CSC 226 FALL 2020
ALGORITHMS AND DATA STRUCTURES II
ASSIGNMENT 1
UNIVERSITY OF VICTORIA

1. In the 3SUM problem, we are given as input an array A of n distinct integers and the goal is to check if there are three integers in A that sum to 0. Design a $O(n^2)$ running time algorithm for 3SUM. Describe your algorithm in pseudocode and show that its running time is $O(n^2)$.

2. Recall the LinearSelect algorithm we learnt in the class. Suppose that we modify the algorithm to use groups of size 3 instead of 7. Show that the modified algorithm does not run in $O(n)$ time.

3. Given an array A, an inversion in A is defined as a pair of indices i and j such that $i < j$ but $A[i] > A[j]$. Design a $O(n \log n)$ running time algorithm for computing the number of inversions in an input array A.

4. **The Master Theorem**

$$\begin{aligned} T(n) &= c \text{ if } n < d \\ &= aT\left(\frac{n}{b}\right) + \Theta(n^c) \text{ if } n \geq d \end{aligned}$$

   (a) If $c < \log_b a$, then $T(n)$ is $\Theta(n^{\log_b a})$.

   (b) If $c = \log_b a$, then $T(n)$ is $\Theta(n^c \log n)$.

   (c) If $c > \log_b a$, then $T(n)$ is $\Theta(n^c)$.

   Solve the following recurrence equations using the Master Theorem given above.

   (a) $T(n) = 16T(n/4) + n^4$

   (b) $T(n) = 125T(n/5) + n^2$

   (c) $T(n) = 64T(n/8) + n^2$

5. An Array A contains $n-1$ distinct integers in the range $[0, n-1]$. That is, there is one number in this range that is not in A. Describe in pseudo-code an $O(n)$ running time algorithm for finding the missing number. You are only allowed to use $O(\log n)$ bits of additional space besides the array A itself.