Module 3

Infinite Impulse Response Filter Design, Part II

Overview

- Allpass filters
  - Pole/zero constellation
  - Transfer function properties
- Efficient structures for allpass filters
- Lowpass filter implementation using parallel allpass filters

Allpass Filters (1 of 2)

- A filter is said to be *allpass* if its magnitude response is constant

$$\left|A\left(e^{j\omega}\right)\right|=1, \quad -\pi\leq\omega<\pi$$

- An allpass transfer function has poles and zeros that are *conjugate reciprocal pairs*

first-order example

$$A(z)=\frac{-\alpha^{*}+z^{-1}}{1-\alpha z^{-1}}$$

Pole: $z=\alpha$

Zero: $z=1/\alpha^{*}$

Allpass Filters (2 of 2)

- Factored form, general case

$$A_M(z) = \prod_{k=1}^{M} \frac{-\alpha_k^* + z^{-1}}{1 - \alpha_k z^{-1}}$$

Poles: $z = \alpha_k$

Zeros: $z = 1/\alpha_k^*$

- Direct form

$$A_M(z) = \frac{a_M^* + a_{M-1}^* z^{-1} + \ldots + a_0^* z^{-M}}{a_0 + a_1 z^{-1} + \ldots + a_M z^{-M}}$$

Structures for Allpass Filters

- Allpass filters may be realized using
  - Cascade of 1st-order and 2nd-order sections
    - Real-valued coefficients result if poles are
      - Complex conjugate pairs or real-valued
      - Zeros are conjugate reciprocals or reciprocals of poles
  - Gray-Markel lattice structure
    - All multipliers have a magnitude less than unity if allpass transfer function is stable
    - Transfer function remains stable and allpass in spite of multiplier quantization

Gray-Markel Lattice Structure (1 of 4)

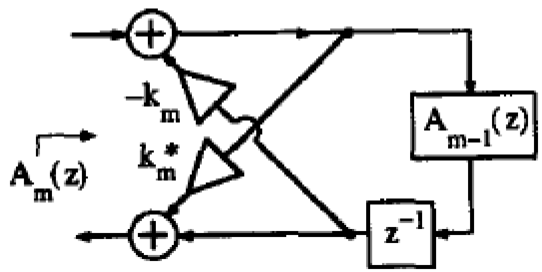

Fig. 5. Lattice filter interpretation of order reduction process for complex all-pass functions. The signals at all nodes are assumed to be complex-valued.

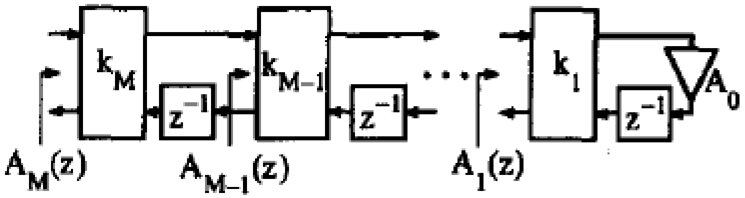

Fig. 3. The cascaded lattice implementation of the all-pass function $A_M(z)$.

Reprinted from: 'The Digital All-Pass Filter: A Versatile Signal Processing Building Block' by P.P. Vaidynathan et al.

Gray-Markel Lattice Structure (2 of 4)

- Synthesis procedure uses the following recursion:

$$z^{-1}A_{m-1}(z)=\frac{A_m(z)-k_m^*}{1-k_m A_m(z)} \quad m=M,M-1,\ldots,1$$

with $k_m = A_m^*(\infty)$

Gray-Markel Lattice Structure (3 of 4)

- Define the direct-form transfer function for the m[th] stage of the recursion as

$$A_m(z) = \frac{a^*_{m,m} + a^*_{m,m-1} z^{-1} + \ldots + a^*_{m,0} z^{-m}}{a_{m,0} + a_{m,1} z^{-1} + \ldots + a_{m,m} z^{-m}}$$

$$k_m = A^*_m(\infty) = \frac{a_{m,m}}{a^*_{m,0}}$$

Gray-Markel Lattice Structure (4 of 4)

- For m=M (1st iteration of recursion)

$$a_{M,m} = a_m, \quad m = 0, 1, \ldots, M$$

$$k_M = A_M^*(\infty) = \frac{a_M}{a_0^*}$$

- In general,

$$a_{m-1,n} = a_{m,n} - k_m a_{m,m-n}^* \quad n = 0, 1, \ldots, m-1$$

Example (1 of 2)

- M=2, real-valued coefficients

$$A_2(z)=\frac{a_2+a_1z^{-1}+a_0z^{-2}}{a_0+a_1z^{-1}+a_2z^{-2}} \qquad k_2=A_2(\infty)=\frac{a_2}{a_0}$$

- 1st iteration of recursion (m=2)

$$z^{-1}A_1(z)=\frac{A_2(z)-k_2}{1-k_2A_2(z)}=\frac{\overbrace{(a_2-k_2a_0)}^{=0}+\overbrace{(a_1-k_2a_1)}^{a_{1,1}}z^{-1}+\overbrace{(a_0-k_2a_2)}^{a_{1,0}}z^{-2}}{\underbrace{(a_0-k_2a_2)}_{a_{1,0}}+\underbrace{(a_1-k_2a_1)}_{a_{1,1}}z^{-1}+\underbrace{(a_2-k_2a_0)}_{=0}z^{-2}}$$

Example (2 of 2)

$$k_1 = A_1(\infty) = \frac{a_{1,1}}{a_{1,0}} = \frac{a_1 - k_2 a_1}{a_0 - k_2 a_2}$$

- 2nd iteration of recursion (m=1)

$$z^{-1}A_0(z) = \frac{A_1(z) - k_1}{1 - k_1 A_1(z)} = \frac{\overbrace{(a_{1,1} - k_1 a_{1,0})}^{=0} + (a_{1,0} - k_1 a_{1,1})z^{-1}}{(a_{1,0} - k_1 a_{1,1}) + \underbrace{(a_{1,1} - k_1 a_{1,0})}_{=0} z^{-1}}$$

$$\Rightarrow A_0(z) = 1$$