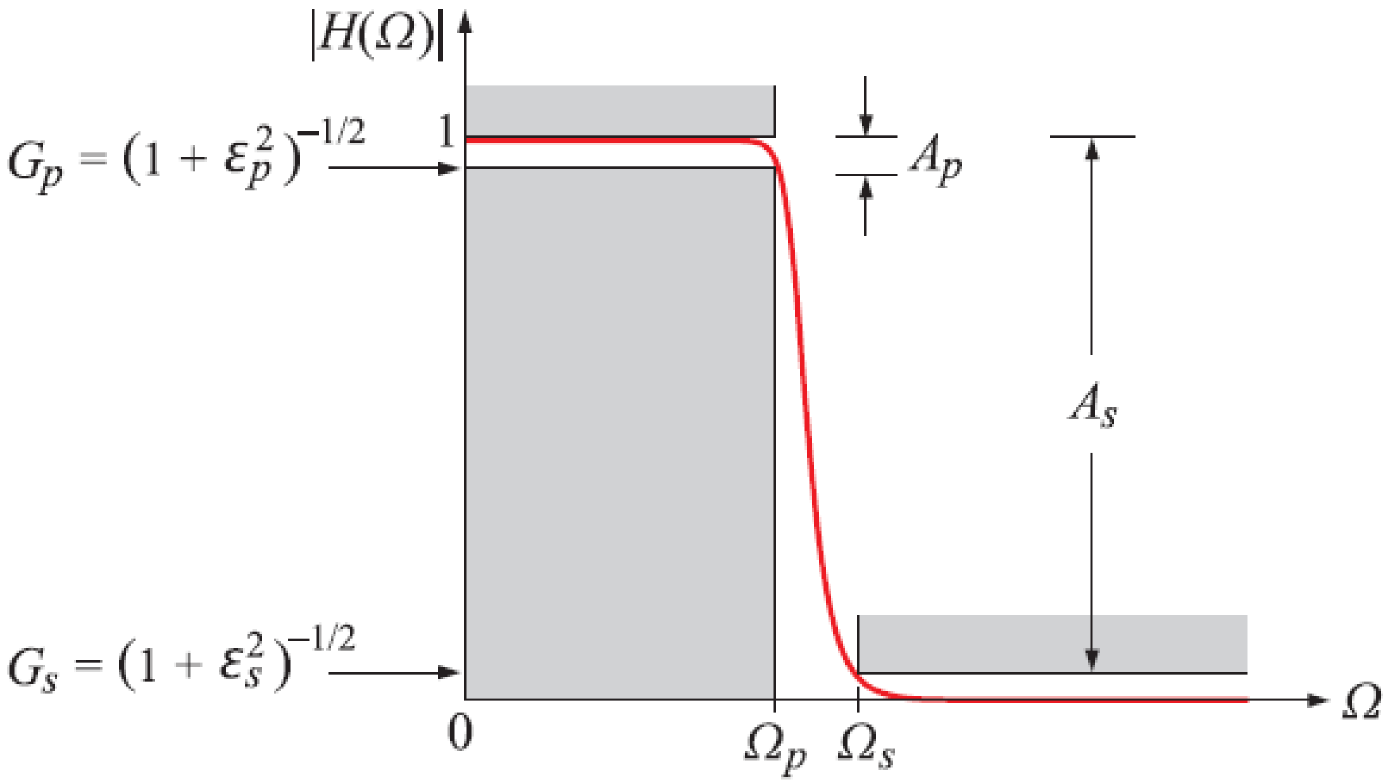
Module 2

Infinite Impulse Response Filter Design, Part I

Overview

- Analog filter types
  - Butterworth
  - Chebyshev I/II
  - Elliptic
  - Bessel
- Bilinear transformation
- Other transformations

General Formulation (1 of 3)



Reprinted from: 'Lecture Notes on Elliptic Filter Design' by Sophocles J. Orfanidis

General Formulation (2 of 3)

- General form for magnitude-squared response

$$|H(j\Omega)|^2 = \frac{1}{1 + \epsilon_p^2 F_N^2\left(\frac{\Omega}{\Omega_p}\right)}$$

- N – filter order
- Ω_p,Ω_s – frequencies of passband/stopband edges
- $\varepsilon_p,\varepsilon_s$ – attenuation parameters for passband/stopband
- $F_N(x)$ – function depending on response type (e.g. Butterworth, Chebyshev I/II, Elliptic)

General Formulation (3 of 3)

- Given passband/stopband attenuation (in dB)

$$\epsilon_p = \sqrt{10^{A_p/10} - 1} \quad \epsilon_s = \sqrt{10^{A_s/10} - 1}$$

- The filter order N can be computed from

$$F_N\left(k^{-1}\right) = k_1^{-1}$$

with $k = \dfrac{\Omega_p}{\Omega_s} \quad k_1 = \dfrac{\epsilon_p}{\epsilon_s}$

Butterworth Response (1 of 3)

- The magnitude response of the Butterworth filter is *maximally flat* at $\Omega=0$
- Magnitude response is monotonic in both passband and stopband
- Poles lie on a circle of radius Ω_c (3dB cutoff frequency) in the s-plane
- No zeros in the finite s-plane (all at $s=\infty$)

Butterworth Response (2 of 3)

- Magnitude-squared response

$$|H(j\Omega)|^2 = \frac{1}{1+\epsilon_p^2\left(\frac{\Omega}{\Omega_p}\right)^{2N}}$$

N – filter order
Ω_p – passband frequency

- Filter order N is computed from

$$N = \frac{\log\left(\epsilon_s/\epsilon_p\right)}{\log\left(\Omega_s/\Omega_p\right)}$$

Butterworth Response (3 of 3)

- Pole locations (left half of s-plane)

$$s_k = j\Omega_c e^{j(2k-1)\frac{\pi}{2N}} \quad k = 1, \ldots, N$$

$$\Omega_c = \epsilon_p^{-1/N}\Omega_p$$ 3dB cutoff frequency

Chebyshev Response

- Chebyshev Type I
  - Equiripple in the passband and monotonic in the stopband
- Chebyshev Type II
  - Equiripple in the stopband and monotonic in the passband

Chebyshev Type I (1 of 3)

- Magnitude-squared response for Chebyshev Type I

$$|H(j\Omega)|^2 = \frac{1}{1 + \epsilon_p^2 C_N^2\left(\frac{\Omega}{\Omega_p}\right)}$$

- Ω_p is the equiripple cutoff frequency
- $C_N(x)$ is the N^{th} order Chebyshev polynomial

Chebyshev Type I (2 of 3)

$$C_N(x) = \begin{cases} \cos\left(N\cos^{-1}x\right), & \text{if } |x| \le 1 \\ \cosh\left(N\cosh^{-1}x\right), & \text{if } |x| > 1 \end{cases}$$

- $C_N(x)$ can be shown to obey the following recursion

$$C_{N+1}(x) = 2xC_N(x) - C_{N-1}(x)$$

with $C_0(x) = 1$, $C_1(x) = x$

Chebyshev Type I (3 of 3)

- Pole locations (left half of s-plane)

$$Re\{s_k\} = -\Omega_p \left[\sinh\left(\frac{1}{N}\sinh^{-1}\left(\frac{1}{\epsilon_p}\right)\right)\sin\left((2k-1)\frac{\pi}{2N}\right)\right]$$

$$Im\{s_k\} = \Omega_p \left[\cosh\left(\frac{1}{N}\sinh^{-1}\left(\frac{1}{\epsilon_p}\right)\right)\cos\left((2k-1)\frac{\pi}{2N}\right)\right]$$

$$k = 1, \ldots, N$$

- Filter order $N = \dfrac{\cosh^{-1}\left(\epsilon_s/\epsilon_p\right)}{\cosh^{-1}\left(\Omega_s/\Omega_p\right)}$

Chebyshev Type II

- Magnitude-squared response for Chebyshev Type II

$$|H(j\Omega)|^2 = \frac{1}{1 + \square_s^2 / C_N^2\left(\dfrac{\Omega_s}{\Omega}\right)}$$

- Filter order N computed using same equation as for Type I
- See ‘Lecture Notes on Elliptic Filter Design’ for pole/zero locations