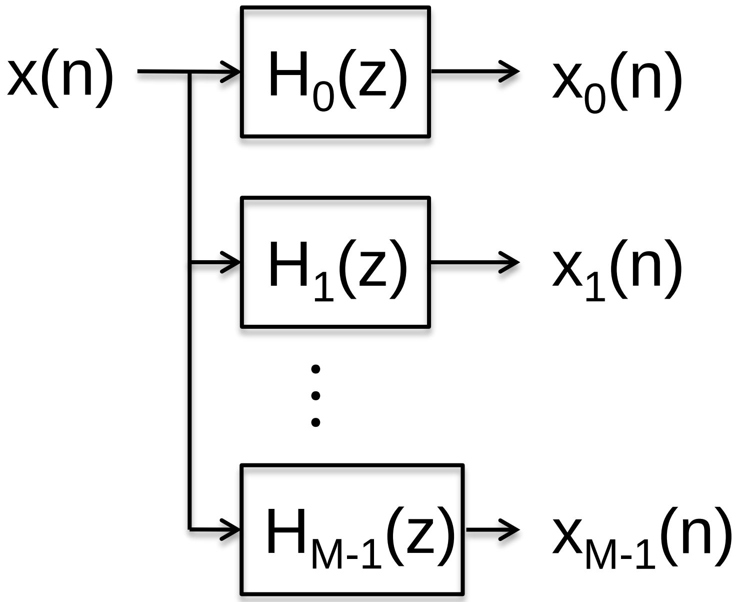
Module 9

Digital Filter Banks, Part I

Overview

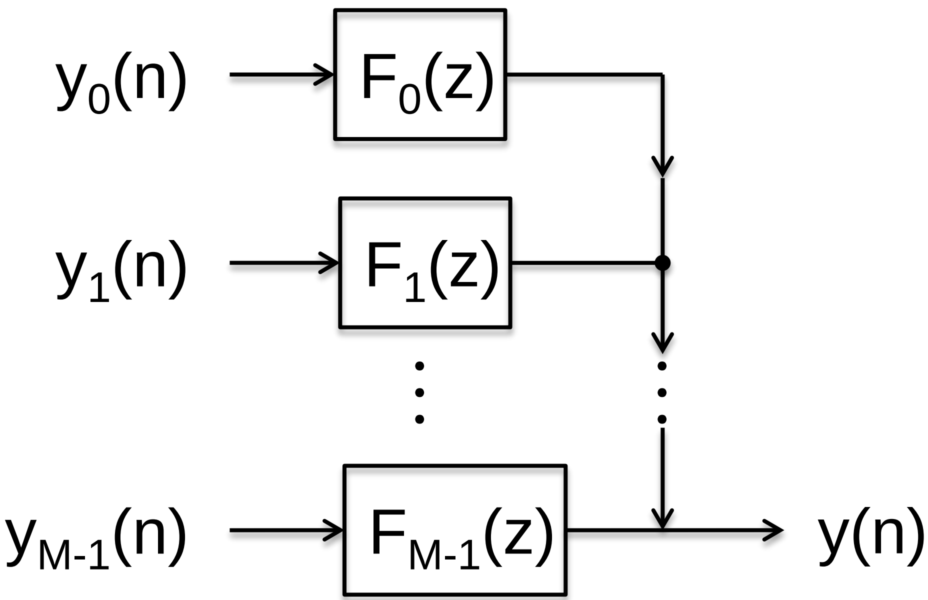
- Analysis filter bank
- Synthesis filter bank
- DFT filter bank
- Polyphase implementation of uniform filter banks
- MATLAB examples

Analysis Filter Bank



- The filters $H_k(z)$ split the input signal into M subband signals $x_k(n)$, $0 \leq k \leq M-1$
- The subbands may be uniform or non-uniform width in frequency
- Since each subband has reduced bandwidth relative to the input, downsampling may also be applied

Synthesis Filter Bank

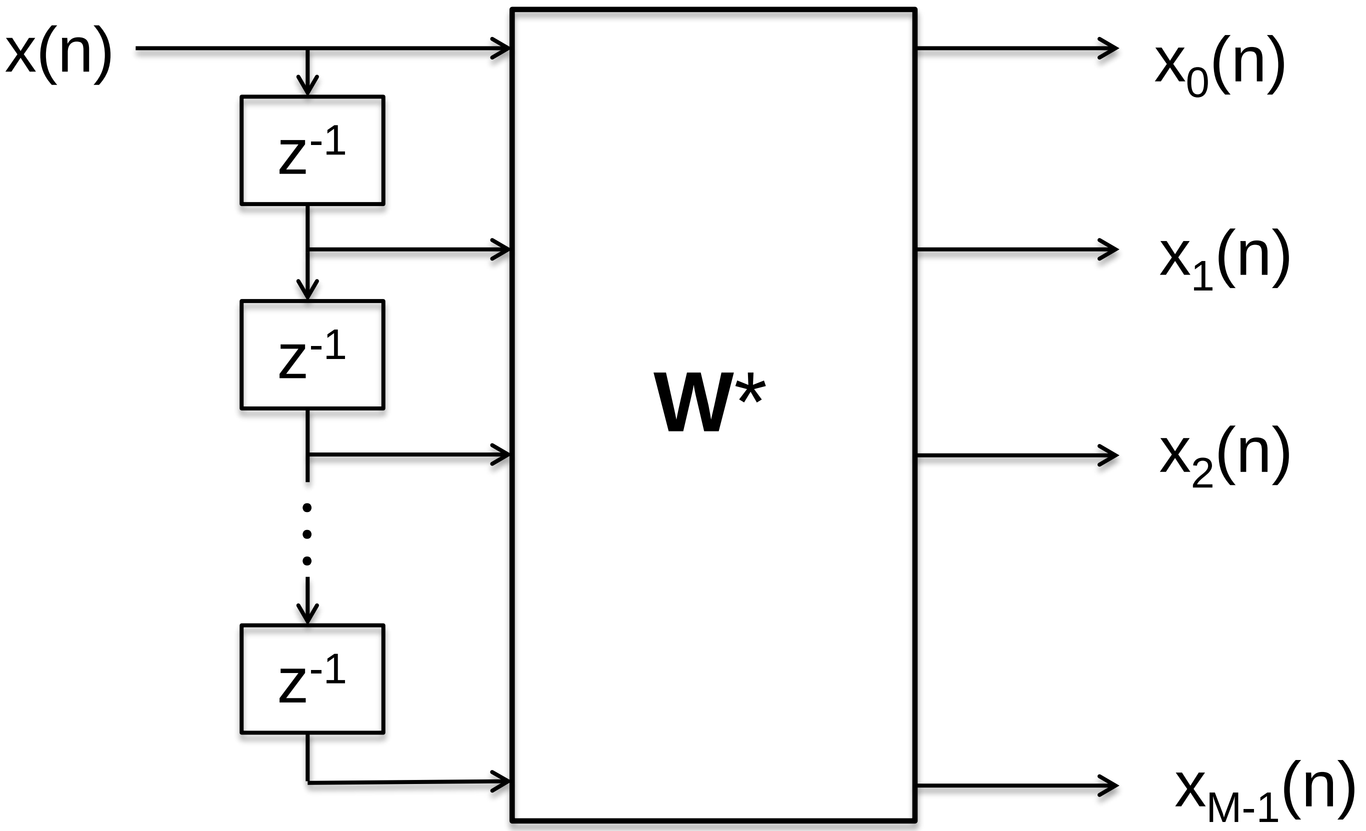


- The synthesis filters $F_k(z)$ are used to combine the subband signals into the output $y(n)$
- Upsampling may be required if the subband signals were downsampled in the analysis filter bank

Filter Bank Applications

- Perceptual encoding of signals (e.g. MPEG)
- Multiresolution signal analysis (e.g. wavelets)
- Audio signal processing
- Channelizers for frequency division multiplexing (FDM) communication systems
- Octave-spaced filter banks using half-band filters are popular

DFT Filter Bank



- **W** is the MxM DFT matrix with elements $W_{km} = \exp(-j2\pi km/M)$ (k^{th} row and m^{th} column), * denotes complex conjugation

DFT Filter Bank

- Let $s_l(n)=x(n-l)$ $0 \le l \le M-1$. The sequences $s_l(n)$ are generated by passing x(n) through and delay chain.
- The subband signals are computed by

$$x_k(n)=\sum_{l=0}^{M-1} s_l(n) W^{-kl} \qquad W=e^{-j2\pi/M}$$

DFT Filter Bank

- The z-transform of the k^th subband signal is

$$X_k(z) = \sum_{l=0}^{M-1} S_l(z) W^{-kl} = \sum_{l=0}^{M-1} W^{-kl}\left[z^{-l} X(z)\right]$$

$$= \underbrace{\left[\frac{1-(zW^k)^{-M}}{1-(zW^k)^{-1}}\right]}_{H_k(z)} X(z)$$

- Each subband filter $H_k(z)$ is a *modulated* version of a prototype filter $H_0(z)$

$$H_k(z) = H_0(zW^k)$$

DFT Filter Bank

- The prototype filter $H_0(z)$ for the DFT filter bank corresponds to a M-sample moving average filter

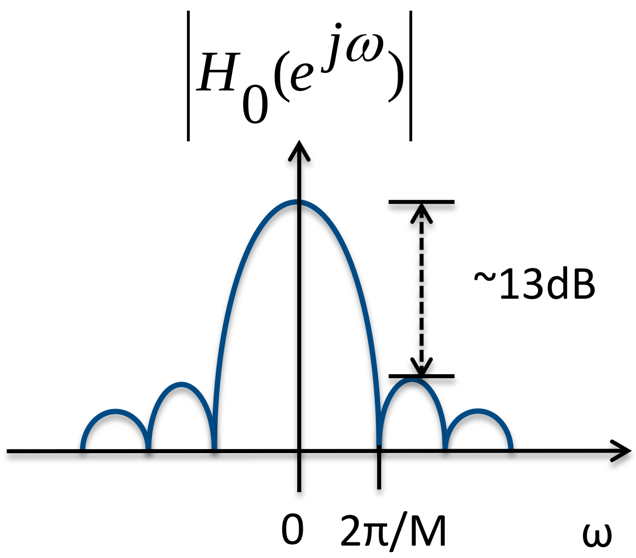
$$H_0(z) = \frac{1-z^{-M}}{1-z^{-1}}$$

$$= 1+z^{-1}+z^{-2}+\cdots+z^{-(M-1)}$$

- The frequency response of $H_0(z)$ is

$$\left|H_0(e^{j\omega})\right| = \left|\frac{\sin \omega M/2}{\sin \omega/2}\right|$$

DFT Filter Bank



- Magnitude response of prototype filter $H_0(z)$
- Width of mainlobe is inversely proportional to M
- Highest sidelobe is ~13dB down
- Other subband filters are modulated versions of $H_0(z)$ with modulation frequency $\omega_k = 2\pi k/M,\ 1 \le k \le M-1$

Polyphase Implementation

- A filter bank in which the analysis filters are related according to $H_k(z) = H_0(zW^k)$ is called a *uniform* filter bank.
- A uniform filter bank may be implemented efficiently using polyphase components. The DFT filter bank is a special case of a uniform filter bank.

Polyphase Implementation

- Representing the prototype filter $H_0(z)$ with a (Type I) polyphase structure produces

$$H_0(z)=\sum_{l=0}^{M-1} z^{-l} E_l(z^M) \quad e_l(n)=h_0(Mn+l) \quad 0 \le l \le M-1$$

- The modulated subband filters are

$$H_k(z)=H_0(zW^k)=\sum_{l=0}^{M-1} \left(zW^k\right)^{-l} E_l(z^M)$$

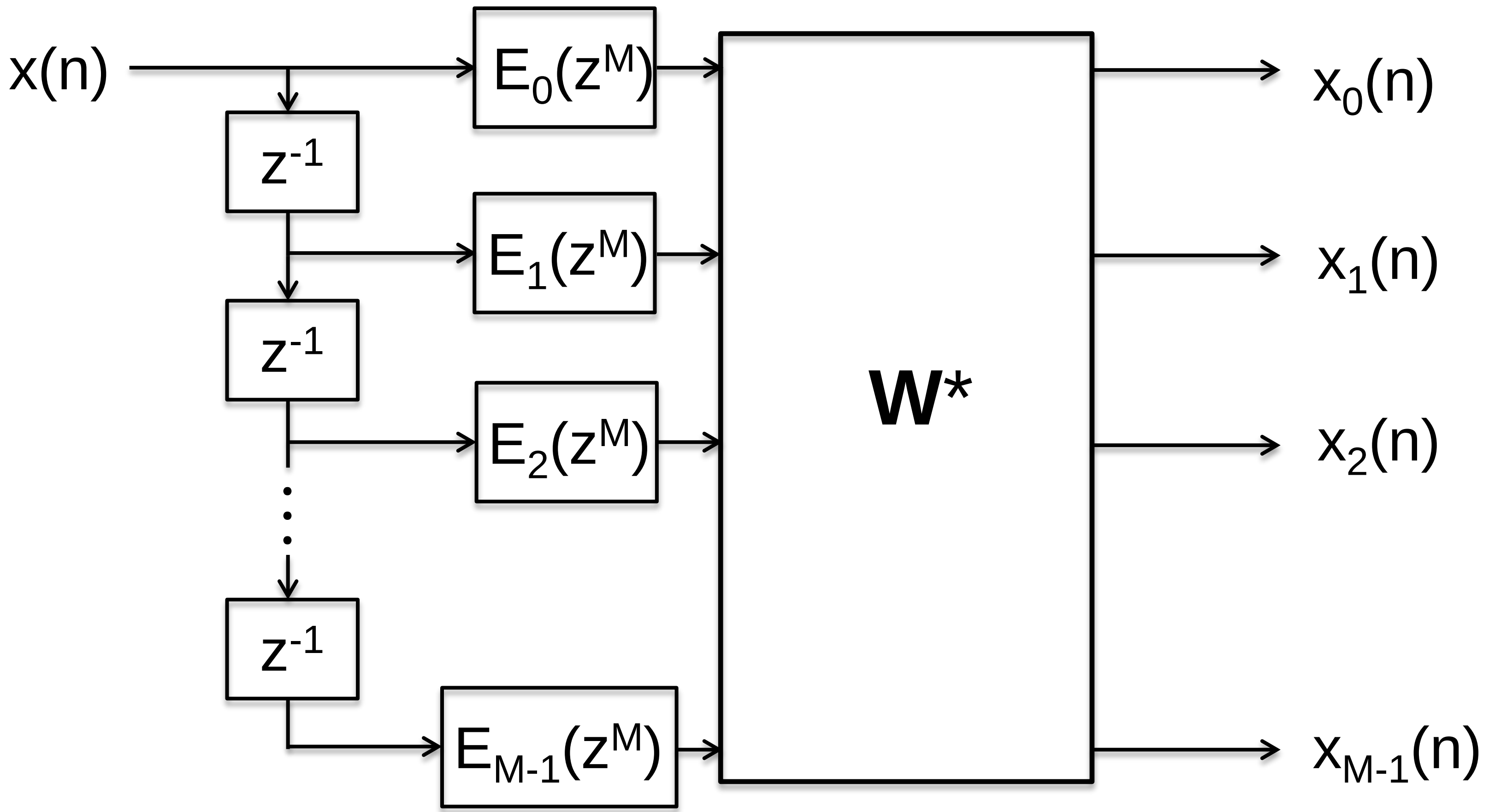
Polyphase Implementation

- The z-transform of the subband output $x_k(n)$ is

$$
\begin{aligned}
X_k(z) &= H_k(z)X(z) \\
&= \sum_{l=0}^{M-1} W^{-kl}\left[z^{-l}E_l(z^M)X(z)\right]
\end{aligned}
$$

- The DFT filter bank results if $E_l(z)=1$ for all l (compare with slide 7)

Uniform Filter Bank Block Diagram



Uniform Filter Bank Block Diagram w/ Downsampling