CSc 225 Assignment 1:
Discrete Mathematics Unit

Due date:

The submission deadline is 11:55pm on Monday, May 18th, 2020.

How to hand it in:

Submit your **assignment.pdf** file through the Assignment 1 link on the CSC225 conneX page.
IMPORTANT: the file submitted **must** have a **.pdf** extension.

Exercises:

1. Given the word UNDERGRADUATE

   a. How many arrangements of the letters are there?

   $$\frac{13!}{2!2!2!2!2!} = \frac{13!}{2^5} = 194,594,400$$

   b. How many arrangements are there with all A's adjacent to one another?

   $$\frac{12!}{2!2!2!2!} = \frac{12!}{2^4} = 29,937,600$$

   c. How many arrangements are there with none of the A's adjacent to one another?

   UNDERGRDUTE

   $$\frac{11!}{2!2!2!2!} = \frac{11!}{2^4} = 2494800$$

   $$\binom{12}{2} = 66 \qquad \therefore 66 \cdot \frac{11!}{2^4} = 164,656,800$$

   d. How many arrangements are there with all of the vowels adjacent to one another?

   $$AAEEUU = \frac{7!}{2!2!} = \frac{7!}{4} = 1260$$

   $$\frac{6!}{2!2!2!} = \frac{6!}{8} = 90$$

   NDRGRDT =

   $$1260 \cdot 90 \cdot 8 = 907200$$

2. Suppose you draw 5 cards from a standard deck of 52.

a. How many ways can you draw exactly 3 clubs?

$13C3 \cdot 39C2 = 211926$

b. How many ways can you draw at least 2 hearts?

$13C2 \cdot 39C3 + 13C3 \cdot 39C2 + 13C4 \cdot 39C1 + 13C5$

$= 953940$

c. How many ways can you draw 3 clubs and 2 hearts?

$13C3 \cdot 13C2 = 22308$

$$\sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

3. Determine the coefficient of x^7y^5 in the following expansions:

a. $(x+y)^{12}$ $n=12,\ k=7$

$\binom{12}{7} x^7 y^{12-7}$

$= 792x^7y^5$

b. $(-4x+3y)^{12}$ $n=12,\ k=7$

$\binom{12}{7}(-4)^7(3)^5$

$792 \cdot -16384 \cdot 243$

$= -3153199104x^7y^5$

c. $(12x-2y)^{12}$ $n=12,\ k=7$

$\binom{12}{7}(12)^7(-2)^5$

$792 \cdot 35831808 \cdot -32$

$= -9.08 \times 10^{11} x^7 y^5$

$\binom{n+r-1}{r}$

4. **Determine the number of integer solutions of $x_1 + x_2 + x_3 + x_4 = 16$, where**

a. $x_i \geq 0,\ 1 \leq i \leq 4$ $n=4,\ r=16$

$\binom{19}{16} = 969$

b. $x_1, x_2 \geq 1,\ x_3, x_4 \geq 3$ $n=4,\ r=8$

$\binom{11}{8} = 165$

c. $x_i \geq -1,\ 1 \leq i \leq 4$

$\binom{23}{20} = 1771$

d. $x_i \geq 1,\ 1 \leq i \leq 3,\ 5 \leq x_4 \leq 7$

0|0|0|00000=8 ; 0|0|0|000000=7 ; 0|0|0|0000000=6

$\binom{10}{8} + \binom{9}{7} + \binom{8}{6}$

$45 + 36 + 28$

$= 109$

5. **As a New Year's Resolution, Ali decides to go for a run at least once a day for the first 5 weeks of the year. To not overdo it, Ali makes sure to not run more than 50 times during this 5-week time period. Show that there must be a period of consecutive days for which Ali goes on exactly 19 runs.**

5 weeks = 35 days

# of runs per day = X_0

$1 \leq X_1 < X_2 < \cdots < X_{35} \leq 50$

strictly increasing

to show that there consecutives days where ali goes on exactly 19 runs:

$20 \leq X_1 + 19 < X_2 + 19 < \cdots < X_{35} + 19 \leq 69$

now consider the 70 number of runs:

$X_1, X_2, \ldots X_{35}, X_1 + 19, X_2 + 19, \ldots, X_{35} + 19$

they are all between 1 and 69, so therefore two of the numbers must be equal.

Because $X_1, X_2, \ldots X_{35}$ and $X_1 + 19, X_2 + 19, \ldots, X_{35} + 19$ are distinct, the two equal numbers from each set are X_i and $X_j + 19$

$\therefore X_i = X_j + 19$, the days $j+1, j+2, \ldots i$ are when ali goes on exatly 19 runs