Unit 3: Foundations for inference

2. Confidence intervals

Sta 104 - Summer 2018, Term 1

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Slides posted at https://www2.stat.duke.edu/courses/Summer18/sta104.001-1/

1. Statistical inference methods based on the CLT depend on the same conditions as the CLT

Always check these in context of the data and the research question!

- 1. Independence: Sampled observations must be independent.
 - * This is difficult to verify, but is more likely if
 - random sampling/assignment is used, and,
 - if sampling without replacement, n < 10% of the population.
- 2. Sample size/skew: Either the population distribution is normal or n>30 and the population distribution is not extremely skewed (the more skewed the distribution, the higher n necessary for the CLT to apply).
 - * This is also difficult to verify for the population, but we can check it using the sample data, and assume that the sample mirrors the population.

▶ Midterm 1 Review tomorrow in lecture

- ▶ Midterm 1 (in-class) Thursday May 31
- ▶ Lab 3 is due Thursday (12:45 pm)

2. Use confidence intervals to estimate population parameters

CI: point estimate ± margin of error

If the parameter of interest is the population mean, and the point estimate is the sample mean,

$$\bar{x} \pm Z^{\star} \frac{s}{\sqrt{n}}$$

Clicker question

What is the critical value (Z^*) for a confidence interval at the 91% confidence level?

- (a) $Z^* = 1.34$
- **b** $Z^* = 1.65$
- **©** $Z^* = 1.70$
- **d** $Z^* = 1.96$
- (a) $Z^* = 2.33$

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4. Calculate the sample size a priori to achieve desired margin of error

$$ME = z^* \frac{s}{\sqrt{n}}$$

So if we know the desired ME, and confidence level (and hence Z^*), and the sample standard deviation, we can solve for n.

1. The confidence level of a confidence interval is the probability that the true population parameter is in the confidence interval you construct for a single sample.

The confidence level is equal to the proportion of random samples that result in confidence intervals that contain the true pop. parameter.

2. A narrower confidence interval is always better.

This is incorrect since the width is a function of both the confidence level and the standard error.

 A wider interval means less confidence.
 This is incorrect since it is possible to make very precise statements with very little confidence.

Application exercise: 3.1 Confidence interval for a single mean

See course website for details.

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Summary of main ideas

- 1. Statistical inference methods based on the CLT depend on the same conditions as the CLT
- 2. Use confidence intervals to estimate population parameters
- 3. Critical value depends on the confidence level
- 4. Calculate the sample size a priori to achieve desired margin of error