$\mathbb{Z}$ Normal Distribution

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Motivation

Western music is discretized so that it is easier to notate.

Discrete, yet infinitely negative and positive features:

- octaves
- dynamics
- tempo
- duration

Normal Distribution

$$\mu \in \mathbb{R},\ \sigma \in \mathbb{R}^+$$

$$p(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$1 = \int_{\mathbb{R}} p(x \mid \mu, \sigma^2)$$

$\mathbb{Z}$ Normal Distribution

$$\mu \in \mathbb{Z},\ \sigma \in \mathbb{R}^+$$

$$p(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$1 = \sum_{\mathbb{Z}} p(x \mid \mu, \sigma^2)$$

$\mathbb{Z}$ Normal Distribution

Sample from $\mathcal{N}_{\mathbb{Z}}$ distribution with a mean $\mu \in \mathbb{Z}$ and a standard deviation of $\sigma^2 \in \mathbb{R}^+$

$$x \sim \mathcal{N}(\mu, \sigma^2)$$
$$\lfloor x \rfloor$$

$\mathbb{Z}$ Normal Distribution

Theorem 1. This sampling method for $\mathcal{N}_{\mathbb{Z}}$ distribution denotes a probability distribution over $\mathbb{Z}$.

Theorem

Proof.

Durations

Most common durations are half, quarter, and eight, but the scale is infinite

Durations can be represented as a pair in $\mathbb{Z} \times \mathbb{N}$ where the first element is the base duration and the second element is the number of dots.

$$
\begin{aligned}
d &\sim \mathcal{N}_{\mathbb{Z}}(\mu, \sigma^2) \\
x &\sim B(n, p) \\
(d, x)
\end{aligned}
$$

Octaves

Tempo

Dynamics