Module 2

Infinite Impulse Response Filter Design, Part I

Overview

- Analog filter types
  - Butterworth
  - Chebyshev I/II
  - Elliptic
  - Bessel
- Bilinear transformation
- Other transformations

Elliptic Response

- Elliptic response is equiripple in both passband and stopband
- Both poles and zeros in finite s-plane
- Filter order as well as poles and zeros are computed from elliptic integrals (see 'Lecture Notes on Elliptic Filter Design' for additional details and a design example)

Bessel Response

- Bessel filter has *maximally flat* group delay response at Ω=0
- Transfer function is $$H(s)=\frac{b_0}{\sum_{k=0}^{N} b_k s^k}$$

with $b_k=\frac{(2N-k)!}{2^{N-k}k!(N-k)!}$

MATLAB Function Summary

- Butterworth
  - *butter, buttap, buttord*
- Chebyshev Type I
  - *cheb1ord, cheby1*
- Chebyshev Type II
  - *cheb2ord, cheby2*
- Elliptic
  - *ellip, ellipap, ellipord*
- Bessel
  - *besself, besselap*

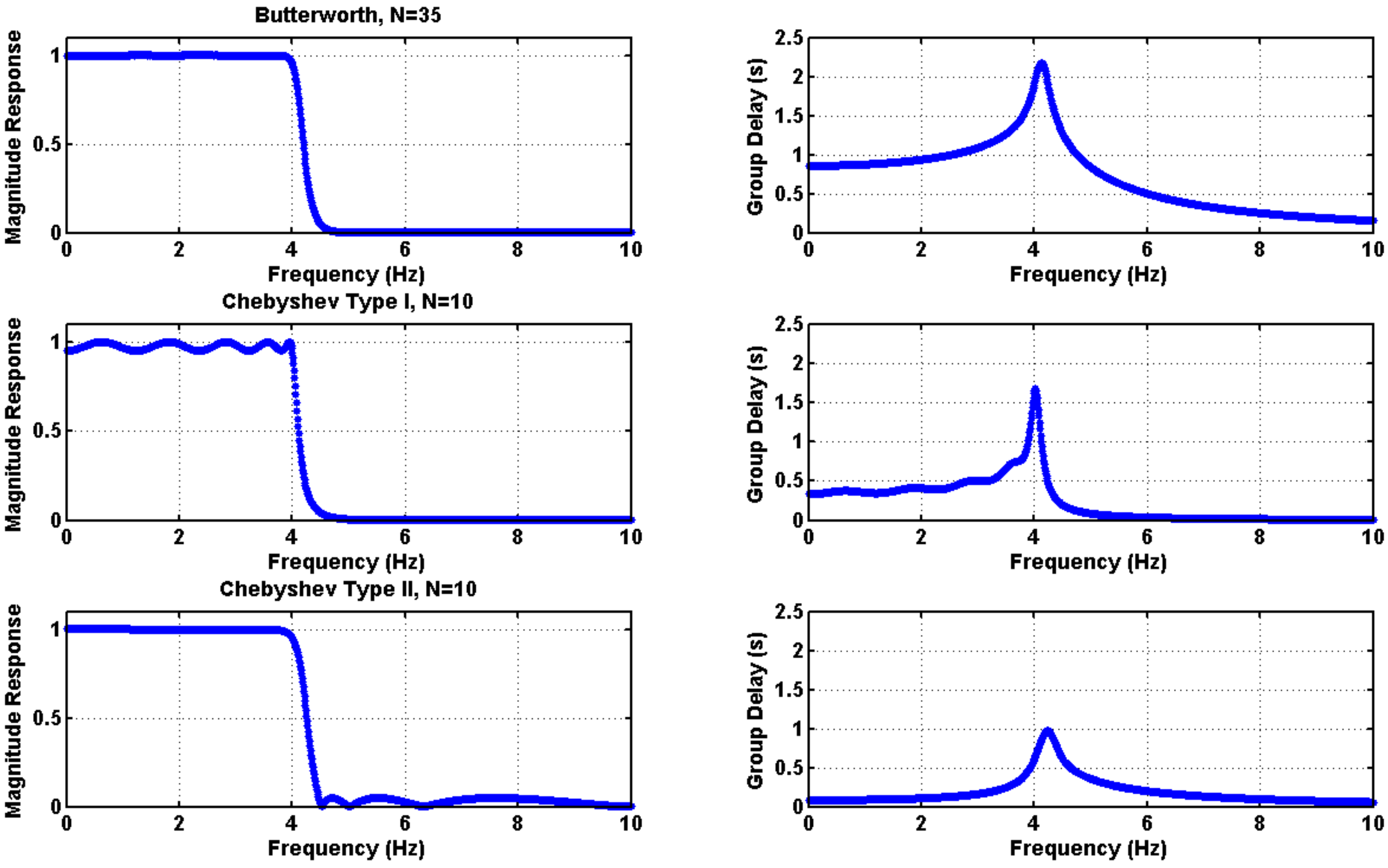
Filter Response Comparison (1 of 4)

- Butterworth filter has maximally flat magnitude response with no ripple in either the passband or stopband
- Chebyshev Type I is equiripple in the passband and monotonic in the stopband
- Chebyshev Type II is equiripple in the stopband and monotonic in the passband

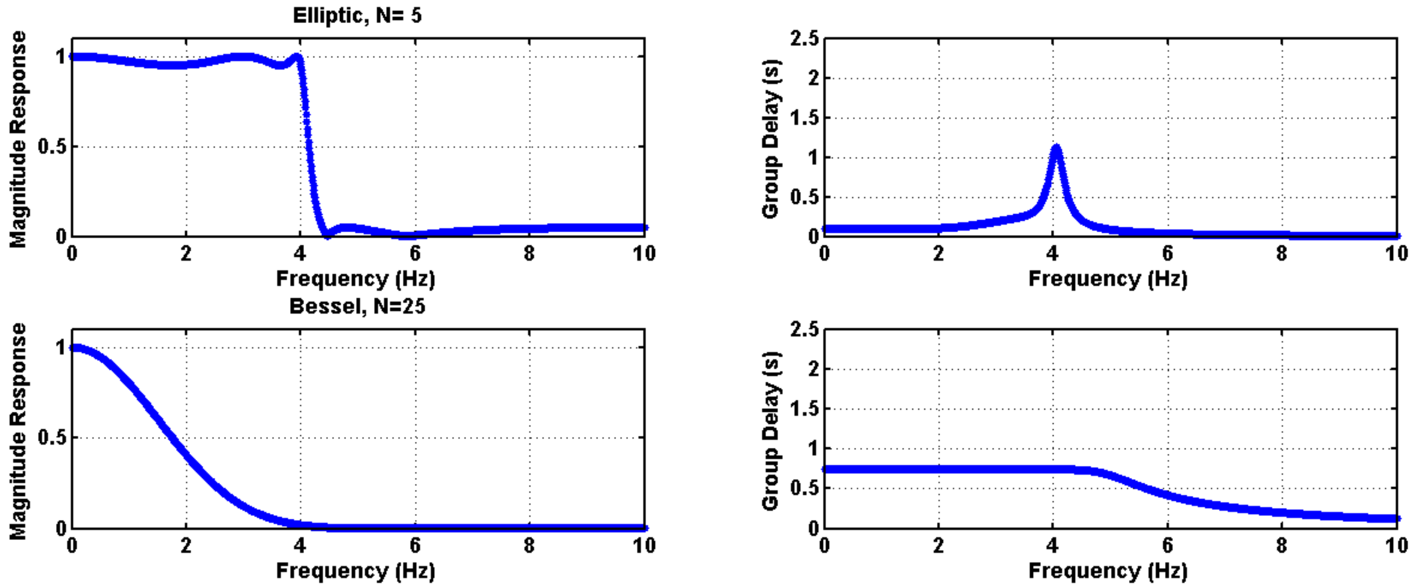
Filter Response Comparison (2 of 4)

- Elliptic filter is most economical in the sense that a given specification can be met with a lower filter order than the other response types
  - Phase response of the elliptic filter is highly non-linear, however
- The Bessel filter offers nearly linear phase response (constant group delay) but has poor magnitude response

Filter Response Comparison (3 of 4)



Filter Response Comparison (4 of 4)



Bilinear Transformation

- A lowpass analog prototype filter is transformed to a lowpass discrete-time filter using the *bilinear transformation*

$$H(z) = H(s)\Big|_{s=\frac{1-z^{-1}}{1+z^{-1}}}$$

- The bilinear transformation maps the poles/zeros of H(s) to H(z) according to

$$z = \frac{1+s}{1-s}$$

s = ∞ maps to z=-1, s = 0 maps to z=1

s=jΩ maps to unit circle (|z|=1)

Bilinear Transformation

- The bilinear transformation introduces *frequency warping* described by

$$\Omega = \tan\left(\frac{\omega}{2}\right)$$

- Passband/stopband frequencies must be *pre-warped* using this expression when designing the analog prototype filter

Other Transformations

- Lowpass→Lowpass, Lowpass→Highpass, Lowpass→Bandpass and Lowpass→Bandstop frequency transformations are given by

$$
\begin{aligned}
&\text{(LP)} \qquad s = \frac{1 - z^{-1}}{1 + z^{-1}}, \qquad && \Omega = \tan\left(\frac{\omega}{2}\right) \\
&\text{(HP)} \qquad s = \frac{1 + z^{-1}}{1 - z^{-1}}, \qquad && \Omega = -\cot\left(\frac{\omega}{2}\right) \\
&\text{(BP)} \qquad s = \frac{1 - 2c_0 z^{-1} + z^{-2}}{1 - z^{-2}}, \qquad && \Omega = \frac{c_0 - \cos\omega}{\sin\omega} \\
&\text{(BS)} \qquad s = \frac{1 - z^{-2}}{1 - 2c_0 z^{-1} + z^{-2}}, \qquad && \Omega = -\frac{\sin\omega}{c_0 - \cos\omega}
\end{aligned}
\tag{86}
$$

where $c_0 = \cos\omega_0$, with ω_0 corresponding to the center of the bandpass or bandstop filter.

Reprinted from: ‘Lecture Notes on Elliptic Filter Design’ by Sophocles J. Orfanidis