TANGENT PLANES AND LINEARIZATION

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Learning Objectives.

- 1. Connect linear approximation of a function of two variables to that of a function of one variable.
- 2. Compute the tangent plane or linearization of a function of two variables at a point.
- 3. Use the linearization to estimate a value of a function.

Motivation and review.

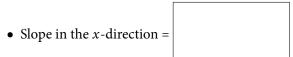
- Recall from Calc I that if we zoom in on the graph of a 'nice' function y = f(x) then it looks like a line.
- But not all functions are *differentiable* or *locally linear*:



- Zoom in on a graph z = f(x, y) and it looks like a *plane*.
- In Calc I we approximated (differentiable) functions by tangent lines;
- In Calc III we approximate (differentiable) functions by tangent planes.

Equation of the tangent plane. We have a function f(x, y) and a point (x_0, y_0) . How do we find the tangent plane of f 'anchored' at (x_0, y_0) ?

What do we need?



- Slope in the *y*-direction =
- Point =

Combine it all to get the equation for a tangent plane:





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go.unl.edu/ZG-zoom-infor-plane



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Example 1. Find an equation for the plane tangent to the graph of

$$f(x,y) = \sqrt{x^2 + y^3}$$

at the point (1, 2, f(1, 2)).

Linearization: using the tangent plane to approximate f. The *linearization* of a function f at (x_0, y_0) is defined to be the linear function whose graph is the tangent plane. That is,

$$L(x, y) =$$

The linearization can be used to approximate f near (x_0, y_0) ; i.e., $L(x, y) \approx f(x, y)$ when $(x, y) \approx (x_0, y_0)$.

The linearization is useful because ...

This is just like the situation in one variable: the tangent line at x_0 approximates f near x_0 .

Bike shop example. Fixed costs total \$10000; each mountain bike sells for \$1500; and each road bike sells for \$2000. The profit is modeled by the function

$$P(x, y) = 1500x + 2000y - 10000.$$

If the bike shop sells 4 fewer mountain bikes and 5 more road bikes, how will that affect their profit?

More generally, on a *plane* with *x*-slope *a* and *y*-slope *b*, shifting the input

$$(x,y) \longmapsto (x + \Delta x, y + \Delta y)$$

results in a change of

in the value of z.

Suppose that we know $f(x_0, y_0)$ but shift the input slightly:

$$(x_0, y_0) \longmapsto (x_0 + \Delta x, y_0 + \Delta y).$$

We can estimate the resulting change Δf in f by the *differential*

$$df =$$
 $\approx \Delta f$.

Example 2. Use differentials to estimate $\sqrt{1.01^2 + 1.97^3}$.



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