Module 4

Finite Impulse Response Filter Design, Part I

Overview

- Ideal lowpass filter and its impulse response
- FIR filter design using windows
- Window design tradeoffs
- Linear phase FIR filters

Ideal Lowpass Filter (1 of 2)

- Frequency response

$$H_{LP}\left(e^{j\omega}\right)=\begin{cases}1, & \text{for } |\omega|\le\omega_c \\ 0, & \text{for } \omega_c<|\omega|\le\pi\end{cases}$$

- Impulse response

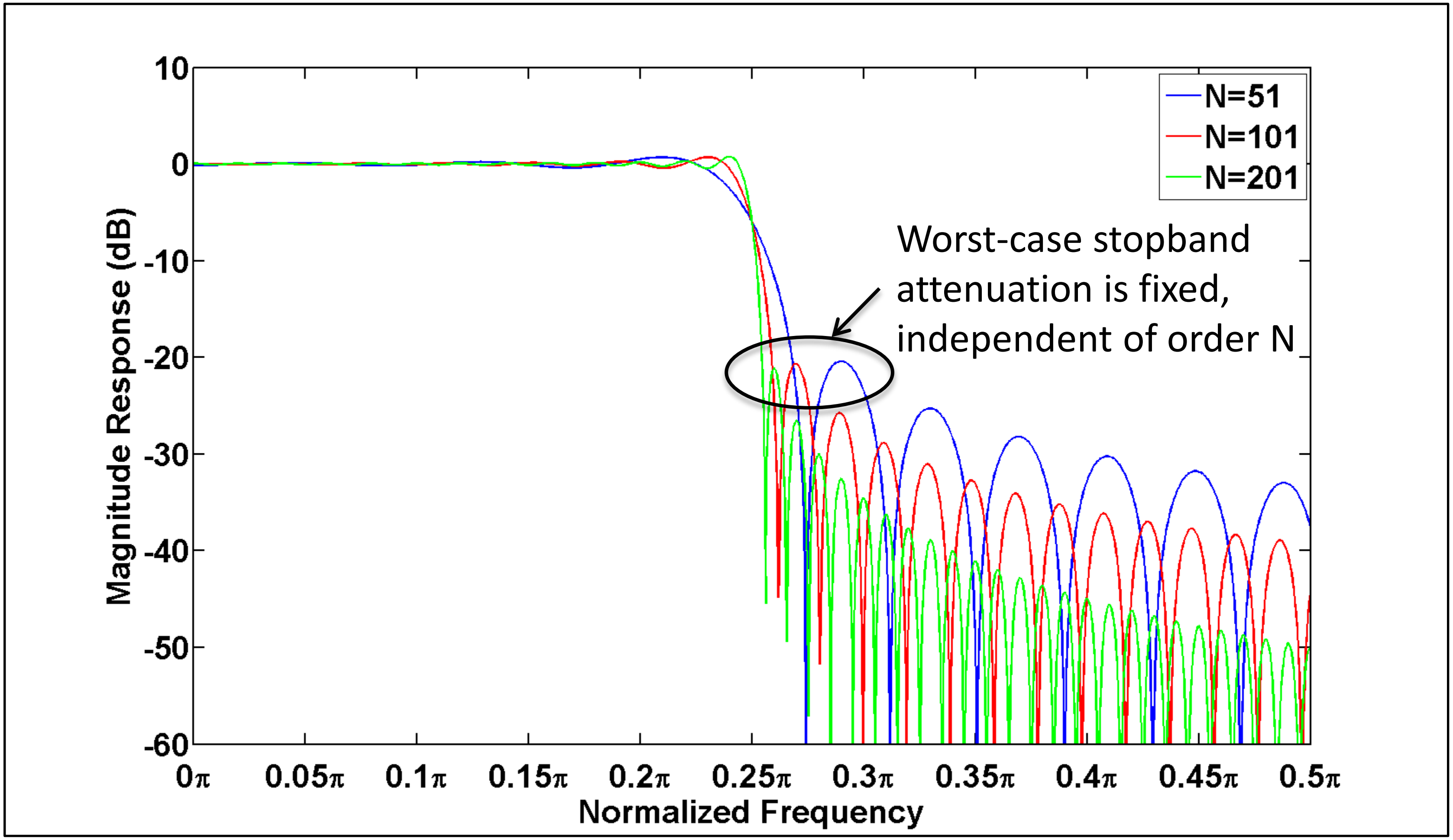
$$h_{LP}(n)=\frac{\sin\omega_c n}{\pi n}\quad -\infty<n<\infty$$

- Impulse response has *infinite* extent

Ideal Lowpass Filter (2 of 2)

- Simply truncating the impulse response (rectangular window) produces a realizable filter, however, the best achievable stopband attenuation is ~21dB, regardless of filter order.
- Use of a window sequence with a gradual taper will mitigate this effect but introduces a tradeoff between transition band width and stopband attenuation.

Lowpass Filter w/ Rectangular Window



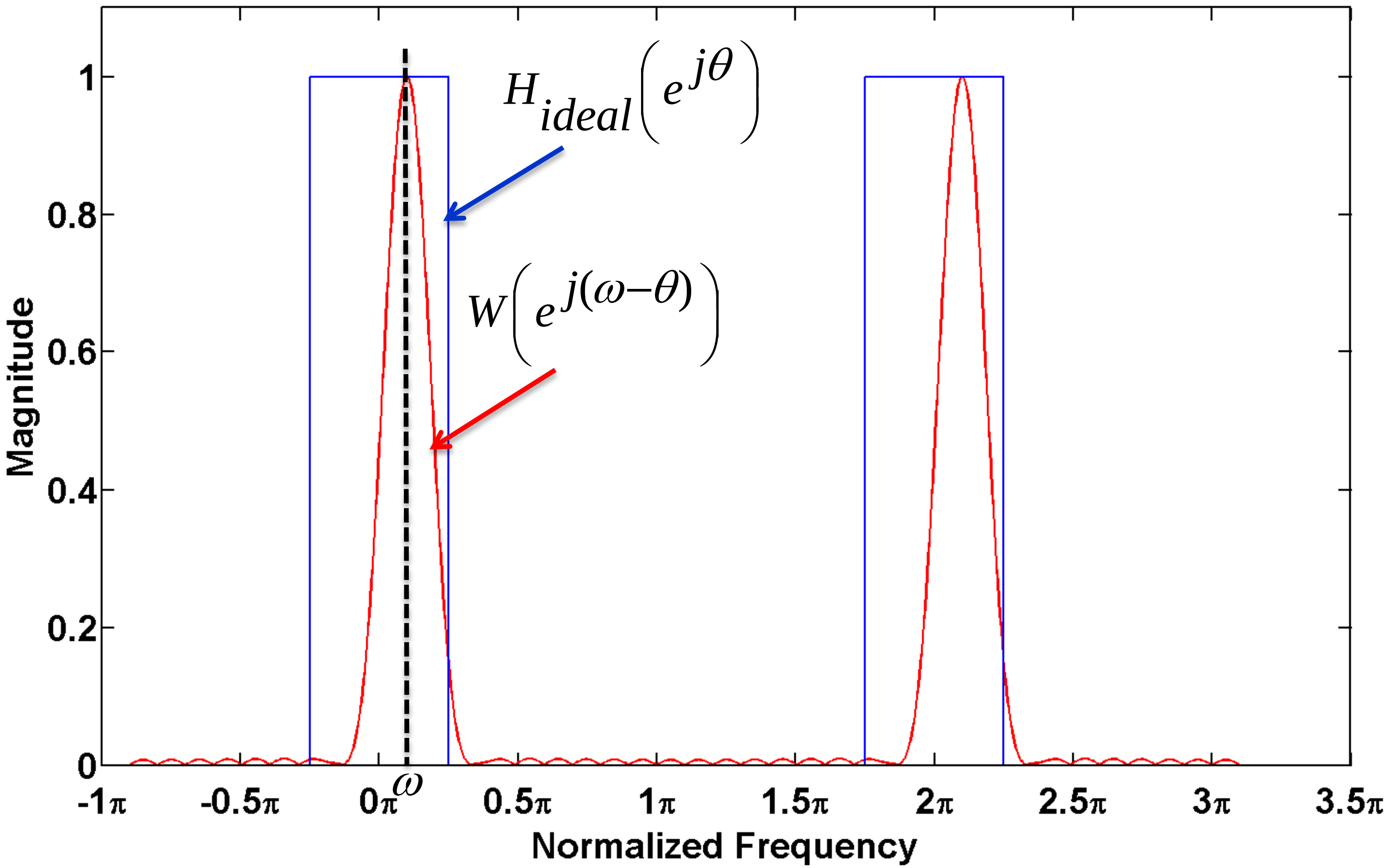
FIR Filter Design using Windows (1 of 2)

$$h(n) = \underbrace{h_{ideal}(n)}_{\text{ideal impulse response}} \times \underbrace{w(n)}_{\text{window sequence}}$$

$$H\left(e^{j\omega}\right) \frac{1}{2\pi} \int_{-\pi}^{\pi} H_{ideal}\left(e^{j\theta}\right) W\left(e^{j(\omega-\theta)}\right) d\theta$$

Frequency response obtained is the *periodic convolution* (in frequency) of the ideal frequency response with the Fourier transform of the window sequence.

Convolution in Frequency



FIR Filter Design using Windows (2 of 2)

- Width of the main lobe of $W(e^{j\omega})$ determines the *width of the transition band* for $H(e^{j\omega})$.
- Sidelobe levels of $W(e^{j\omega})$ determine the amount of *passband ripple* and the *stopband attenuation* for $H(e^{j\omega})$.
- Selection of a window sequence involves a tradeoff between these parameters.

Common Windows (1 of 2)

- Rectangular $w(n)=1, \quad 0 \le n \le N-1$
- Hamming $w(n)=0.54-0.46\cos\left(\dfrac{2\pi n}{N-1}\right), \quad 0 \le n \le N-1$
- Dolph-Chebyshev
- Kaiser $w(n)=I_0\left(\beta\sqrt{1-\left(\dfrac{2n}{N-1}-1\right)^2}\right)/I_0(\beta), \quad 0 \le n \le N-1$

$I_0(x)$ is the modified zeroth-order Bessel function

Common Windows (2 of 2)