Optimizing Number of Fire-Fighters During Special Events

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***Abstract*—This paper uses linear programming to optimize the number of fire-fighter staff available for special events. The initial model is derived from Stevanovic, et al [1]. This model successfully allocates numbers of fire-fighters across shifts in a way that reduces total cost. The model is able to achieve optimization for a single event, but it does not provide solutions for staffing when multiple events are considered. In order to address this, we introduce a linearized maximin model to optimize staffing for multiple events while considering budgeting constraints.**

Keywords—optimization; linear programming; fire-fighter; model

I. Introduction

Organizations often need to make several decisions in regards to staffing and scheduling. These decisions are influenced by many factors such as the number of staff available, the cost of each staff member, shift duration, and the number of staff required for successful operations. Fire departments and the scheduling of fire-fighters is no exception.

Fire departments are responsible for staffing fire-fighters so that there is a sufficient amount of personnel to respond to emergencies around the clock. In addition to the staffing of these day to day operations, they commonly provide additional services for special events where the presence of fire-fighters is warranted. Special events include community events, music concerts, sporting events, and the like.

In the case of scheduling for special events, the main decision to be made is how many fire-fighters to schedule for each shift. To make this decision, the number of fire-fighters available and the number of fire-fighters required for discrete time periods must be considered. This decision should minimize the total cost of staffing a given special event.

II. Preliminary Model

A. *The Need for a Model*

Consider the day-to-day operations of a fire department in a northwestern state. Fire departments are depended upon to protect life and property from fire and other disasters. Standard operations in this regard require trained firefighters on duty to respond to emergencies that can occur at any time. Adequate staff must be provided to handle a range of emergencies that are uncertain in time, location, and frequency. Along with this staffing, monitoring of social and special events where incidents are more likely to occur is also needed.

B. *Decisions*

In order to provide coverage for these special events, station commanders need to decide on how to distribute personnel to cover special events and maintain a regular watch for emergencies. Station commanders can split normally available personnel to cover the events and can call upon additional personnel to work on an overtime basis. So the number of firefighters covering special events are the decision variables.

C. *Constraints*

Initial main constraints ensure that the minimum number of fire-fighters requested by the special event is staffed at any given time. Additional constraints are added so that the number of fire-fighters scheduled at the special event does not exceed the number of fire-fighters available for that shift. A final main constraint is added so that the total number of fire-fighters scheduled for the special event is less than or equal to the total number of fire-fighters available to the commander. This constraint considers fire-fighters that are not available due to mandated "off" time, and those absent due to vacation and illness. Lastly, nonnegativity constraints are placed on all decision variables.

D. *Objective*

The objective is to minimize the total cost across all shifts. Preferred solutions would be the allocation of personnel, which lowers costs.

E. *Parameter Values*

Parameter values come directly from the organizers of the special event. Event coordinators were able to request a specific number of fire-fighters for specific time periods based on experience from years prior.

F. *Model*

We reflect upon the model as presented by Stevanovic, et al., to minimize the cost of employing fire-fighters while providing an adequate staffing level to meet ordinary functions and provide sufficient coverage for special events [1]. The main parameters and the model as presented provide a summary of the elements required to make an informed staffing decision. These factors are defined as:

$x_i \triangleq$ number of fire-fighters working on shift i

$n_i \triangleq$ minimum number of fire-fighters required for special events on shift i

$N_i \triangleq$ minimum number of fire-fighters required for ordinary functions on shift i

$N \triangleq$ total number of available fire-fighters that the fire station can rely on

The model, as presented in the paper, is:

$$min\ f(x) = x_1 + x_2 \quad (1)$$

$$subject\ to:\ x_1 \geq n_i \quad (2)$$

$$x_2 \geq n_i \quad (3)$$

$$x_1 + x_2 \geq n_i \quad (4)$$

$$x_1 \geq N_1 \quad (5)$$

$$x_2 \geq N_2 \quad (6)$$

$$x_1 + x_2 \leq N \quad (7)$$

$$x_1, x_2 \geq 0 \quad (8)$$

An additional model is considered if the objective is not to simply minimize total number staffed, but instead to minimize total cost.

$C_i \triangleq$ cost for the daily allowance for one fire-fighter on shift i

$$min\ f(x) = C_1 x_1 + C_2 x_2 \quad (9)$$

$$subject\ to:\ x_1 \geq n_i \quad (2)$$

$$x_2 \geq n_i \quad (3)$$

$$x_1 + x_2 \geq n_i \quad (4)$$

$$x_1 \geq N_1 \quad (5)$$

$$x_2 \geq N_2 \quad (6)$$

$$x_1 + x_2 \leq N \quad (7)$$

$$x_1, x_2 \geq 0 \quad (8)$$

III. Alternative Model Proposal

A. Evaluation of Preliminary Model

The preliminary model can validly address the needs of decision-makers if all constraints are satisfied. Constraints ensure that there is an adequate amount of fire-fighters available. Once this is established, the model can allocate the number of fire-fighters for each shift in such a way that minimizes cost for a single special event.

While the model presented by Stevanovic, et al., provides insight into the problem of day-to-day staffing, it does not consider how this fits into the larger picture of projection and budgeting on an annual basis. To even consider the problem of day-to-day staffing, the staffing levels are determined by internal and external factors that provide the pool of firefighters available for work. However, the day-to-day staffing requirements feed back into the budgeting and annual staffing process, coupling the processes together.

B. Decisions

The model proposed by Stevanovic, et al. can be used to help decision-makers determine day-to-day staffing requirements, but first annual considerations must be made. The decision variables would continue to be the number of fire-fighters to staff for special events. However, changing the scope of this variable would allow us to make decisions for an annual time frame. Instead of deciding the number of fire-fighters for each shift, we would be deciding the number of fire-fighters to allocate to each special event in a given year.

C. Constraints

An assumption made by Stevanovic et al. is that a fire-station is able to supply the number of fire-fighters requested by a given special event. Primary sources at Gwinnett County Fire Department indicate that staffing for special events such as concerts and local football games is not mandatory [2]. Even for staffing of day to day operations, some fire stations are unable to meet minimum staffing requirements as suggested by the National Fire Protection Association (NFPA) due to budget constraints [3]. When considering that working special events may be voluntary, as well as the possibility of staff shortages, it is important to acknowledge that main constraints 2-4 may not be realistic.

Extrapolating the model to cover every special event within a year requires new main constraints that accommodate this scope. An annual budget constraint allocated for special events would be needed. The maximum number of available fire-fighters for each special event would be another constraint required for the model.

D. Objective

The preliminary model minimizes total cost while meeting the staffing demands of special events. However, the cost being minimized does not ensure that it is a cost that is affordable. If a special event requested a large enough number of fire-fighters, the cost of staffing could exceed what a budget allows for, regardless if there was a model to minimize said cost.

An alternative model could allow decision-makers to control this type of cost. The total cost of special events for a given year could be planned for in an annual budget. If this value were predetermined, a new objective would be to allocate funds evenly for all special events in a given year. This could be achieved using a linearized maximin model. The model would maximize the total expenditure of the budget set aside for special events so that they are staffing them to the best of their ability. Yet, it would be done in a way that maximizes the minimum amount each event is allocated.

E. Parameter Values

Initial parametric values are derived from the Eastside Fire & Rescue (EF&R) 2017 Budget and can be found in Table 1. EF&R provides life and property protection services for King County and the cities of Issaquah, North Bend, and Sammamish in Washington state. Every year EF&R develops an annual budget needed to cover all operational expenses through the year. Staffing levels are a significant portion of the annual budget at 87% of total operating expenses [4].

Table 1: Eastside Fire & Rescue 2017 Budget		
Suppression Division Budget		
Salaries	$13,611,894.00	
Benefits	$4,412,567.00	
Total	$18,024,461.00	
2017 Operating Expenses Budget		
Salaries	$15,653,184.00	
Benefits	$5,699,102.00	
Total	$21,352,286.00	
2017 Operating Expenses Budget for Overtime		$1,012,444.00
Suppression % of Total Salaries + Benefits		84.4%
Suppression Estimated Overtime Budget		$854,651.23
Suppression Division Total FTEs		121
Working Hours/Year		1980
Suppression Division Hourly Rate (mean for 121 FTEs @ 1980 working hours/year)		$75.23
Suppression Division Overtime Rate (overtime rate does not include benefits)		$56.82
Suppression Division Est. Available Overtime Hours/Year		15,043

F. Model

$j \triangleq$ event number

$c_j \triangleq$ cost of each fire-fighter working event j

$x_j \triangleq$ number of fire-fighters working event j

$n \triangleq$ number of events

$B \triangleq$ total budget for special events

$N \triangleq$ total number of fire-fighters available for each special event

$$max\ f \quad (10)$$

$$subject\ to:\ c_j x_j \geq f\ \ \forall j \quad (11)$$

$$\sum_{j=1}^{n} c_j x_j \leq B \quad (12)$$

$$x_j \leq N\ \ \forall j \quad (13)$$

$$x_j \geq 0\ \forall j \quad (14)$$

This model addresses decision making in terms of an annual time frame rather than for individual events. However, parameter c_j, the cost of each fire-fighter working event j becomes problematic when considering that there may be upwards of hundreds of events, each costing a different amount depending on their respective duration.

The model can be altered to anticipate event duration differences by indexing decision variables by time duration rather than an individual event. The revised annual model is a maximin model intended to evenly distribute the allocated budget for events evenly across shift durations:

$j \triangleq$ type of event defined by its duration

$x_j \triangleq$ number of fire-fighters working event of duration j

$r \triangleq$ hourly rate in dollars for events

$d_j \triangleq$ duration of event in hours

$c_j \triangleq$ number of events of duration j

$n \triangleq$ number of durations for events

$B \triangleq$ total budget for special events

$N \triangleq$ total number of fire-fighters available for each special event

$$max\ f \quad (10)$$

$$subject\ to:\ r d_j c_j x_j \geq f\ \ \forall j \quad (15)$$

$$\sum_{j=1}^{n} r d_j c_j x_j \leq B \quad (16)$$

$$x_j \leq N\ \ \forall j \quad (13)$$

$$x_j \geq 0\ \forall j \quad (14)$$

Consider EF&R has a fixed annual budget, which determines the number of fire-fighters available in that year. A first assumption is that the number of firefighters is determined solely by the budget, which was informed by historical data of demands for staffing to deal with fire suppression and rescue efforts [4]. It is also assumed that available staff is sized to account for absences because of training, sick leave, and vacation. From the data provided, we determine an hourly cost per fire-fighter, which is fixed by year. There is no need to account for salary differentials in a scheduling model as those costs are spread over the entire force for the entire year [4].

A further assumption is that a fixed portion of the overtime operating budgets is available to cover events. In anticipation of not being able to fully staff each event the staffing goal would be to provide as much staff as possible to each event while ensuring the entire available budget is spent by the end of the year.

Additional assumptions:1) events are accounted for as fixed durations in hours, 2, 4, 6, or 8-hour events. 2) there is historical data or statistical data available for the number of events and event durations, estimated as 767, 246, 91, and 13 events, respectively, for the cited durations. 3) Hourly overtime rate of $56.82. 4) Annual budget for events of $428,000. 5) Number of available fire-fighters for events is 60 [2][4].

The model is structured for the relevant data to be input to cover the entire department or a single station. Provision is also made that the model can be updated as the year progresses and provides forecasting for the remainder of the year based on current budgetary expenditures.

IV. AMPL and Sensitivity Analysis

Upon solving the model with the AMPL application, the optimal solutions are shown in Table 2:

Table 2: Initial Optimal Solutions

Decision Variable	*Shift Duration (hrs)*	*Number of Fire-Fighters*
x_1	2	1
x_2	4	2
x_3	6	3
x_4	8	18

An initial observation of the results of this model shows that the optimal solution for two hour events is an allocation of one fire-fighter. Although different Emergency Medical Services models exist, an accepted team model of a two rescuer minimum has been advocated for both basic life support and advanced life support [5]. Accounting for this introduces additional main constraints where the number of fire-fighters being staffed at each event duration should be greater than or equal to two:

$$x_j \geq 2 \ \forall j \ (17)$$

The addition of these constraints yield the results in Table 3:

Table 3.1: Optimal Solutions and Sensitivity Analysis

Decision Variable	*Shift (hrs)*	*Number of Fire-Fighters*			
x_1	2	2			
x_2	4	2			
x_3	6	2			
x_4	8	12			
Constraint (16)					
Dual Variable	*Slack*	*Lower*	*Current*	*Upper*	
0	\$8,895.68	\$419,104	\$428,000	+infinity	
Constraints (13)					
j	*Dual Variable*	*Slack*	*Lower*	*Current*	*Upper*
1	0	58	2	60	+infinity
2	0	58	2	60	+infinity
3	0	58	2	60	+infinity
4	0	48	12	60	+infinity
Constraints (17)					
j	*Dual Variable*	*Slack*	*Lower*	*Current*	*Upper*
1	0	58	-infinity	2	2
2	0	58	-infinity	2	2
3	0	58	-infinity	2	2
4	10	48	-infinity	2	12

Because the maximin model objective function (10) is the single variable *f*, sensitivity analysis will not reveal insights on the costs associated with each decision variable. Analysis of the decision variables is unique in that what can be displayed from the AMPL application does not reveal true effects associated with the cost of decision variables. The AMPL application assumes all costs of decision variables to be zero as their respective coefficients are technically zero due to them not being present in the objective function (10).

Sensitivity analysis for main constraint (13) shows that the number of fire-fighters available is not an active constraint. All dual variable values equal \$0/per change in the number of fire-fighters available, indicating that any change in the number of fire-fighters will yield no change in our objective function value for all other model attributes held constant. The slackness indicates that we do not approach the maximum amount of fire-fighters available likely due to the associated cost constraints being more limiting. Practically we see that there is little chance of being short-handed for events.

Main constraints (17) all correspond to a dual variable value of \$0/per change in the minimum number of fire-fighters scheduled. Additionally, there is no slack for 2, 4, and 6-hour events, and their current value of "2" shows that these constraints have met the upper bound they can take before they would theoretically change the optimal value. At first glance, it may be perplexing why the dual variable values are zero. Recall that variable costs with respect to the objective function (10) are technically zero. The right-hand side of these constraints can take the upper bound with a dual variable value of zero, yet this bound still represents the value that, if exceeded, will change the optimal value. We note that increasing the right-hand side more than "2" for these constraints will make these constraints more restrictive, thus tightening and changing the outcome of the model.

The model shows that there is slack within constraint (16) with regard to budget limitations. The value being considered is the right-hand side (budget), and sensitivity analysis shows that its current value is \$428,000 with a dual variable value of \$0/per change in the amount of the current budget. The range indicates that as long as the budget is between \$419,104 and positive infinity, the results will be the same for the current dual variable value, meaning that between this range, the optimal solution will remain optimal as it will not change the objective function value. Considering this, a decision could be made to lower the budget from \$428,000 to \$419,104 so that the remaining \$8,896 (the slack the value indicated) could be used elsewhere.

Identifying the slackness indicated in constraint (16) could lead to alternative decisions as well. The model attempts to use as much of the allocated budget as possible, yet a considerable amount is left over (~\$8,896). It is possible to increase the value of certain decision variables depending on their cost. As noted previously, this cost cannot be derived from the objective function (10), yet it can be obtained on an annual basis arithmetically as referenced in Table 4:

Table 4: Annual Cost per Fire-Fighter				
Decision Variable	***Hourly Rate***	***Shift (hrs)***	***Number of Events***	***Variable Cost***
x_1	$56.82	2	767	$87,161.88
x_2	$56.82	4	246	$55,910.88
x_3	$56.82	6	91	$31,023.72
x_4	$56.82	8	13	$5,909.28

Considering these costs, we see that we would be able to add one more fire-fighter at a cost of $5,909.28 annually for 13 events (each 8-hours duration). This cost is within our given slack, so it will not violate constraint (16). Adding a fire-fighter to any of the other events with different shift durations will result in exceeding the budget.

The unique relationship between the objective function (10) and constraints (15) places limitations on both AMPL sensitivity analysis and qualitative analysis, mainly because the right-hand side of constraints (15) is *f*, which is technically a variable rather than a parameter.

To evaluate the results completely, we must look at the interplay between the objective function (10) and constraints (15) simultaneously. The overall objective is to maximize *f*, yet *f* is constrained by always being less than the annual cost associated with the four categories of interest (Table 4). This means that the greatest value that *f* can take is limited by the category with the lowest overall cost. Unfortunately, in an effort to determine *f*, with respect to variables x_j, the model favors an equitable allocation of the annual cost for each category. With the annual cost of each shift category being somewhat equal, we see predictable model behavior. If the annual cost per fire-fighter is relatively low, the x_j variable for the number of fire-fighters will be increased to meet the annual cost being allocated. Alternatively, if the annual cost per fire-fighter is high, the model will assign fewer fire-fighters to that category.

This behavior is confirmed in both iterations of the model. Table 2 and Table 3 show that significantly greater numbers of fire-fighters are assigned to events with a duration of 8 hours. Table 4 shows that the cost of each variable is dominated by the number of events.

In an effort to distribute the number of fire-fighters more evenly across different shift durations, constraints can be added to limit the differences in fire-fighter allocation between different shifts. Specifically, ensuring that there is never more than a six fire-fighter difference between 6 or 8-hour shifts and 2 or 4-hour shifts:

$$-x_1 + x_4 \leq 6 \; (18)$$

$$-x_1 + x_3 \leq 6 \; (19)$$

$$-x_2 + x_4 \leq 6 \; (20)$$

$$-x_2 + x_3 \leq 6 \; (21)$$

Adding constraints (18-21) produces results in Table 3.2:

Table 3.2: Revised Optimal Solutions and Sensitivity Analysis					
Decision Variable	***Shift (hrs)***	***Number of Fire-Fighters***			
x_1	2	2			
x_2	4	2			
x_3	6	2			
x_4	8	8			
Constraint (16)					
Dual Variable	***Slack***	***Lower***	***Current***	***Upper***	
0	$32,532.80	$395,467	$428,000	+infinity	
Constraints (13)					
j	***Dual Variable***	***Slack***	***Lower***	***Current***	***Upper***
1	0	58	2	60	+infinity
2	0	58	2	60	+infinity
3	0	58	2	60	+infinity
4	0	52	8	60	+infinity
Constraints (17)					
j	***Dual Variable***	***Slack***	***Lower***	***Current***	***Upper***
1	0	58	-infinity	2	2
2	0	58	-infinity	2	2
3	0	58	-infinity	2	2
4	6	48	-infinity	2	8

Results show that the optimal solution remains except for x_4, the number of fire-fighters scheduled for 8-hour events, which dropped from “12” to “8.” Given that constraint (16) did not change during this model revision, it is clear to tell that in order to satisfy constraints (18-21), a significant portion of the budget is now unused. This is confirmed in that the slackness for the right-hand side of constraint (16) is $32,532.80, indicating we could reduce our budget by this value and maintain the optimal value achieved. With this slackness, we could now make a choice to add one more fire-fighter to 6-hour events since the cost of doing so would be $31,023.72 (Table 4) without violating current constraints.

An additional consideration is that both publicly sanctioned and commercial events may provide compensation towards the costs of firefighter staffing of the event [2]. In this case, the rate in our model would be adjusted according to the payment differential. In the case that an event pays $20/hr, the rate in our model would be adjusted to $36.82. To illustrate how this may affect optimal solutions, we will assume that approximately half of the events will not offer any compensation and half will pay $20/hr. Different hourly rates mean that the cost of decision variables will now be different as well and are shown in Table 4.1.

Table 4.1: Annual Cost per Fire-Fighter (with additional rates)				
Decision Variable	***Hourly Rate***	***Shift (hrs)***	***Number of Events***	***Variable Cost***
x_1	$56.82	2	384	$42,637.76
x_2	$56.82	4	123	$27,955.44
x_3	$56.82	6	46	$15,682.32
x_4	$56.82	8	7	$3,181.92
x_5	$36.82	2	383	$28,204.12
x_6	$36.82	4	123	$18,115.44
x_7	$36.82	6	45	$9,941.40
x_8	$36.82	8	6	$1,767.36

Although there continues to be quite a difference in decision variable costs for the original variables, constraints (18-21) ensure that the difference between 8 or 6-hour events and 2 or 4-hour events will not exceed six fire-fighters. Seeing the decision variable cost for the new four variables, we can predict that the model will disproportionately allocate fire-fighters to x_8. To prevent this, main constraints (22-25) are introduced to ensure that for each duration, the difference allocated between paying events and nonpaying events does not exceed two fire-fighters:

$$-x_1 + x_5 \leq 2 \ (22)$$

$$-x_2 + x_6 \leq 2 \ (23)$$

$$-x_3 + x_7 \leq 2 \ (24)$$

$$-x_4 + x_8 \leq 2 \ (25)$$

Table 3.3 shows the final optimal solutions and sensitivity analysis results. No significant insights were derived from constraints (13) and (17). We see that there is not an excessive allocation to decision variable x_8, despite its cost being significantly lower. Additionally, we continue to see slackness in constraint (16), this time being $21,255.60. With this slackness, we see increased freedom in decision making, as decision variables x_3, x_4, x_6, x_7, x_8 could all be increased (some by more than one fire-fighter) without violating constraint (16). Again, an alternative decision could be to reduce the budget by $21,255.60.

Table 3.3: Final Optimal Solutions and Sensitivity Analysis		
Decision Variable	***Shift (hrs)***	***Number of Fire-Fighters***
x_1	2	3
x_2	4	3
x_3	6	2
x_4	8	9
x_5	2	2
x_6	4	2
x_7	6	2
x_8	8	11

Table 3.3: Final Optimal Solutions and Sensitivity Analysis				
Constraint (16)				
Dual Variable	***Slack***	***Lower***	***Current***	***Upper***
0	$21,255.60	$406,744	$428,000	+infinity

V. Project Contributions

Project Contributions		
Task Description	***J. Wilson***	***Z. Douglas***
Initial Paper Search and Review	24	12
Initial Model Build	0.5	1
Phone Interview	-	0.5
Data Review and Parameter Estimation	6	1
Draft Write-Up	12	12
Model Refinement	30	30
Final Write-Up	12	16
Total Hours	84.5	72.5
Total E-mail Replies		135
Total Phone Meetings		2

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