CSC 226 FALL 2020
ALGORITHMS AND DATA STRUCTURES II
ASSIGNMENT 3
UNIVERSITY OF VICTORIA

1. Let (x, y, w) denote the edge $\{x, y\}$ with weight w. The graph for the parts (a)and (b) is given by nodes $V = \{A, B, C, D, E\}$ and weighted edges
$(A, B, 7), (A, C, 5), (A, D, 1), (B, C, 4), (B, D, 7), (B, E, 1), (C, E, 3), (D, E, 2)$.

   a) Show how to construct a minimum spanning tree using Prim's algorithm. List the edges and nodes in order of when the edge is added to the tree. Give the initial values of $D(v)$ for each node v. The first node in T is A. Each time a node is added to T, give the D values which have changed as a result.

   b) Show how to construct a minimum spanning tree using Kruskal's algorithm, using the disjoint set data structure with weighted union and path compression to decide if the endpoints of a edge are in different connect components.
   i) List the edges and nodes in order of when the edge is added to the tree.
   ii) Give a picture of the disjoint date structure initially and a picture showing how it changes each time it changes.

2. Will Prim's algorithm and Kruskal's algorithm still work correctly if the graph contains edges with negative edge weights? Explain your answer.

3. Suppose all edge weights in a graph are integers in the range from 1 to $|V|$ where V is the set of vertices. How fast can you make Kruskal's algorithm run? Explain.

4. Show how to solve the single source shortest path problem of the graph in the lecture slides on Dijkstra's algorithm, slide 11 using Dijkstra's algorithm, when the source node is g. Give the initial values of $D(v)$ for each node v. Each time a node is pulled into the cloud, give the D values which have changed as a result.

5. Show how to solve the single source shortest path problem of the graph in the lecture slides on Bellman-Ford's algorithm, slide 3 using the Belllman Ford algorithm, when the source node is b. Give the sequence of D values for all the nodes initially and then just give changes to D values which occurs, by giving the node affected and its new D value, in order of which they occur. In each round, consider the edges in lexicographic order.

6. Give a simple example for which Dijkstra's algorithm produces a wrong answer when there are negative edge weights.