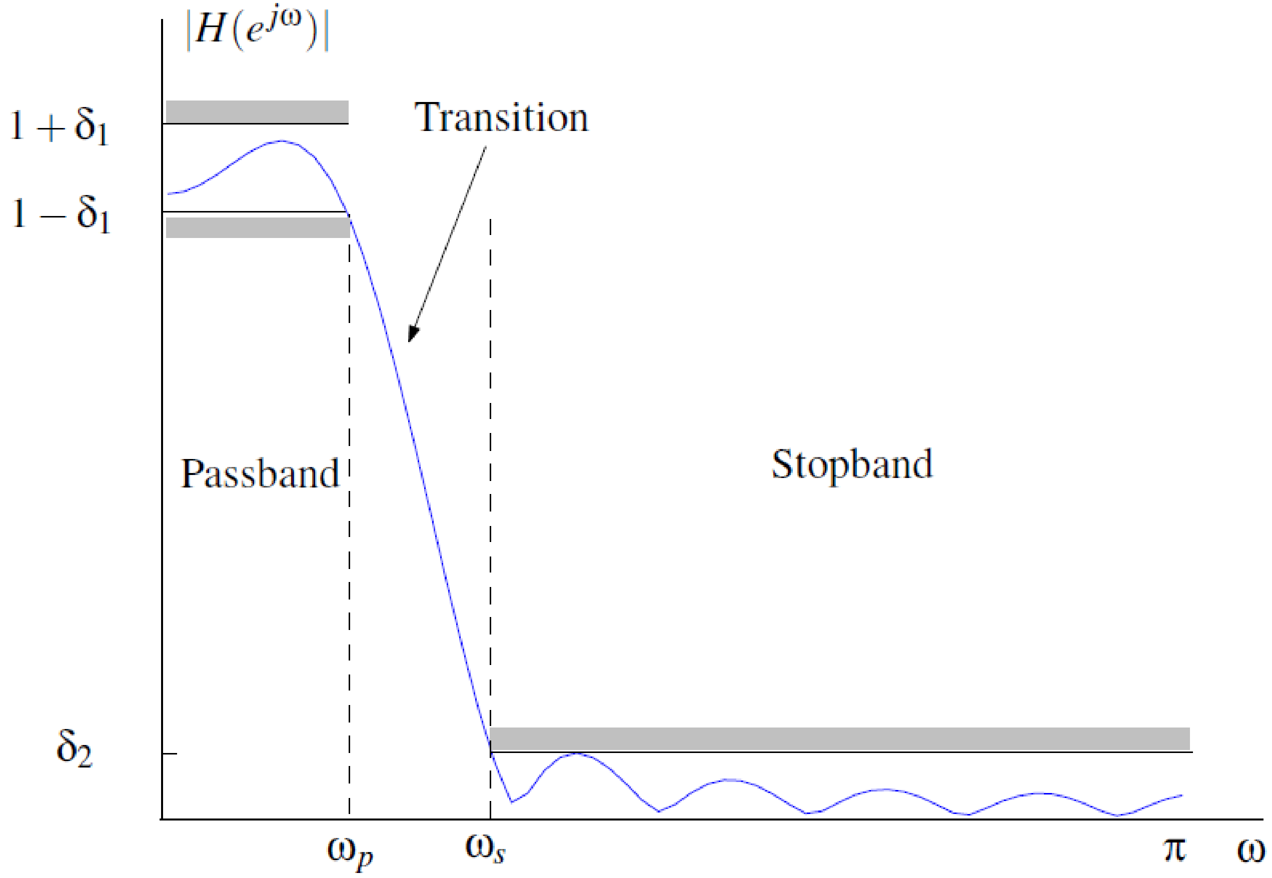
Module 5

Finite Impulse Response Filter Design, Part II

Overview

- Equiripple FIR filter design
  - Parks-McClellan algorithm details
  - Design example
- Eigenfilter design
  - Optimization criteria
  - Design examples

Approximation to Ideal Lowpass Filter



Parks-McClellan Algorithm

- A numerical approach for equiripple FIR filter design
  - Approximation error is distributed uniformly in the passband and stopband
  - Allows independent control of passband and stopband ripple
- Implemented in MATLAB function *firpm*

Algorithm Details

- Consider the Type I *zero phase* FIR filter

$h(n)=h(-n),\ -M/2\leq n\leq M/2$ even symmetry, M even

- The frequency response is

$$H\left(e^{j\omega}\right) = \sum_{n=-M/2}^{M/2} h(n)e^{-j\omega n}$$

$$= h(0)+\sum_{n=1}^{M/2} 2h(n)\cos\omega n$$

Algorithm Details

- Using the Chebyshev polynomial, the cosine term in the frequency response may be expressed as $\cos(\omega n) = C_n(\cos\omega)$
- $C_n(x)$ is the n^{th} order Chebyshev polynomial defined by the recursion

$$C_0(x) = 1, \quad C_1(x) = x$$

$$C_{n+1}(x) = 2x\,C_n(x) - C_{n-1}(x)$$

Algorithm Details

- The frequency response can now be expressed as

$$H\left(e^{j\omega}\right) = \sum_{n=0}^{L} a_n (\cos\omega)^n \qquad L = M/2$$

which is a polynomial in $\cos\omega$

- The FIR filter design problem has been recast into a problem of polynomial approximation.

Algorithm Details

- Optimization objective is to minimize the maximum approximation error in the passband and stopband (*minimax* criterion)
  - o Different weighting functions may be applied to the passband and stopband
- **Alternation theorem** (see references) states that the optimizing polynomial must have at least L+2 alternations. At these frequencies the frequency response is exactly equal to the maximum allowable ripple.
  - o Alternations occur at ω_p and ω_s and either $\omega=0,\pi$ or both

Estimating the Filter Order

- The filter order M can be estimated from

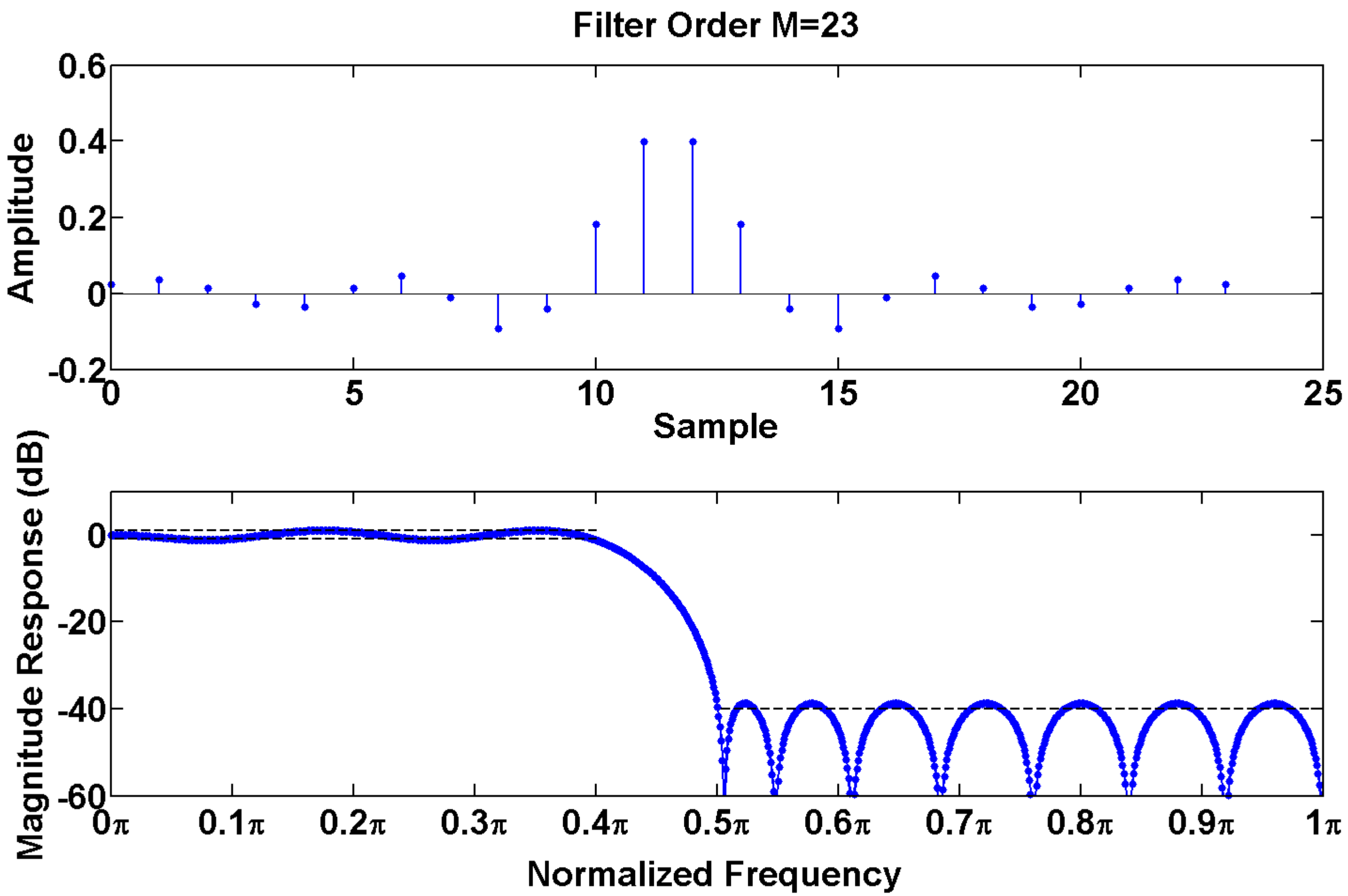
$$M = \frac{-10\log_{10}(\delta_1\delta_2) - 13}{2.324\Delta\omega}$$

- Implemented by MATLAB function *firpmord*

Design Example

- Using the Parks-McClellan algorithm, design a FIR filter with the following specifications:
    - $\omega_p = 0.4\pi$, $\omega_s = 0.5\pi$
    - $A_p = 1\text{dB}$, $A_s = 40\text{dB}$
- Filter order is estimated to be M=23
    - Order is actually *underestimated* in this case
    - M=27 is required to meet specifications

Design Example



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Design Example