Module 3

Infinite Impulse Response Filter Design, Part II

Overview

- Allpass filters
  - o Pole/zero constellation
  - o Transfer function properties
- Efficient structures for allpass filters
- Lowpass filter implementation using parallel allpass filters

Lowpass Filter Implementation

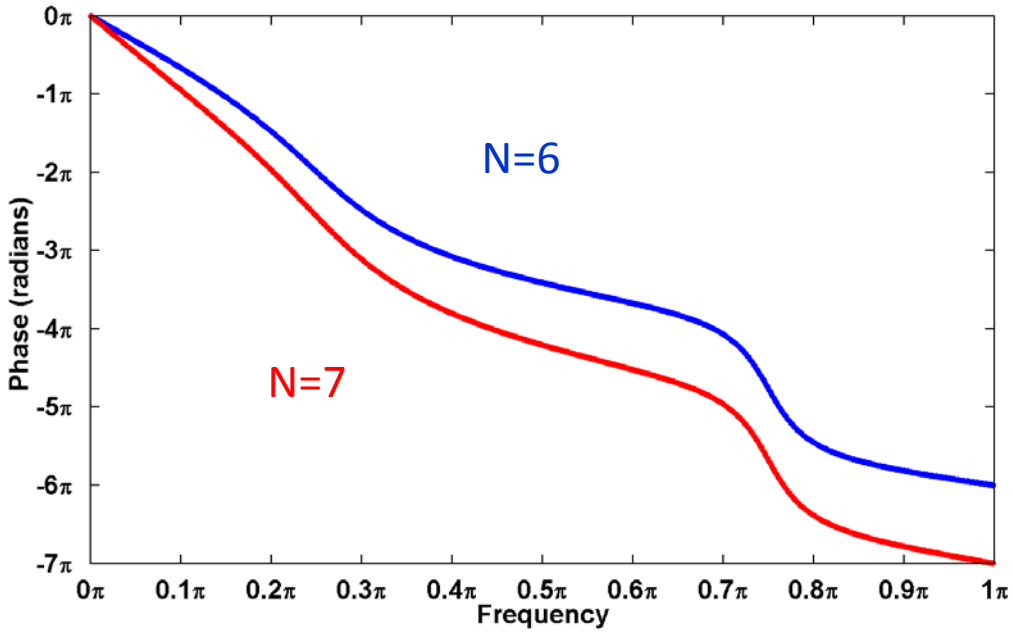
- Lowpass filters can be implemented as a parallel connection of allpass sections

$$H_{LP}(z)=\frac{A_N(z)+A_M(z)}{2}$$

- N+M must be *odd*
- We consider allpass sections with real coefficients only

Allpass Phase Response

- A stable allpass filter of order N has a phase response that decreases monotonically from 0 to –Nπ as ω varies from 0 to π



Parallel Allpass Filters

$$H_{LP}(e^{j\omega}) = \frac{A_N(e^{j\omega}) + A_M(e^{j\omega})}{2}$$

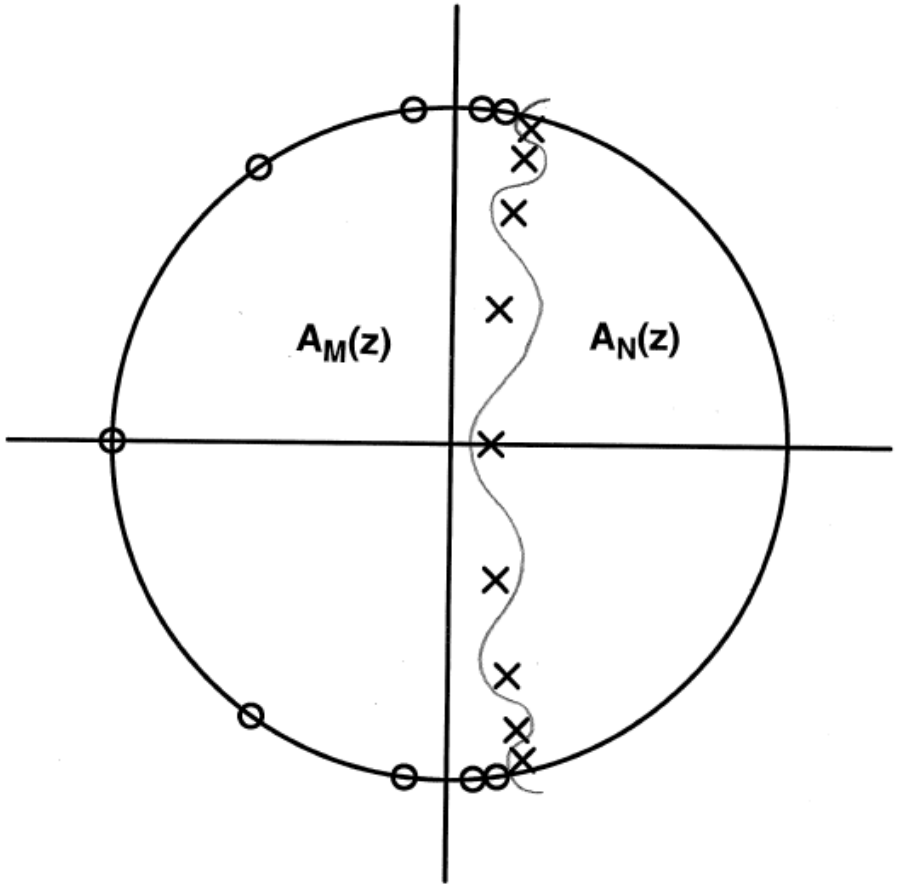
$$= \exp\left[\frac{j}{2}\left(\phi_M(\omega)+\phi_N(\omega)\right)\right]\cos\left[\frac{1}{2}\left(\phi_M(\omega)-\phi_N(\omega)\right)\right]$$

- *Passband* when $\phi_M(\omega)-\phi_N(\omega) \approx 0, 2\pi, 4\pi, \ldots$
- *Stopband* when $\phi_M(\omega)-\phi_N(\omega) \approx \pi, 3\pi, 5\pi, \ldots$

Design Procedure

- Design the discrete-time filter to meet specifications. The filter order M+N must be odd.
- Choose poles for $A_M(z)$ and $A_N(z)$ according to the *pole interlace property* (see next slide).
- Select the structures for realizing each allpass section (e.g. cascade, Gray-Markel)

Pole Interlace Property for $A_M(z)$ and $A_N(z)$

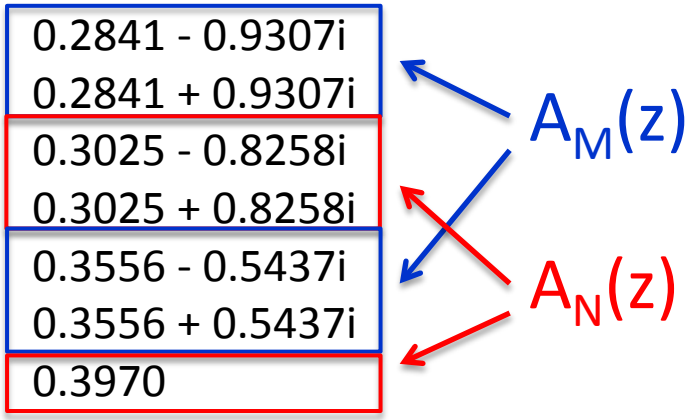


Design Example (1 of 3)

- Design a lowpass elliptic filter and implement as a parallel connection of allpass sections
  - o $\omega p=0.4\pi$, $\omega s=0.45\pi$ passband/stopband edge frequencies
  - o Rp=0.1dB, Rs=40dB passband/stopband attenuation
- Required filter order is 7. Take M=4, N=3.

Design Example (2 of 3)

- Poles for lowpass elliptic filter

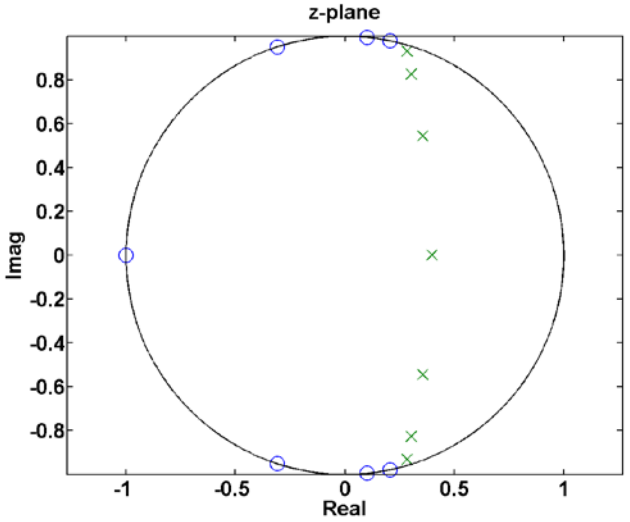


- Allpass sections

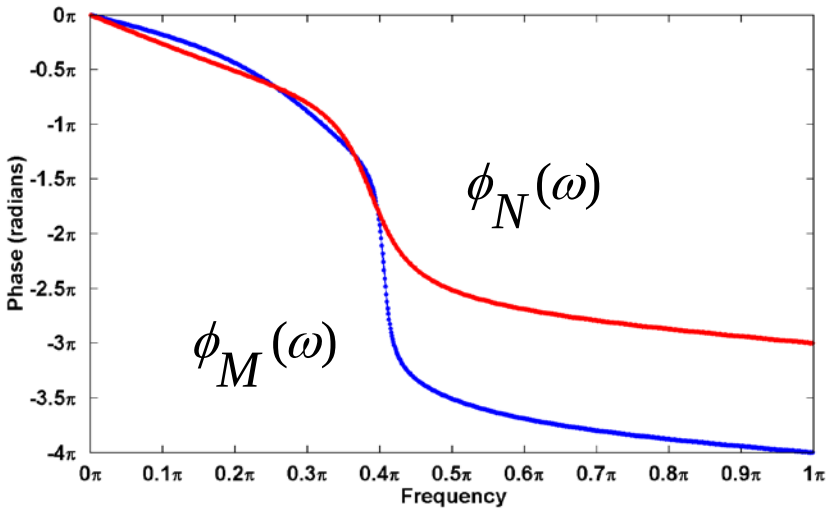
$$A_M(z)=\frac{0.3997-0.9134z^{-1}+1.7732z^{-2}-1.2795z^{-3}+1.0000z^{-4}}{1.0000-1.2795z^{-1}+1.7732z^{-2}-0.9134z^{-3}+0.3997z^{-4}}$$

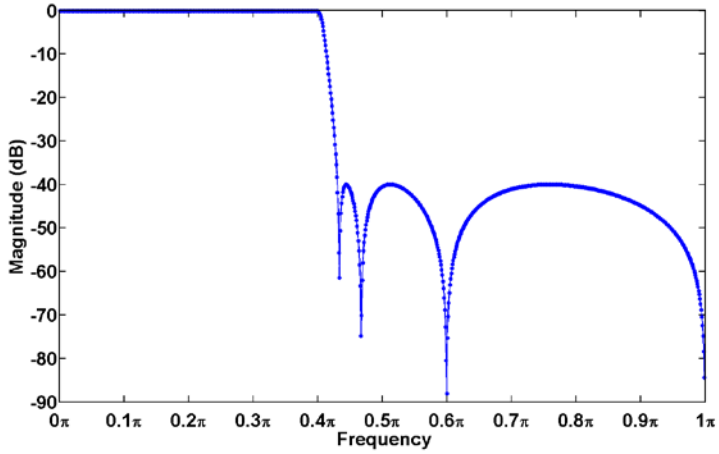
$$A_N(z)=\frac{-0.3071+1.0135z^{-1}-1.0020z^{-2}+1.0000z^{-3}}{1.0000-1.0020z^{-1}+1.0135z^{-2}-0.3071z^{-3}}$$

Design Example (3 of 3)



Pole/zero plot





Magnitude response for parallel connection of allpass sections

Highpass Filter Implementation

- A highpass filter may obtained by subtracting the allpass sections rather than adding them

$$H_{HP}(z)=\frac{A_N(z)-A_M(z)}{2}$$

- $H_{LP}(z)$ and $H_{HP}(z)$ form a *power complementary pair*

$$\left|H_{LP}\left(e^{j\omega}\right)\right|^2+\left|H_{HP}\left(e^{j\omega}\right)\right|^2=1$$

Power Complementary Pair