Unit 6: Introduction to linear regression

1. Introduction to regression

STA 104 - Summer 2017

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Slides posted at

http://www2.stat.duke.edu/courses/Summer17/sta104.001-1/

- MT 2 grades have been posted today!

The CDC monitors the physical activity level of Americans. A recent survey on a random sample of 23,129 Americans yielded a 95% confidence interval of 61.1% to 62.9% for the proportion of Americans who walk for at least 10 minutes per day. Which is the most accurate statement?

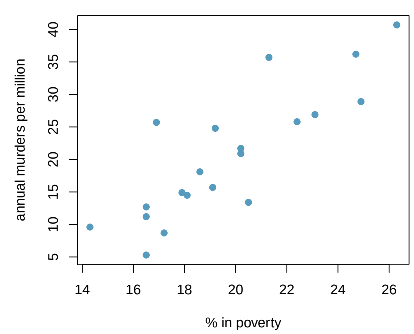
- A. 95% of random samples of 23,129 Americans will yield confidence intervals between 61.1% and 62.9%.
- B. This interval does not support the claim that less than 50% of Americans walk at least 10 minutes per day.
- C. We are 95% confident that each American walks for at least 10 minutes per day on 61.1% to 62.9% of the days.
- D. Between 61.1% and 62.9% of random samples of 23,129 Americans are expected to yield confidence intervals that contain the true proportion of Americans who walk for at least 10 minutes per day.
- E. 95% of the time the true proportion of Americans who walk for at least 10 minutes per day is between 61.1% to 62.9%.

- So far we have worked with single numerical and categorical variables, and explored relationships between numerical and categorical, and two categorical variables.
- In this unit we will learn to quantify the relationship between two numerical variables, as well as modeling numerical response variables using a numerical or categorical explanatory variable.
- In the next unit we'll learn to model numerical variables using many explanatory variables at once.

Clicker question

Which of the following is the best guess for the correlation between annual murders per million and percentage living in poverty?

(a) -1.52
(b) -0.63
(c) -0.12
(d) 0.02
(e) 0.84

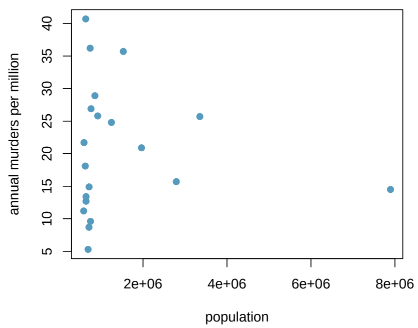


Guessing the correlation

Clicker question

Which of the following is the best guess for the correlation between annual murders per million and population size?

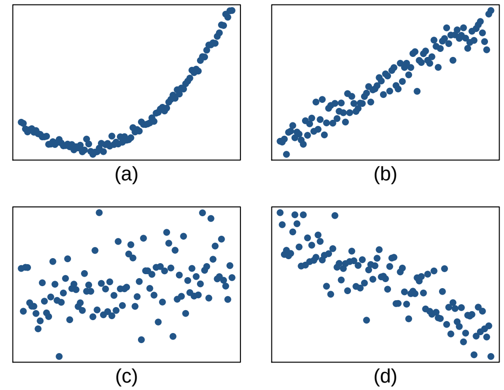
(a) -0.97
(b) -0.61
(c) -0.06
(d) 0.55
(e) 0.97



Assessing the correlation

Clicker question

Which of the following is has the strongest correlation, i.e. correlation coefficient closest to +1 or -1?

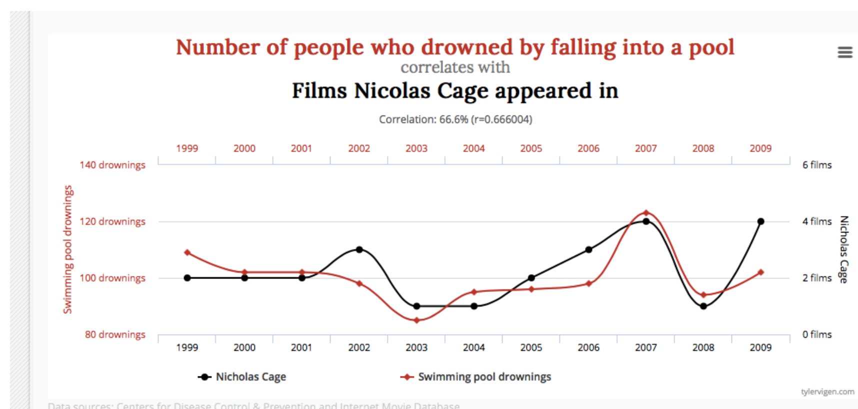


Play the game!

To sharpen your correlation guessing abilities:
http://guessthecorrelation.com/

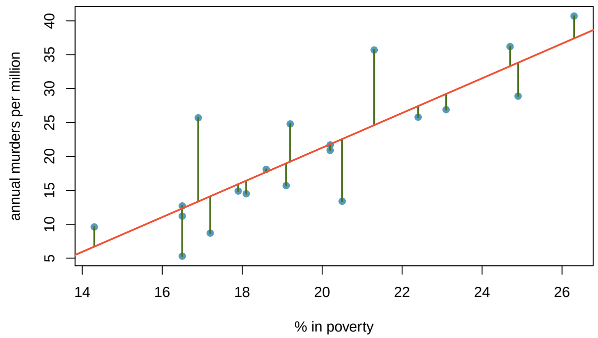
Spurious correlations

Remember: correlation does not always imply causation!
http://www.tylervigen.com/



(2) Least squares line minimizes squared residuals

- Residuals are the leftovers from the model fit, and calculated as the difference between the observed and predicted y:
  $e_i = y_i - \hat{y}_i$
- The least squares line minimizes squared residuals:
  - Population data: $\hat{y} = \beta_0 + \beta_1 x$
  - Sample data: $\hat{y} = b_0 + b_1 x$



(3) Interpreting the last squares line

- *Slope:* For each unit increase in x, y is expected to be higher/lower on average by the slope.

$$b_1 = \frac{s_y}{s_x} R$$

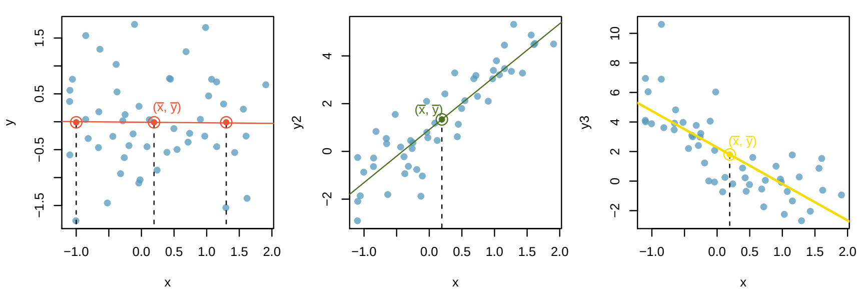
- *Intercept:* When $x = 0$, y is expected to equal the intercept.

$$b_0 = \bar{y} - b_1 \bar{x}$$

  - The calculation of the intercept uses the fact the a regression line **always** passes through $(\bar{x}, \bar{y})$.

Why does the regression line **always** pass through $(\bar{x}, \bar{y})$?

- If there is no relationship between x and y ($b_1 = 0$), the best guess for $\hat{y}$ for any value of x is $\bar{y}$.
- Even when there is a relationship between x and y ($b_1 \neq 0$), the best guess for $\hat{y}$ when $x = \bar{x}$ is still $\bar{y}$.



Application exercise: 6.1 Linear model

See course website for details

Clicker question

What is the interpretation of the slope?

(a) Each additional percentage in those living in poverty increases number of annual murders per million by 2.56.

(b) For each percentage increase in those living in poverty, the number of annual murders per million is expected to be higher by 2.56 on average.

(c) For each percentage increase in those living in poverty, the number of annual murders per million is expected to be lower by 29.91 on average.

(d) For each percentage increase annual murders per million, the percentage of those living in poverty is expected to be higher by 2.56 on average.

Clicker question

Suppose you want to predict annual murder count (per million) for a series of districts that were not included in the dataset. For which of the following districts would you be most comfortable with your prediction?

A district where % in poverty =

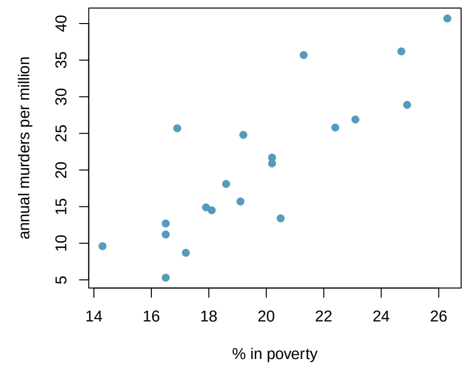
(a) 5%

(b) 15%

(c) 20%

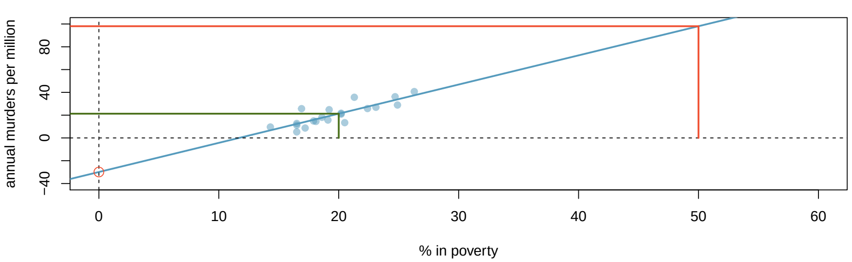
(d) 26%

(e) 40%



A note about the intercept

Sometimes the intercept might be an extrapolation: useful for adjusting the height of the line, but meaningless in the context of the data.



Calculating predicted values

By hand: $\widehat{murder} = -29.91 + 2.56\ poverty$

The predicted number of murders per million per year for a county with 20% poverty rate is:

$$\widehat{murder} = -29.91 + 2.56 \times 20 = 21.29$$

In R:

```
# load data
murder <- read.csv("https://stat.duke.edu/~mc301/data/murder.csv")
# fit model
m_mur_pov <- lm(annual_murders_per_mil ~ perc_pov, data = murder)
# create new data
newdata <- data.frame(perc_pov = 20)
# predict
predict(m_mur_pov, newdata)
```

```
       1
21.28663
```

1. Correlation coefficient describes the strength and direction of the linear association between two numerical variables
2. Least squares line minimizes squared residuals
3. Interpreting the least squares line
4. Predict, but don't extrapolate