Module 4

Finite Impulse Response Filter Design, Part I

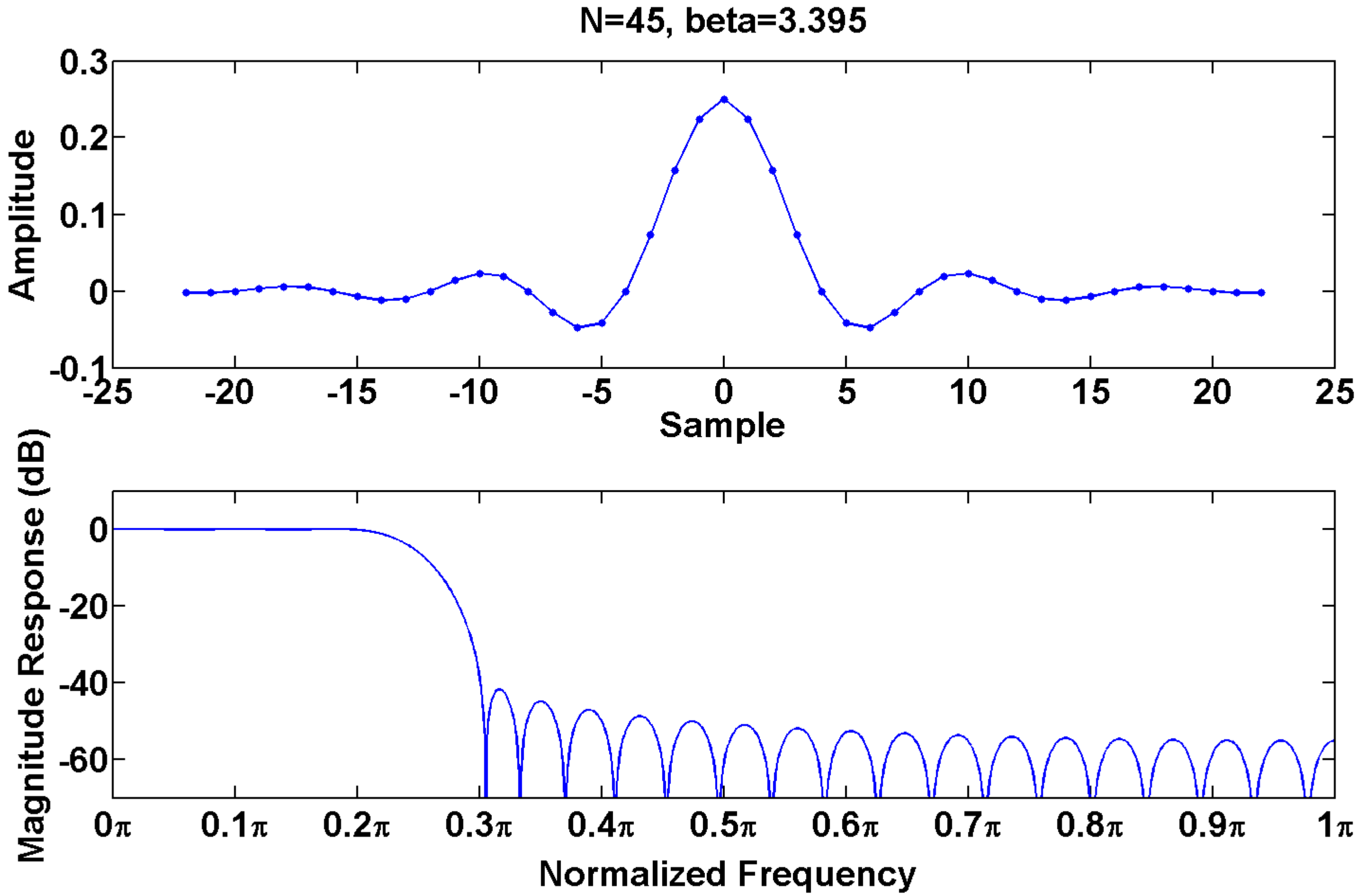
Overview

- Ideal lowpass filter and its impulse response
- FIR filter design using windows
- Window design tradeoffs
- Linear phase FIR filters

Design Example (1 of 2)

- Design a lowpass filter using the Kaiser window with
  - $\omega_p = 0.2\pi$
  - $\omega_s = 0.3\pi$
  - Stopband attenuation of 40dB
- Using the MATLAB function *kaiserord,* we find N=45 and β=3.395. The values of the window sequence are then found from the MATLAB function *kaiser*.

Design Example (2 of 2)



FIR Filter Design Summary

- The window technique may be applied to any filter response for which the impulse response can be determined analytically.
- This includes the following filter types
  - o Lowpass
  - o Highpass
  - o Bandpass, Bandstop

Linear Phase FIR Filters

- Even or odd symmetry of the impulse response is sufficient to ensure linear phase response.

$$h(n) = h(M-n),\ 0 \le n \le M \quad \text{even symmetry}$$

$$h(n) = -h(M-n),\ 0 \le n \le M \quad \text{odd symmetry}$$

Note that the length of the impulse response is M+1

- There are 4 cases to consider:
  - Type I – M even, even symmetry
  - Type II –M odd, even symmetry
  - Type III – M even, odd symmetry
  - Type IV – M odd, odd symmetry

Zeros at z=1,-1

Type	M	Symmetry	H(1)	H(-1)
I	Even	Even	No restriction	No restriction
II	Odd	Even	No restriction	H(-1)=0
III	Even	Odd	H(1)=0	H(-1)=0
IV	Odd	Odd	H(1)=0	No restriction

z=1 corresponds to $\omega=0$ (DC)

z=-1 corresponds to $\omega=\pi$ (Nyquist)

Type III, zero at z=1

$$H(z)\big|_{z=1} = \sum_{m=0}^{M} h(m)$$

$$= \sum_{m=0}^{M/2-1} h(m) + h(M/2) + \sum_{m=M/2+1}^{M} h(m)$$

$$= h(M/2) + \sum_{m=0}^{M/2-1} \left(h(m) + h(M-m) \right)$$

- For Type III (M even, odd symmetry)

$$h(m) = -h(M-m) \text{ and } h(M/2) = 0 \quad \Rightarrow H(1) = 0$$

Type II, zero at z=-1

$$H(z)\big|_{z=-1} = \sum_{m=0}^{M} h(m)(-1)^m$$

$$= \sum_{m=0}^{(M-1)/2} h(m)(-1)^m + \sum_{m=(M-1)/2}^{M} h(m)(-1)^m$$

$$= \sum_{m=0}^{(M-1)/2} \big(h(m) - h(M-m)\big)(-1)^m$$

- For Type II (M odd, even symmetry)

$$h(m) = h(M-m) \quad \Rightarrow H(-1) = 0$$

Linear Phase Filter Summary

- Impulse response symmetry (or anti-symmetry) imposes restrictions on the filter response at z=1 and/or z=-1.
- *Type I has no restrictions.*
- All other types have restrictions at either z=1 (Types III/IV) or z=-1 (Types II/III) which limit their usage.