Module 7

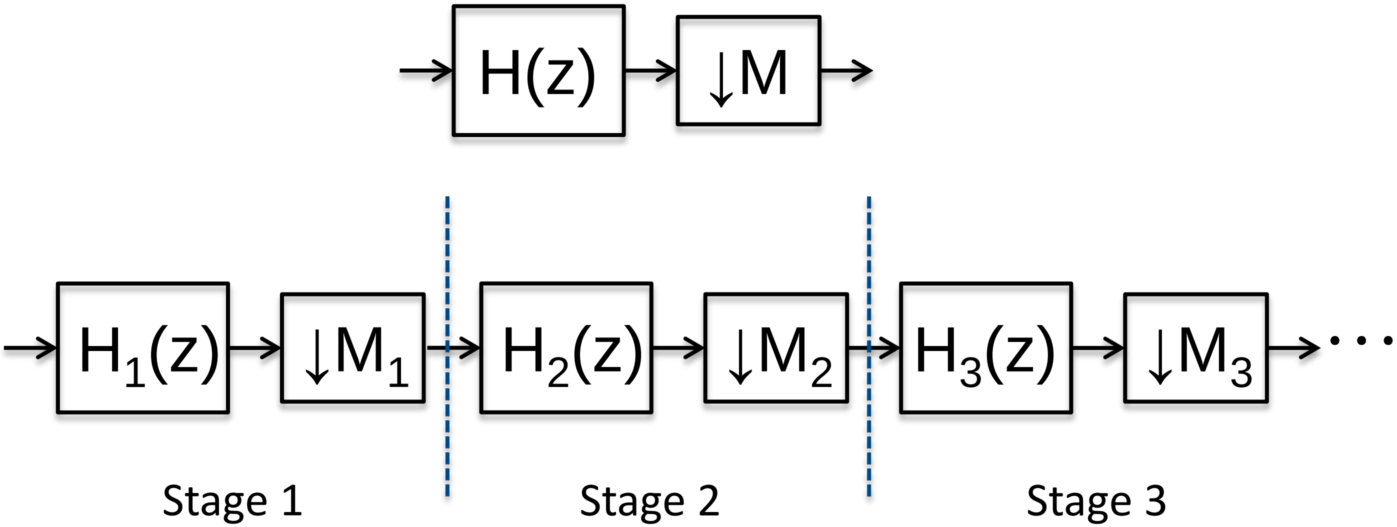
Upsampling and Downsampling, Part II

Overview

- Multistage downsampling concept
- Interpolated FIR (IFIR)
- IFIR design example
- Other stretch factors
- Multistage decimator design
- Multistage design example

Multistage Downsampler

- An M-fold downsampler can often be implemented more efficiently by splitting the operation into multiple stages



Multistage Downsampler

- Key questions
  - How to factor $M=M_1 x M_2 x M_3 \ldots$?
  - How to arrange the stages?
- We begin by introducing *Interpolated FIR* (IFIR) filters which are useful for the design of narrowband lowpass filters

Interpolated FIR (M=2)

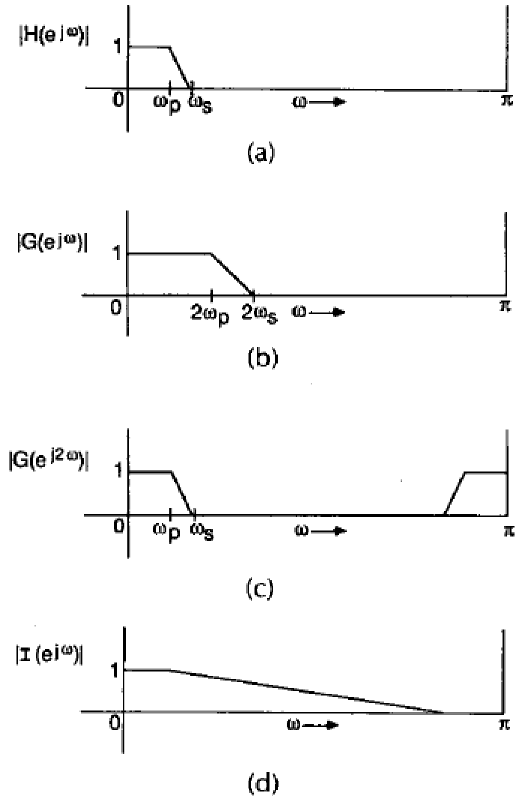


Fig. 42. Illustrating IFIR approach for narrow-band FIR design. (a) Low-pass filter with desired response. (b) Twofold stretched response. (c) Replacing each delay with two delays. (d) Removing unwanted passband by use of low-pass filter *I*(*z*).

The desired frequency response is first *stretched* by a factor of 2.

The frequency response of the interpolated filter has the desired passband and stopband edge frequencies, but has an unwanted image at ω=π.

The filter I(z) removes the unwanted image at ω=π.

Interpolated FIR

$$\rightarrow \boxed{H(z)} \rightarrow \;\equiv\; \rightarrow \boxed{I(z)} \rightarrow \boxed{G(z^2)} \rightarrow$$

Since G(z) is a stretched version of the desired response H(z), its transition band is wider (2x in this example) and therefore it requires less computation.

I(z) removes the undesired image from $G(z^2)$ and has a wide transition band.

Interpolated FIR

- For an equiripple linear-phase filter designed using the Parks-McClellan algorithm, the filter order N is estimated by

$$N = \frac{-10\log_{10}(\delta_1\delta_2) - 13}{2.324\Delta\omega}$$

δ_1 and δ_2 are the passband and stopband ripple parameters, respectively, and $\Delta\omega$ is the width of the transition band.

Adjusting the Ripple Sizes

- For two lowpass filters with passband and stopband ripple parameters (α_1,α_2) and (β_1,β_2), what can be said about the passband/stopband ripple for the cascade connection?

$$(1+\alpha_1)(1+\beta_1)\approx 1+(\alpha_1+\beta_1) \quad \text{passband}$$

$$\alpha_2\beta_2 \le \max(\alpha_2,\beta_2) \quad \text{stopband}$$

- We can choose $\alpha_1=\beta_1=\delta_1/2$ and $\alpha_2=\beta_2=\delta_2$

Design Example

- Specifications for H(z)
  - o $\omega_p = 0.09\pi$, $\omega_s = 0.11\pi$
  - o $\delta_1=0.02$, $\delta_2=0.001$
- The required filter order for direct implementation of H(z) (equiripple design) is N=233.

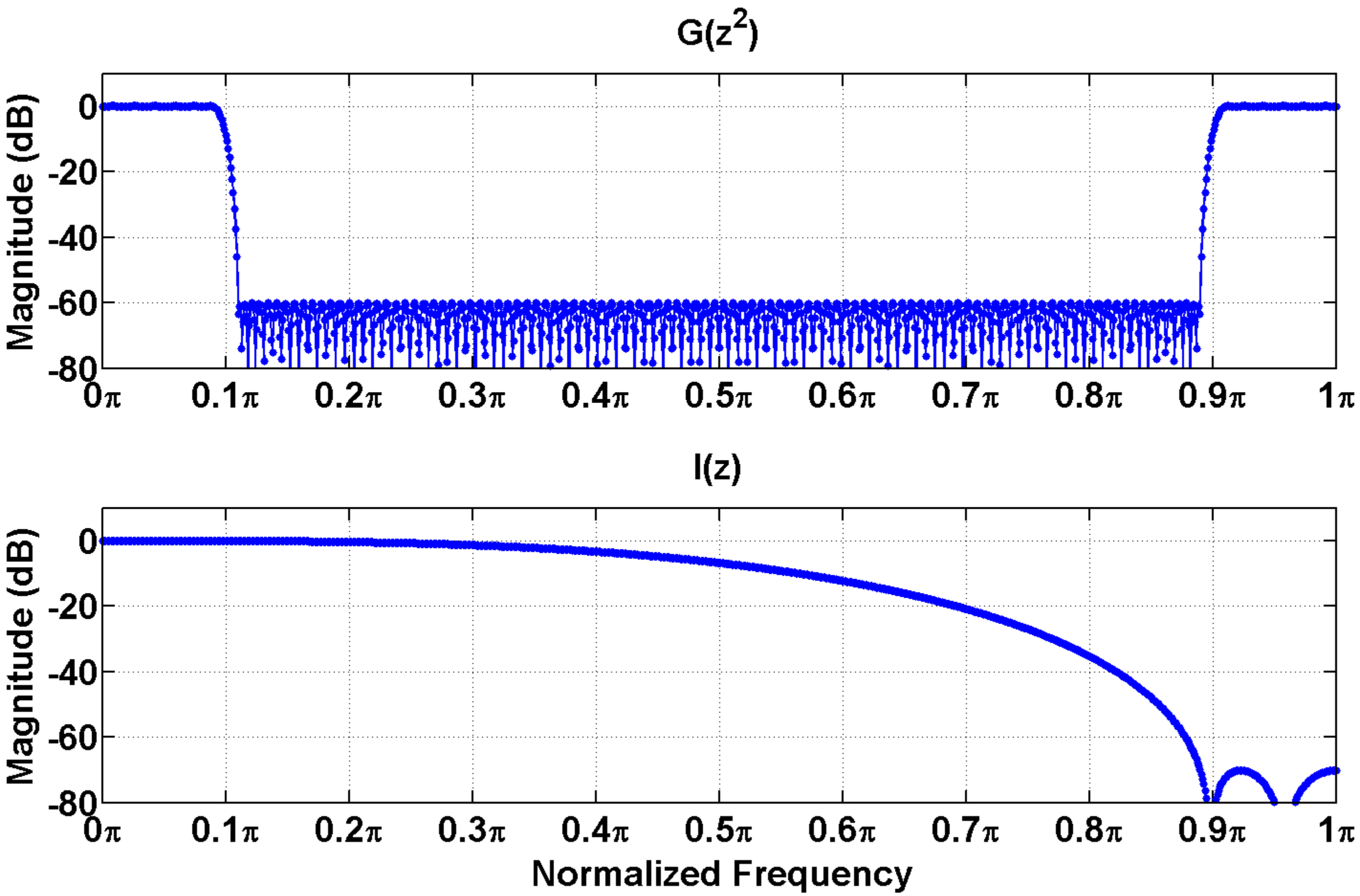
Design Example

- If we use the IFIR method with a stretch factor of 2x
  - Specifications for G(z)
    - $\omega_p = 0.18\pi$, $\omega_s = 0.22\pi$
    - $\delta_1=0.01$, $\delta_2=0.001$

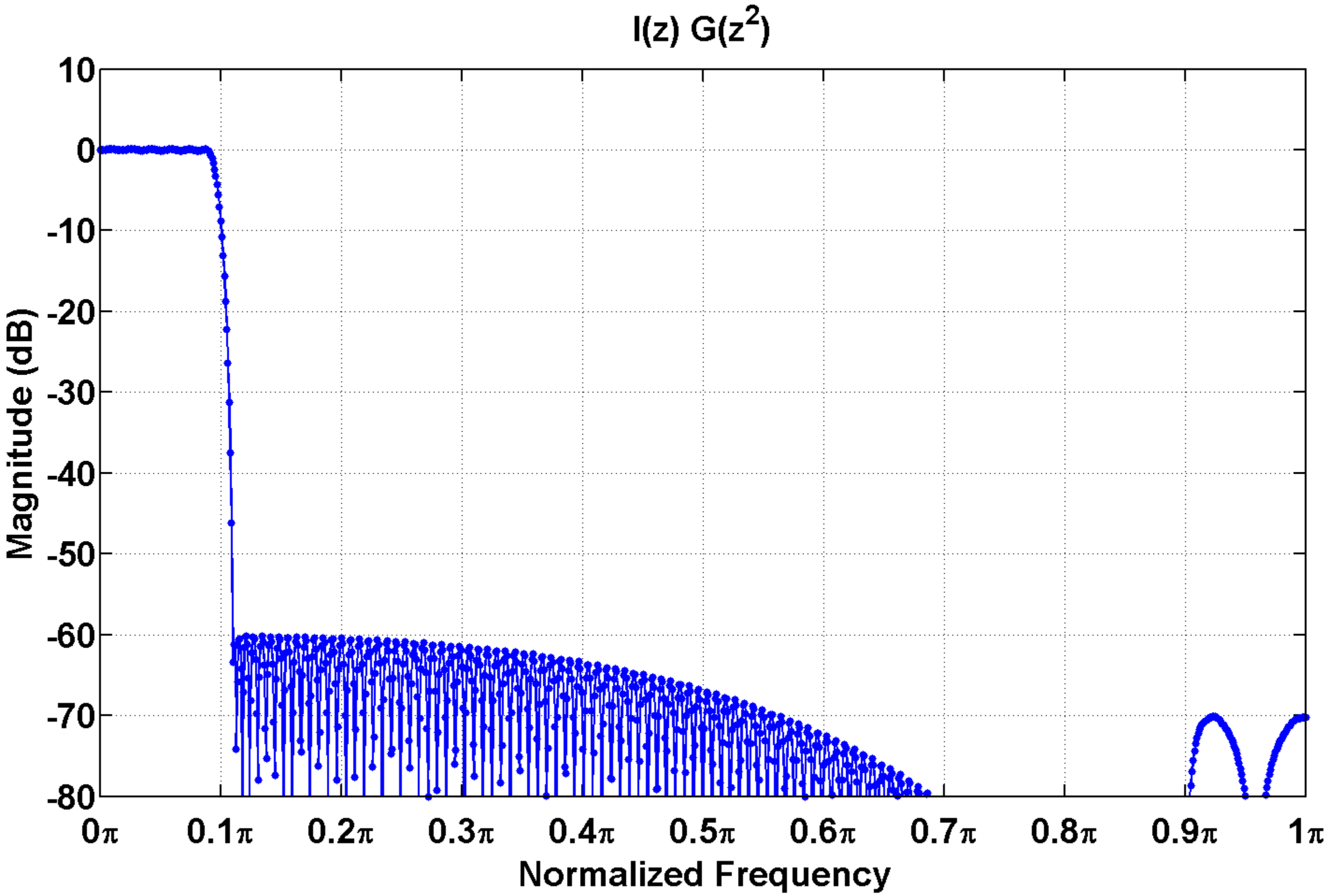
    $\Rightarrow$ filter order for G(z) is $N_G=131$
  - Specifications for I(z)
    - $\omega_p = 0.09\pi$, $\omega_s = \pi-0.11\pi$
    - $\delta_1=0.01$, $\delta_2=0.001$

    $\Rightarrow$ filter order for I(z) is $N_I=6$

Design Example



Design Example



Stretch Factors >2

- We can use a stretch factor M_1>2. In this case, the filter parameters for G(z) and I(z) are as follows:
  - G(z)
    - $M_1\omega_p$, $M_1\omega_s$
    - $\delta_1/2$, δ_2
  - I(z)
    - ω_p, $2\pi/M_1-\omega_s$
    - $\delta_1/2$, δ_2

← Passband/stopband edge frequencies stretched by M_1

← Stopband edge frequency set to reject images of $G(z^{M_1})$

Stretch Factors >2

- Total computational cost is $N_G + N_I$ where:

$$N_G = \frac{D(\delta_1/2, \delta_2)}{M_1(\omega_s - \omega_p)} \qquad N_I = \frac{D(\delta_1/2, \delta_2)}{\frac{2\pi}{M_1} - (\omega_s + \omega_p)}$$

$$D(\delta_1, \delta_2) = \frac{-10\log_{10}(\delta_1\delta_2) - 13}{2.324}$$

- By varying the stretch factor M_1, we can minimize the computational cost