

Unit 3: Foundations for inference

3. Hypothesis tests

Sta 104 - Summer 2018, Term 2

Duke University, Department of Statistical Science

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Slides posted at <https://www2.stat.duke.edu/courses/Summer18/sta104.001-2/>

- ▶ Performance Assessment 3 is due today at 11:55 pm
- ▶ Problem Set 3 is due today at 11:55 pm
- ▶ Readiness Assessment 4 is Thursday in-class
- ▶ Lab 4 is due Thursday at 12:45 pm

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1. Use hypothesis tests to make decisions about population parameters

Hypothesis testing framework:

1. Set the hypotheses.
2. Check assumptions and conditions.
3. Calculate a *test statistic* and a p-value.
4. Make a decision, and interpret it in context of the research question.

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Hypothesis testing for a population mean

1. Set the hypotheses
 - $H_0 : \mu = \text{null value}$
 - $H_A : \mu < \text{or } > \text{ or } \neq \text{null value}$
2. Check assumptions and conditions
 - Independence: random sample/assignment, 10% condition when sampling without replacement
 - Sample size / skew: $n \geq 30$ (or larger if sample is skewed), no extreme skew
3. Calculate a *test statistic* and a p-value (draw a picture!)

$$Z = \frac{\bar{x} - \mu}{SE}, \text{ where } SE = \frac{s}{\sqrt{n}}$$

4. Make a decision, and interpret it in context of the research question
 - If p-value $< \alpha$, reject H_0 , data provide evidence for H_A
 - If p-value $> \alpha$, do not reject H_0 , data do not provide evidence for H_A

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Application exercise: 3.2 Hypothesis testing for a single mean

See course website for details.

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Common misconceptions about hypothesis testing

1. P-value is the probability that the null hypothesis is true
A p-value is the probability of getting a sample that results in a test statistic as or more extreme than what you actually observed (and in favor of the null hypothesis) if in fact the null hypothesis is correct. It is a conditional probability, conditioned on the null hypothesis being correct.
2. A high p-value confirms the null hypothesis.
A high p-value means the data do not provide convincing evidence for the alternative hypothesis and hence that the null hypothesis can't be rejected.
3. A low p-value confirms the alternative hypothesis.
A low p-value means the data provide convincing evidence for the alternative hypothesis, but not necessarily that it is confirmed.

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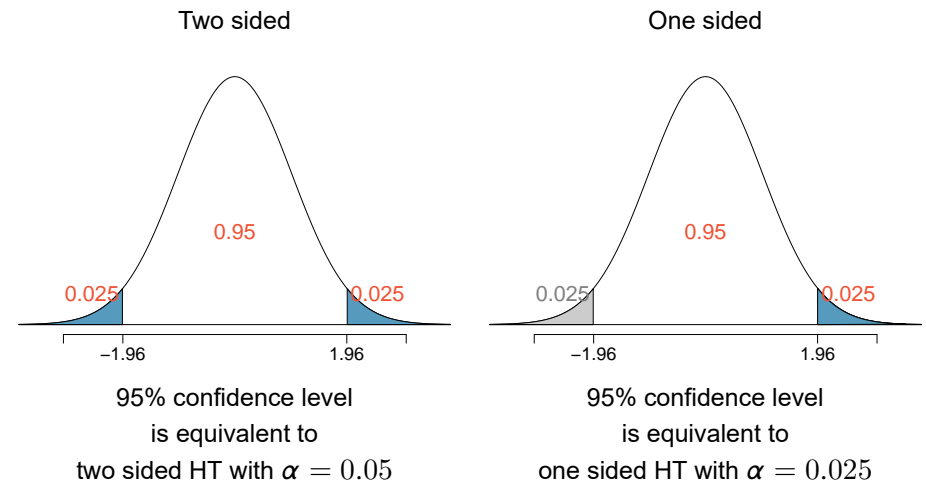
Clicker question

Which of the following is the correct interpretation of the p-value from App Ex 3.2?

- (a) The probability that average GPA of Duke students has changed since 2001.
- (b) The probability that average GPA of Duke students has not changed since 2001.
- (c) The probability that average GPA of Duke students has not changed since 2001, if in fact a random sample of 63 Duke students this year have an average GPA of 3.58 or higher.
- (d) The probability that a random sample of 63 Duke students have an average GPA of 3.58 or higher, if in fact the average GPA has not changed since 2001.
- (e) The probability that a random sample of 63 Duke students have an average GPA of 3.58 or higher or 3.16 or lower, if in fact the average GPA has not changed since 2001.

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2. Hypothesis tests and confidence intervals at equivalent significance/confidence levels should agree



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Clicker question

What is the confidence level for a confidence interval that is equivalent to a two-sided hypothesis test at the 1% significance level? *Hint: Draw a picture and mark the confidence level in the center.*

- (a) 0.80
- (b) 0.90
- (c) 0.95
- (d) 0.98
- (e) 0.99

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Clicker question

What is the confidence level for a confidence interval that is equivalent to a one-sided hypothesis test at the 1% significance level? *Hint: Draw a picture and mark the confidence level in the center.*

- (a) 0.80
- (b) 0.90
- (c) 0.95
- (d) 0.98
- (e) 0.99

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Clicker question

A 95% confidence interval for the average normal body temperature of humans is found to be (98.1 F, 98.4 F). Which of the following is true?

- (a) The hypothesis $H_0 : \mu = 98.2$ would be rejected at $\alpha = 0.05$ in favor of $H_A : \mu \neq 98.2$.
- (b) The hypothesis $H_0 : \mu = 98.2$ would be rejected at $\alpha = 0.025$ in favor of $H_A : \mu > 98.2$.
- (c) The hypothesis $H_0 : \mu = 98$ would be rejected using a 90% confidence interval.
- (d) The hypothesis $H_0 : \mu = 98.2$ would be rejected using a 99% confidence interval.

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3. Results that are statistically significant are not necessarily practically significant

Clicker question

All else held equal, will p-value be lower if $n = 100$ or $n = 10,000$?

- (a) $n = 100$
- (b) $n = 10,000$

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		Decision	
		fail to reject H_0	reject H_0
Truth	H_0 true	✓	Type 1 Error, α
	H_A true	Type 2 Error, β	Power, $1 - \beta$

- ▶ A **Type 1 Error** is rejecting the null hypothesis when H_0 is true: α
 - For those cases where H_0 is actually true, we do not want to incorrectly reject it more than 5% of those times
 - Increasing α increases the Type 1 error rate, hence we prefer to small values of α
- ▶ A **Type 2 Error** is failing to reject the null hypothesis when H_A is true: β
- ▶ **Power** is the probability of correctly rejecting H_0 , and hence the complement of the probability of a Type 2 Error: $1 - \beta$

1. Use hypothesis tests to make decisions about population parameters
2. Hypothesis tests and confidence intervals at equivalent significance/confidence levels should agree
3. Results that are statistically significant are not necessarily practically significant
4. Hypothesis tests are prone to decision errors