Module 5

Finite Impulse Response Filter Design, Part II

Overview

- Equiripple FIR filter design
  - Parks-McClellan algorithm details
  - Design example
- Eigenfilter design
  - Optimization criteria
  - Design examples

Eigenfilter Design

- Optimization approach in which the *mean-square error* over the passband and stopband is minimized.
- Allows relative weighting of passband and stopband errors
- Impulse response is related to the *eigenvector* of a quadratic form

Algorithm Details

- Consider the Type I linear phase FIR filter

$$h(n) = h(N-n),\ 0 \le n \le N \quad \text{even symmetry, N even}$$

- The frequency response is

$$H\left(e^{j\omega}\right) = e^{-j\omega N/2}\left[h(N/2) + \sum_{n=1}^{N/2} 2h(N/2-n)\cos\omega n\right]$$

$$= e^{-j\omega M} H_R(\omega) \quad M = N/2$$

Algorithm Details

$$H_R(\omega)=\sum_{n=0}^{M} b_n \cos \omega n \qquad b_n=\begin{cases} h(M) & \text{n=0} \\ 2h(M-n) & \text{n=1,...,M} \end{cases}$$

- $H_R(\omega)$ can be written as a dot product of two vectors:

$$H_R(\omega)=\mathbf{b}^T \mathbf{c}(\omega) \qquad \text{where}$$

$$\mathbf{b}=\left[b_0,b_1,\ldots,b_M\right]^T \quad \mathbf{c}(\omega)=[1,\cos\omega,\cos 2\omega,\ldots,\cos M\omega]^T$$

Algorithm Details

- Define the ideal amplitude response

$$D(\omega)=\begin{cases} 1 & 0\le\omega\le\omega_p \\ 0 & \omega_s\le\omega\le\pi \end{cases}$$

- The mean-square error for the *stopband* is

$$\begin{aligned}\delta_s &= \frac{1}{\pi}\int_{\omega_s}^{\pi}\left[D(\omega)-H_R(\omega)\right]^2 d\omega \\ &= \frac{1}{\pi}\int_{\omega_s}^{\pi}\mathbf{b}^T\mathbf{c}(\omega)\mathbf{c}^T(\omega)\mathbf{b}\,d\omega=\mathbf{b}^T\mathbf{P}_s\mathbf{b}\end{aligned}$$

Algorithm Details

- The matrix $\mathbf{P}_s$ is defined as

$$\mathbf{P_s} = \frac{1}{\pi}\int_{\omega_s}^{\pi} \mathbf{c}(\omega)\mathbf{c}^T(\omega)\,d\omega$$

- The mean-square error for the *passband* is

$$\begin{aligned}\delta_p &= \frac{1}{\pi}\int_0^{\omega_p}\left[H_R(0)-H_R(\omega)\right]^2 d\omega \\ &= \frac{1}{\pi}\int_0^{\omega_p}\mathbf{b}^T(\mathbf{1}-\mathbf{c}(\omega))(\mathbf{1}-\mathbf{c}(\omega))^T\mathbf{b}\,d\omega = \mathbf{b}^T\mathbf{P_p}\mathbf{b}\end{aligned}$$

Algorithm Details

- The matrix $\mathbf{P}_p$ is defined as

$$\mathbf{P_p} = \frac{1}{\pi}\int_0^{\omega_p} (\mathbf{1} - \mathbf{c}(\omega))(\mathbf{1} - \mathbf{c}(\omega))^T \, d\omega$$

- Both $\mathbf{P}_s$ and $\mathbf{P}_p$ can be computed analytically

Algorithm Details

- We consider a *weighted combination* of the mean-square passband and stopband errors

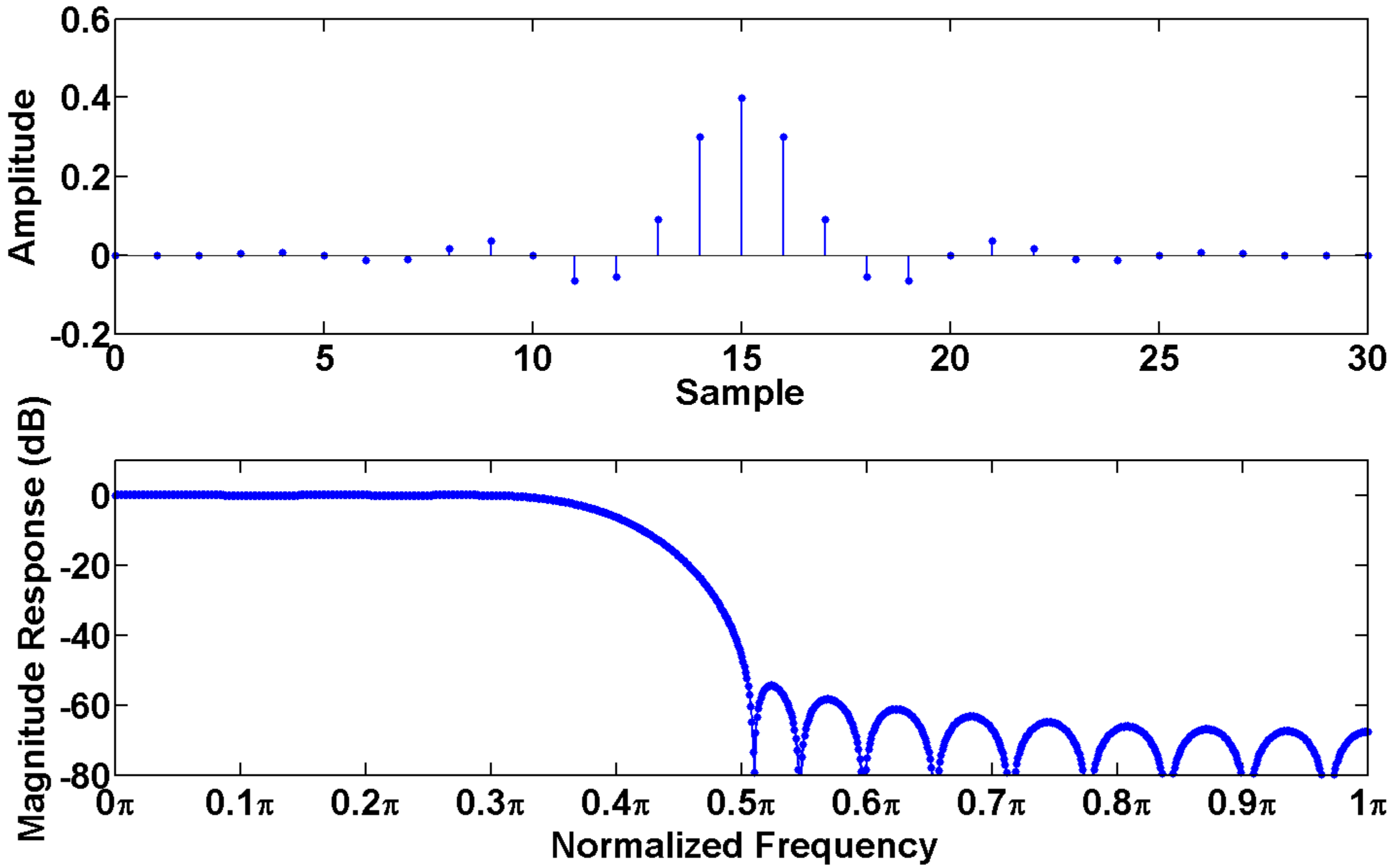
$$\delta = \alpha\delta_s + (1-\alpha)\delta_p \quad \mathbf{P} = \alpha\mathbf{P}_s + (1-\alpha)\mathbf{P}_p \quad 0 \le \alpha \le 1$$

- Optimization problem is to minimize δ subject to $\mathbf{b}^T\mathbf{b} = 1$. The solution is given by the *eigenvector* of **P** with the smallest *eigenvalue*.

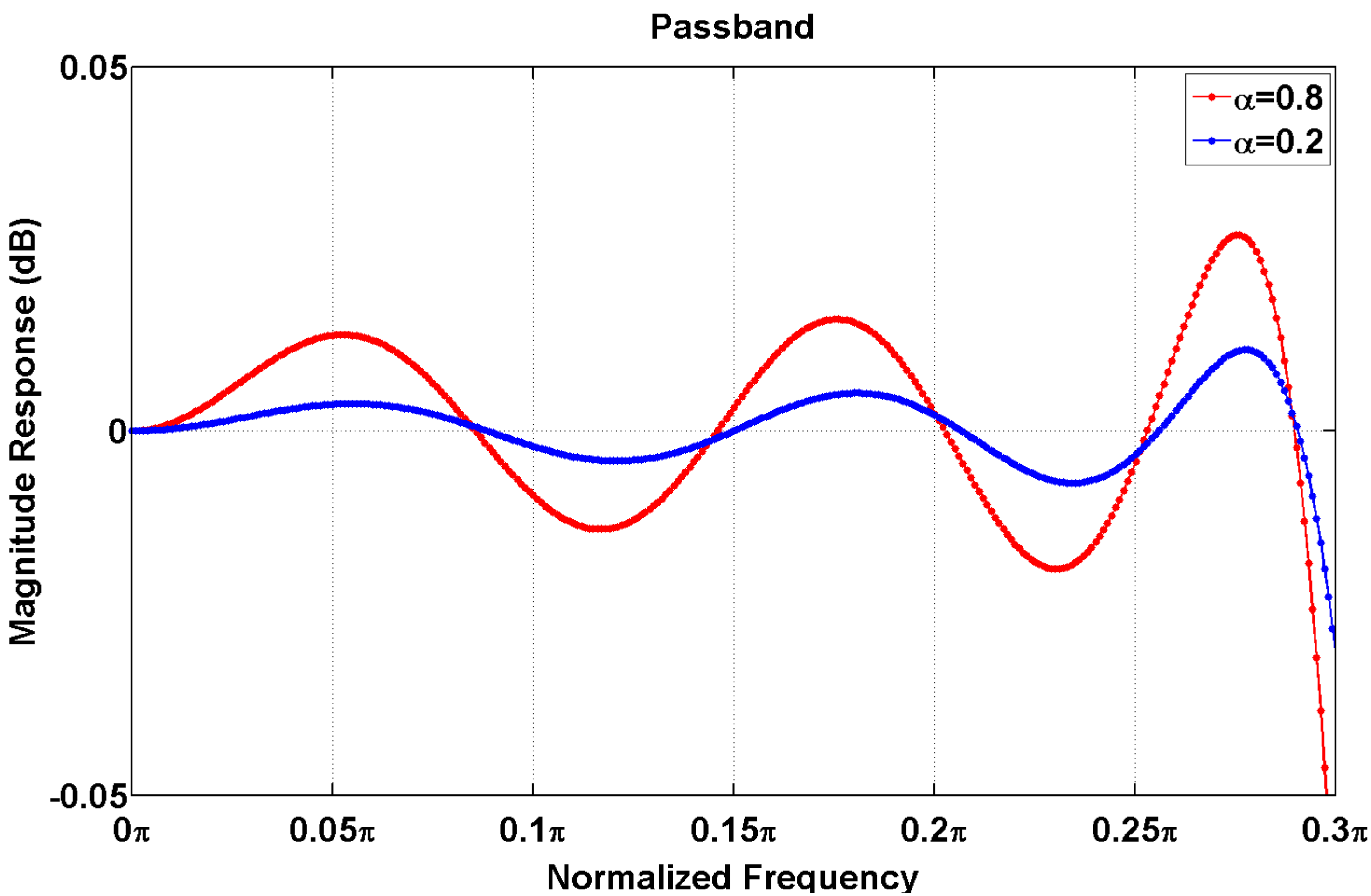
Design Example

- Design a lowpass eigenfilter with
  - o $\omega_p = 0.3\pi$, $\omega_s = 0.5\pi$
  - o $N = 30$, $\alpha = 0.2$
- The matrices $\mathbf{P}_s$ and $\mathbf{P}_p$ can be computed in closed form. Using the MATLAB function *eig,* we can find the eigenvectors and eigenvalues of **P**. The optimal vector **b** is found as the eigenvector corresponding to the smallest eigenvalue. The impulse response is then found from **b**.

Design Example (α=0.2)



Passband Comparison for α=0.2,0.8



Stopband Comparison for α=0.2,0.8