ybar.o)
  theta.b = mvrnorm(1,mu.n.b,Ln.b)
  theta.o = mvrnorm(1,mu.n.o,Ln.o)

  theta.b.post = rbind(theta.b.post,theta.b)
  theta.o.post = rbind(theta.o.post,theta.o)
}
```

```

### Update Sigma.b, Sigma.o
Sn.b = S.b.0 + (t(blue) - c(theta.b)) %*% t(t(blue) - c(theta.b))
Sn.o = S.o.0 + (t(orange) - c(theta.o)) %*% t(t(orange) - c(theta.o))
Sigma.b = solve(rWishart(1,nu.0 + n.b, solve(Sn.b))[,1])
Sigma.o = solve(rWishart(1,nu.0 + n.o, solve(Sn.o))[,1])
Sigma.b.post = rbind(Sigma.b.post,c(Sigma.b))
Sigma.o.post = rbind(Sigma.o.post,c(Sigma.o))
}

```

## Part B

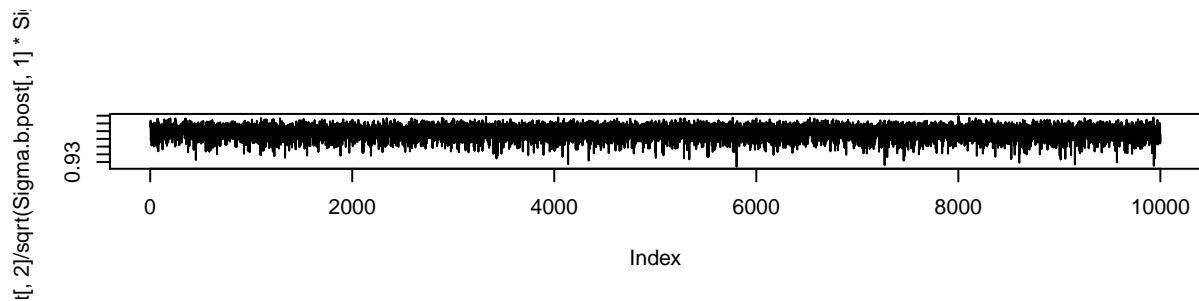
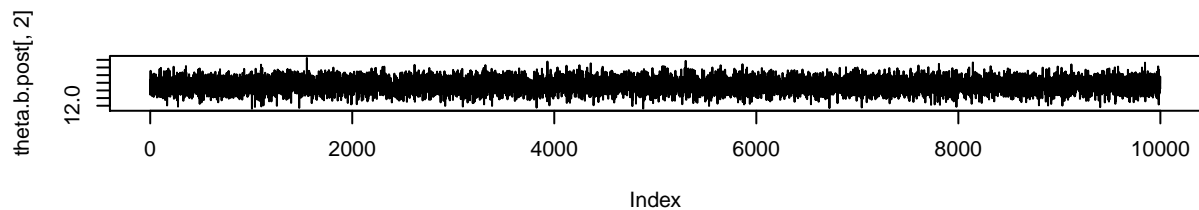
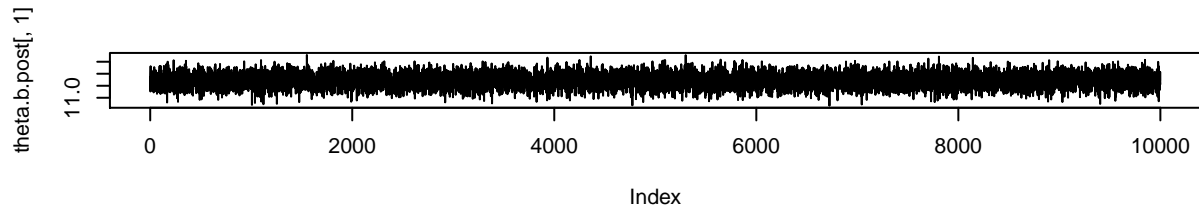
```

theta.b.post = as.data.frame(theta.b.post)
names(theta.b.post) = c("theta1", "theta2")

theta.o.post = as.data.frame(theta.o.post)
names(theta.o.post) = c("theta1", "theta2")

par(mfrow = c(3,1))
plot(theta.b.post[,1], type = "l")
plot(theta.b.post[,2], type = "l")
plot(Sigma.b.post[,2] / sqrt(Sigma.b.post[,1]*Sigma.b.post[,4]), type = "l")

```

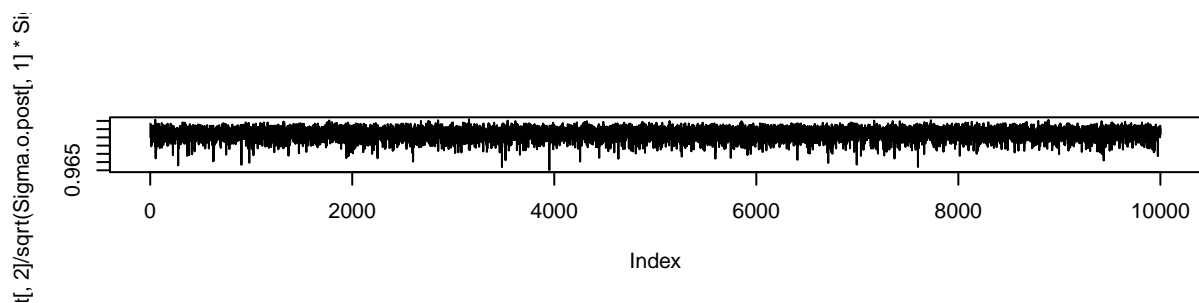
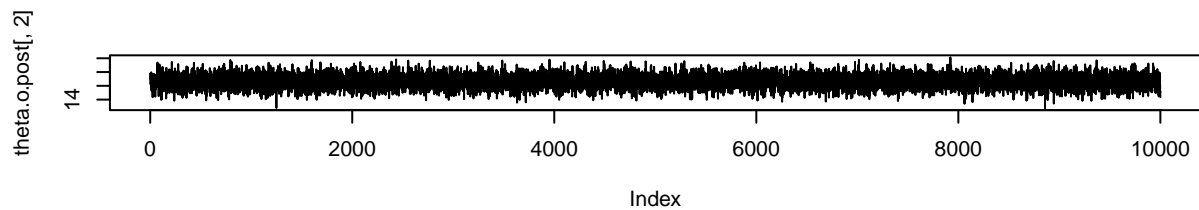
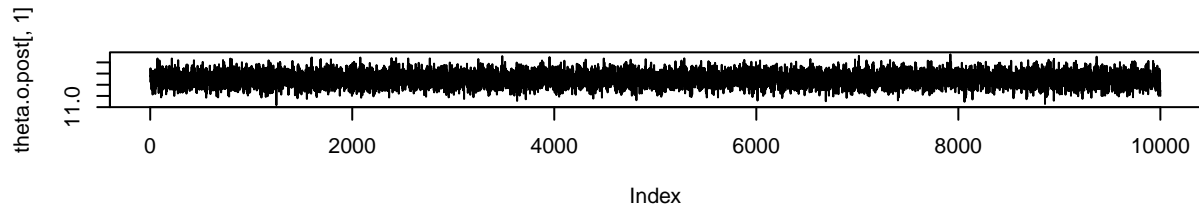


```

par(mfrow = c(3,1))
plot(theta.o.post[,1], type = "l")

```

```
plot(theta.o.post[,2], type = "l")
plot(Sigma.o.post[,2] / sqrt(Sigma.o.post[,1]*Sigma.o.post[,4]), type = "l")
```



```
both.theta.post = cbind(theta.b.post, theta.o.post)
```

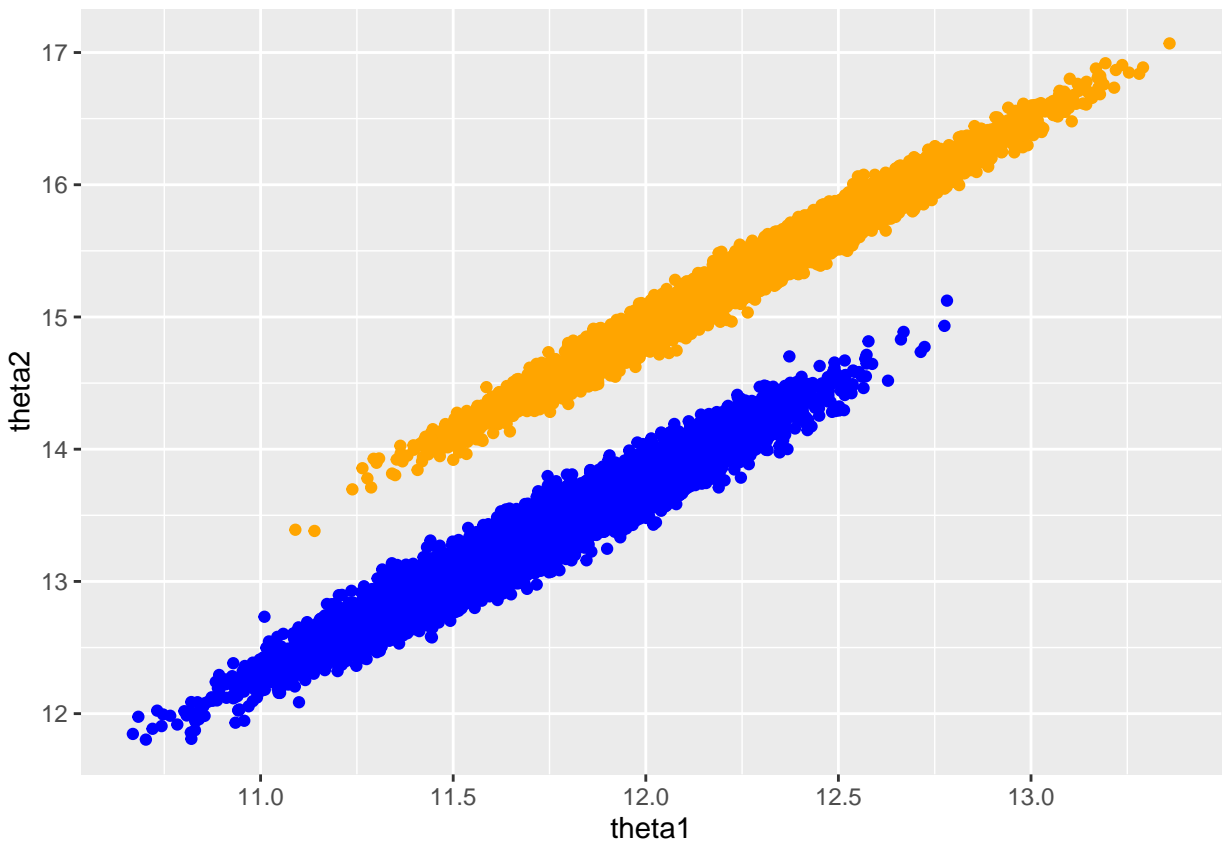
```
## Warning in data.row.names(row.names, row_si, i): some row.names duplicated:
```

```
## 2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,67,68,69,70,71,72,73,74,75,76,77,78,79,80,81,82,83,84,85,86,87,88,89,90,91,92,93,94,95,96,97,98,99,100,101,102,103,104,105,106,107,108,109,110,111,112,113,114,115,116,117,118,119,120,121,122,123,124,125,126,127,128,129,130,131,132,133,134,135,136,137,138,139,140,141,142,143,144,145,146,147,148,149,150,151,152,153,154,155,156,157,158,159,160,161,162,163,164,165,166,167,168,169,170,171,172,173,174,175,176,177,178,179,180,181,182,183,184,185,186,187,188,189,190,191,192,193,194,195,196,197,198,199,200,201,202,203,204,205,206,207,208,209,210,211,212,213,214,215,216,217,218,219,220,221,222,223,224,225,226,227,228,229,230,231,232,233,234,235,236,237,238,239,240,241,242,243,244,245,246,247,248,249,250,251,252,253,254,255,256,257,258,259,260,261,262,263,264,265,266,267,268,269,270,271,272,273,274,275,276,277,278,279,280,281,282,283,284,285,286,287,288,289,290,291,292,293,294,295,296,297,298,299,300,301,302,303,304,305,306,307,308,309,310,311,312,313,314,315,316,317,318,319,320,321,322,323,324,325,326,327,328,329,330,331,332,333,334,335,336,337,338,339,340,341,342,343,344,345,346,347,348,349,350,351,352,353,354,355,356,357,358,359,360,361,362,363,364,365,366,367,368,369,370,371,372,373,374,375,376,377,378,379,380,381,382,383,384,385,386,387,388,389,390,391,392,393,394,395,396,397,398,399,400,401,402,403,404,405,406,407,408,409,410,411,412,413,414,415,416,417,418,419,420,421,422,423,424,425,426,427,428,429,430,431,432,433,434,435,436,437,438,439,440,441,442,443,444,445,446,447,448,449,450,451,452,453,454,455,456,457,458,459,460,461,462,463,464,465,466,467,468,469,470,471,472,473,474,475,476,477,478,479,480,481,482,483,484,485,486,487,488,489,490,491,492,493,494,495,496,497,498,499,500,501,502,503,504,505,506,507,508,509,510,511,512,513,514,515,516,517,518,519,520,521,522,523,524,525,526,527,528,529,530,531,532,533,534,535,536,537,538,539,540,541,542,543,544,545,546,547,548,549,550,551,552,553,554,555,556,557,558,559,560,561,562,563,564,565,566,567,568,569,570,571,572,573,574,575,576,577,578,579,580,581,582,583,584,585,586,587,588,589,590,591,592,593,594,595,596,597,598,599,600,601,602,603,604,605,606,607,608,609,610,611,612,613,614,615,616,617,618,619,620,621,622,623,624,625,626,627,628,629,630,631,632,633,634,635,636,637,638,639,640,641,642,643,644,645,646,647,648,649,650,651,652,653,654,655,656,657,658,659,660,661,662,663,664,665,666,667,668,669,670,671,672,673,674,675,676,677,678,679,680,681,682,683,684,685,686,687,688,689,690,691,692,693,694,695,696,697,698,699,700,701,702,703,704,705,706,707,708,709,710,711,712,713,714,715,716,717,718,719,720,721,722,723,724,725,726,727,728,729,730,731,732,733,734,735,736,737,738,739,740,741,742,743,744,745,746,747,748,749,750,751,752,753,754,755,756,757,758,759,760,761,762,763,764,765,766,767,768,769,770,771,772,773,774,775,776,777,778,779,780,781,782,783,784,785,786,787,788,789,790,791,792,793,794,795,796,797,798,799,800,801,802,803,804,805,806,807,808,809,810,811,812,813,814,815,816,817,818,819,820,821,822,823,824,825,826,827,828,829,830,831,832,833,834,835,836,837,838,839,840,841,842,843,844,845,846,847,848,849,850,851,852,853,854,855,856,857,858,859,860,861,862,863,864,865,866,867,868,869,870,871,872,873,874,875,876,877,878,879,880,881,882,883,884,885,886,887,888,889,890,891,892,893,894,895,896,897,898,899,900,901,902,903,904,905,906,907,908,909,910,911,912,913,914,915,916,917,918,919,920,921,922,923,924,925,926,927,928,929,930,931,932,933,934,935,936,937,938,939,940,941,942,943,944,945,946,947,948,949,950,951,952,953,954,955,956,957,958,959,960,961,962,963,964,965,966,967,968,969,970,971,972,973,974,975,976,977,978,979,980,981,982,983,984,985,986,987,988,989,990,991,992,993,994,995,996,997,998,999,1000,1001,1002,1003,1004,1005,1006,1007,1008,1009,1010,1011,1012,1013,1014,1015,1016,1017,1018,1019,1020,1021,1022,1023,1024,1025,1026,1027,1028,1029,1030,1031,1032,1033,1034,1035,1036,1037,1038,1039,1040,1041,1042,1043,1044,1045,1046,1047,1048,1049,1050,1051,1052,1053,1054,1055,1056,1057,1058,1059,1060,1061,1062,1063,1064,1065,1066,1067,1068,1069,1070,1071,1072,1073,1074,1075,1076,1077,1078,1079,1080,1081,1082,1083,1084,1085,1086,1087,1088,1089,1090,1091,1092,1093,1094,1095,1096,1097,1098,1099,1100,1101,1102,1103,1104,1105,1106,1107,1108,1109,1110,1111,1112,1113,1114,1115,1116,1117,1118,1119,1120,1121,1122,1123,1124,1125,1126,1127,1128,1129,1130,1131,1132,1133,1134,1135,1136,1137,1138,1139,1140,1141,1142,1143,1144,1145,1146,1147,1148,1149,1150,1151,1152,1153,1154,1155,1156,1157,1158,1159,1160,1161,1162,1163,1164,1165,1166,1167,1168,1169,1170,1171,1172,1173,1174,1175,1176,1177,1178,1179,1180,1181,1182,1183,1184,1185,1186,1187,1188,1189,1190,1191,1192,1193,1194,1195,1196,1197,1198,1199,1200,1201,1202,1203,1204,1205,1206,1207,1208,1209,1210,1211,1212,1213,1214,1215,1216,1217,1218,1219,1220,1221,1222,1223,1224,1225,1226,1227,1228,1229,1230,1231,1232,1233,1234,1235,1236,1237,1238,1239,1240,1241,1242,1243,1244,1245,1246,1247,1248,1249,1250,1251,1252,1253,1254,1255,1256,1257,1258,1259,1260,1261,1262,1263,1264,1265,1266,1267,1268,1269,1270,1271,1272,1273,1274,1275,1276,1277,1278,1279,1280,1281,1282,1283,1284,1285,1286,1287,1288,1289,1290,1291,1292,1293,1294,1295,1296,1297,1298,1299,1300,1301,1302,1303,1304,1305,1306,1307,1308,1309,1310,1311,1312,1313,1314,1315,1316,1317,1318,1319,1320,1321,1322,1323,1324,1325,1326,1327,1328,1329,1330,1331,1332,1333,1334,1335,1336,1337,1338,1339,1340,1341,1342,1343,1344,1345,1346,1347,1348,1349,1350,1351,1352,1353,1354,1355,1356,1357,1358,1359,1360,1361,1362,1363,1364,1365,1366,1367,1368,1369,1370,1371,1372,1373,1374,1375,1376,1377,1378,1379,1380,1381,1382,1383,1384,1385,1386,1387,1388,1389,1390,1391,1392,1393,1394,1395,1396,1397,1398,1399,1400,1401,1402,1403,1404,1405,1406,1407,1408,1409,1410,1411,1412,1413,1414,1415,1416,1417,1418,1419,1420,1421,1422,1423,1424,1425,1426,1427,1428,1429,1430,1431,1432,1433,1434,1435,1436,1437,1438,1439,1440,1441,1442,1443,1444,1445,1446,1447,1448,1449,1450,1451,1452,1453,1454,1455,1456,1457,1458,1459,1460,1461,1462,1463,1464,1465,1466,1467,1468,1469,1470,1471,1472,1473,1474,1475,1476,1477,1478,1479,1480,1481,1482,1483,1484,1485,1486,1487,1488,1489,1490,1491,1492,1493,1494,1495,1496,1497,1498,1499,1500,1501,1502,1503,1504,1505,1506,1507,1508,1509,1510,1511,1512,1513,1514,1515,1516,1517,1518,1519,1520,1521,1522,1523,1524,1525,1526,1527,1528,1529,1530,1531,1532,1533,1534,1535,1536,1537,1538,1539,1540,1541,1542,1543,1544,1545,1546,1547,1548,1549,1550,1551,1552,1553,1554,1555,1556,1557,1558,1559,1560,1561,1562,1563,1564,1565,1566,1567,1568,1569,1570,1571,1572,1573,1574,1575,1576,1577,1578,1579,1580,1581,1582,1583,1584,1585,1586,1587,1588,1589,1590,1591,1592,1593,1594,1595,1596,1597,1598,1599,1600,1601,1602,1603,1604,1605,1606,1607,1608,1609,1610,1611,1612,1613,1614,1615,1616,1617,1618,1619,1620,1621,1622,1623,1624,1625,1626,1627,1628,1629,1630,1631,1632,1633,1634,1635,1636,1637,1638,1639,1640,1641,1642,1643,1644,1645,1646,1647,1648,1649,1650,1651,1652,1653,1654,1655,1656,1657,1658,1659,1660,1661,1662,1663,1664,1665,1666,1667,1668,1669,1670,1671,1672,1673,1674,1675,1676,1677,1678,1679,1680,1681,1682,1683,1684,1685,1686,1687,1688,1689,1690,1691,1692,1693,1694,1695,1696,1697,1698,1699,1700,1701,1702,1703,1704,1705,1706,1707,1708,1709,1710,1711,1712,1713,1714,1715,1716,1717,1718,1719,1720,1721,1722,1723,1724,1725,1726,1727,1728,1729,1730,1731,1732,1733,1734,1735,1736,1737,1738,1739,1740,1741,1742,1743,1744,1745,1746,1747,1748,1749,1750,1751,1752,1753,1754,1755,1756,1757,1758,1759,1760,1761,1762,1763,1764,1765,1766,1767,1768,1769,1770,1771,1772,1773,1774,1775,1776,1777,1778,1779,1780,1781,1782,1783,1784,1785,1786,1787,1788,1789,1790,1791,1792,1793,1794,1795,1796,1797,1798,1799,1800,1801,1802,1803,1804,1805,1806,1807,1808,1809,1810,1811,1812,1813,1814,1815,1816,1817,1818,1819,1820,1821,1822,1823,1824,1825,1826,1827,1828,1829,1830,1831,1832,1833,1834,1835,1836,1837,1838,1839,1840,1841,1842,1843,1844,1845,1846,1847,1848,1849,1850,1851,1852,1853,1854,1855,1856,1857,1858,1859,1860,1861,1862,1863,1864,1865,1866,1867,1868,1869,1870,1871,1872,1873,1874,1875,1876,1877,1878,1879,1880,1881,1882,1883,1884,1885,1886,1887,1888,1889,1890,1891,1892,1893,1894,1895,1896,1897,1898,1899,1900,1901,1902,1903,1904,1905,1906,1907,1908,1909,1910,1911,1912,1913,1914,1915,1916,1917,1918,1919,1920,1921,1922,1923,1924,1925,1926,1927,1928,1929,1930,1931,1932,1933,1934,1935,1936,1937,1938,1939,1940,1941,1942,1943,1944,1945,1946,1947,1948,1949,1950,1951,1952,1953,1954,1955,1956,1957,1958,1959,1960,1961,1962,1963,1964,1965,1966,1967,1968,1969,1970,1971,1972,1973,1974,1975,1976,1977,1978,1979,1980,1981,1982,1983,1984,1985,1986,1987,1988,1989,1990,1991,1992,1993,1994,1995,1996,1997,1998,1999,2000,2001,2002,2003,2004,2005,2006,2007,2008,2009,2010,2011,2012,2013,2014,2015,2016,2017,2018,2019,2020,2021,2022,2023,2024,2025,2026,2027,2028,2029,2030,2031,2032,2033,2034,2035,2036,2037,2038,2039,2040,2041,2042,2043,2044,2045,2046,2047,2048,2049,2050,2051,2052,2053,2054,2055,2056,2057,2058,2059,2060,2061,2062,2063,2064,2065,2066,2067,2068,2069,2070,2071,2072,2073,2074,2075,2076,2077,2078,2079,2080,2081,2082,2083,2084,2085,2086,2087,2088,2089,2090,2091,2092,2093,2094,2095,2096,2097,2098,2099,2100,2101,2102,2103,2104,2105,2106,2107,2108,2109,2110,2111,2112,2113,2114,2115,2116,2117,2118,2119,2120,2121,2122,2123,2124,2125,2126,2127,2128,2129,2130,2131,2132,2133,2134,2135,2136,2137,2138,2139,2140,2141,2142,2143,2144,2145,2146,2147,2148,2149,2150,2151,2152,2153,2154,2155,2156,2157,2158,2159,2160,2161,2162,2163,2164,2165,2166,2167,2168,2169,2170,2171,2172,2173,2174,2175,2176,2177,2178,2179,2180,2181,2182,2183,2184,2185,2186,2187,2188,2189,2190,2191,2192,2193,2194,2195,2196,2197,2198,2199,2200,2201,2202,2203,2204,2205,2206,2207,2208,2209,2210,2211,2212,2213,2214,2215,2216,2217,2218,2219,2220,2221,2222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```

NULL

NULL

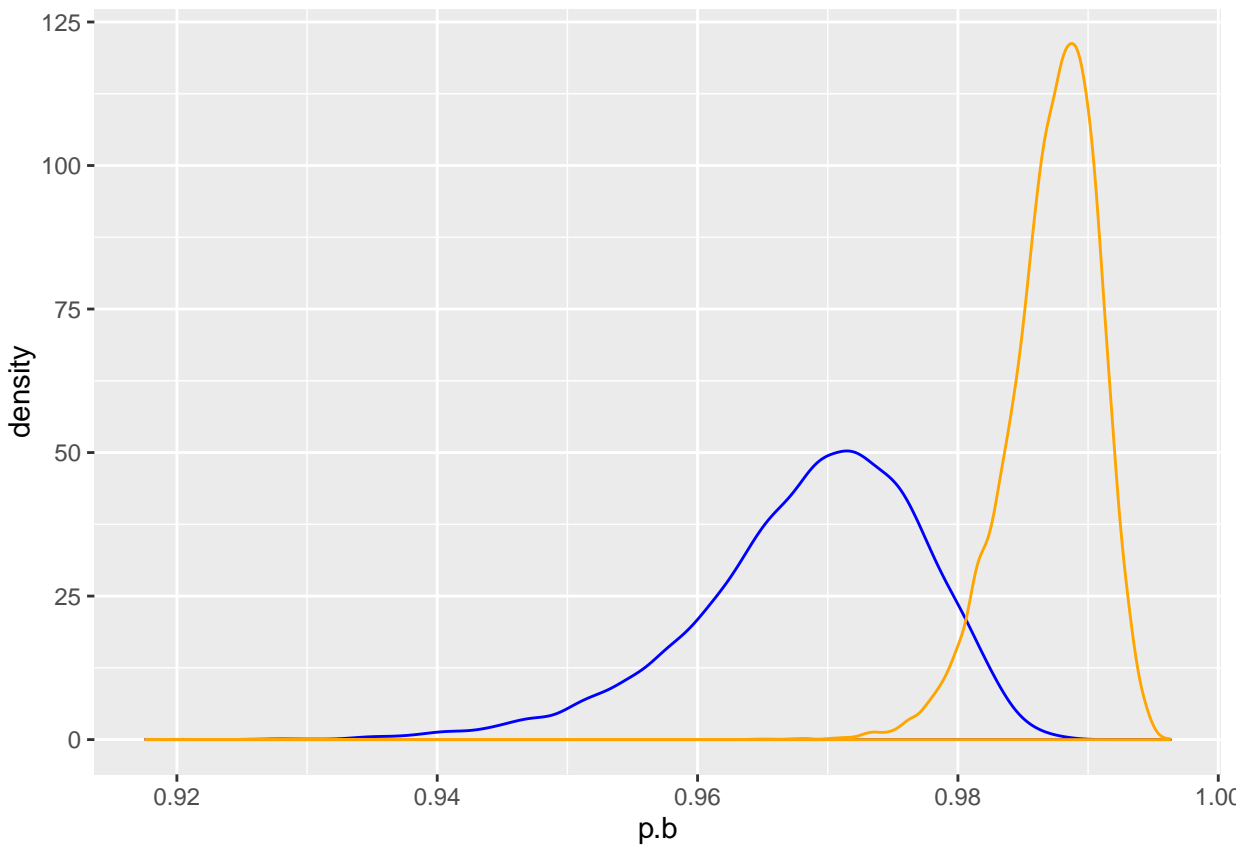
```
ggplot(data = theta.b.post, aes(x = theta1, y = theta2)) + geom_point(color = "blue") + geom_point(data
```



## Part C

```
p.b = Sigma.b.post[,2] / sqrt(Sigma.b.post[,1] * Sigma.b.post[,4])
p.o = Sigma.o.post[,2] / sqrt(Sigma.o.post[,1]*Sigma.o.post[,4])

ggplot(data = as.data.frame(p.b) , aes(x = p.b)) + geom_density(color = "blue") + geom_density(data = a
```



```
mean(p.b < p.o)
```

```
## [1] 0.9902
```

## Exercise 7.5

### Part A

```
inter.exp = read.table("/home/grad/zmw5/Fall 2016/STA 601/sta601/HW/interexp.dat", header = TRUE)
apply(inter.exp,2,mean)
```

```
## yA yB
```

```
## NA NA
```

```
theta = sapply(inter.exp,mean,na.rm = TRUE)
```

```
thetaA = theta[1]
```

```
thetaB = theta[2]
```

```
## Calculate Sigma
```

```
sapply(inter.exp,var)
```

```
## yA yB
```

```
## NA NA
```

```
sigma2 = sapply(inter.exp,var,na.rm = TRUE)
```

```
sigma2.A = sigma2[1]
```

```
sigma2.B = sigma2[2]
## Calculate correlation matrix
complete.rho = cor(inter.exp, use = "complete")
rho = complete.rho[1,2]
```

## Part B

```
A.missingB = inter.exp[is.na(inter.exp[,2]),1]
B.missingA = inter.exp[is.na(inter.exp[,1]),2]

impute.B = thetaB + (A.missingB - thetaA)* rho *sqrt(sigma2.B / sigma2.A)
impute.A = thetaA + (B.missingA - thetaB) * rho * sqrt(sigma2.A / sigma2.B)

imp.data = inter.exp
imp.data[is.na(imp.data[,2]),2] = impute.B
imp.data[is.na(imp.data[,1]),1] = impute.A

t.results = t.test(imp.data[,1],imp.data[,2], paired = TRUE)
t.results$conf.int

## [1] -0.9850730 -0.2383347
## attr(,"conf.level")
## [1] 0.95
```

## Part C

I will use the unit information prior

```
ybar = apply(inter.exp,2,mean,na.rm = TRUE)
complete = which(complete.cases(inter.exp))
## Prior on Sigma
S = (t(inter.exp[complete,]) - ybar) %*% t(t(inter.exp[complete,]) - ybar)/length(complete)
nu.0 = nrow(S) + 2
n = nrow(inter.exp)

n.iter = 10000

y.A.samps = y.B.samps = matrix(0, nrow = n.iter,ncol = n)
theta.post = matrix(0,nrow=n.iter, ncol = 2)
names(theta.post) = c("thetaA","thetaB")

#Starting values
Y = imp.data
Sigma = S
theta = ybar

miss.A = which(is.na(inter.exp$yA))
miss.B = which(is.na(inter.exp$yB))

for(i in 1:n.iter){
  # Update theta
  y.bar.samp = apply(Y,2,mean)
```

```

theta = mvrnorm(1,y.bar.samp,Sigma / (n+1))
theta.post[i, ] <- theta

# Update Sigma
Sn<- S + ( t(Y)-c(theta) )%*%t( t(Y)-c(theta) )
Sigma<-solve(rWishart(1, nu.0+n, solve(Sn))[, , 1])
sigma2.A <- Sigma[1, 1]
sigma2.B <- Sigma[2, 2]
rho <- Sigma[1,2] / sqrt(sigma2.A*sigma2.B)

Y[miss.A,1] = rnorm(length(miss.A),(rho*sqrt(sigma2.A/sigma2.B))*(Y[miss.A,"yB"] - theta[2]),sqrt(sigma2.A))
Y[miss.B,2] = rnorm(length(miss.B),(rho*sqrt(sigma2.B/sigma2.A))*(Y[miss.B,"yA"] - theta[1]), sqrt(sigma2.B))

y.A.samps[i, ] <- Y[, 1]
y.B.samps[i, ] <- Y[, 2]
}

theta.diff = theta.post[,1] - theta.post[,2]
cred.int = quantile(theta.diff, c(.025,.975))
cred.int

##      2.5%      97.5%
## -3.603018  3.864774

```

The credible interval of interest is -3.6030181, 3.8647738, which means that there is a .95 probability that  $\theta_A - \theta_B|y$  lies in th interval -3.6030181, 3.8647738. # Exercise 8.2

## Part A

```

n.a = n.b = 16
y.bar.a = 75.2
s.a = 7.3
y.bar.b = 77.5
s.b = 8.1

delta0 = seq(-4,4,by = 2)
tau2.0 = c(10,50,100,500)

```