Homework for Module I

1. Consider the sequence $x(n) = \alpha^{|n|}$ with α complex. Find its z-transform $X(z)$ and associated region of convergence. Under what conditions does the Fourier transform exist?
2. Consider the non-linear discrete-time system $y(n) = [x(n)]^{\beta}$ with β a positive integer. Compute analytically the Fourier transform of the output for arbitrary input and $\beta = 2$. Evaluate $y(n)$ directly for $x(n) = \cos(\omega_0 n)$ and $\beta = 2,3$. What conclusions can you make regarding the bandwidth properties of the non-linear system $y(n) = [x(n)]^{\beta}$ for arbitrary β?
3. Consider the system block diagram and input as shown in the following sketch. Compute $Y(e^{j\omega})$ for the following cases: $\omega_c = \frac{\pi}{2}$ and M=2,3. Comment on the effects of the M-sample compressor for each value of M. Is the information in $X(e^{j\omega})$ preserved in both cases?