

# RENSSELAER MECHATRONICS

## DC Motor Parameter Identification

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### Part 1: Motor Steady state response

#### Objectives:

- Perform different experiments on a DC motor to determine its motor parameters

#### Background Information:

A DC motor can be modeled with an electrical system coupled to a mechanical system by a magnetic field. The equation for the electrical system is:

$$V = L \frac{di}{dt} + Ri + K_b \omega$$

Where:

- $K_b$  – back emf constant
- $R$  – armature resistance
- $L$  – inductance
- $i$  – the current through the motor windings

The coupling is seen from the voltage generated by the spinning motor  $K_b \omega$ . The equation for the electrical system is

$$K_t i = J \frac{d\omega}{dt} + b\omega$$

- $K_t$  – torque constant
- $b$  – viscous damping coefficient
- $J$  – armature Inductance (or combined motor and load Inertia if load is attached)
- $\omega$  – the velocity of the motor shaft

In the steady-state (no change with time) the equations become:

$$V_{ss} = Ri_{ss} + K_b \omega_{ss}$$

$$K_t i_{ss} = b \omega_{ss}$$

These equations, with the steady state current and voltage measurements can be used to determine the motor parameters.

## Parameter Estimation Background

Resistance: If there is access to the motor leads this can be measured with a multimeter.

Torque and Back EMF constants:

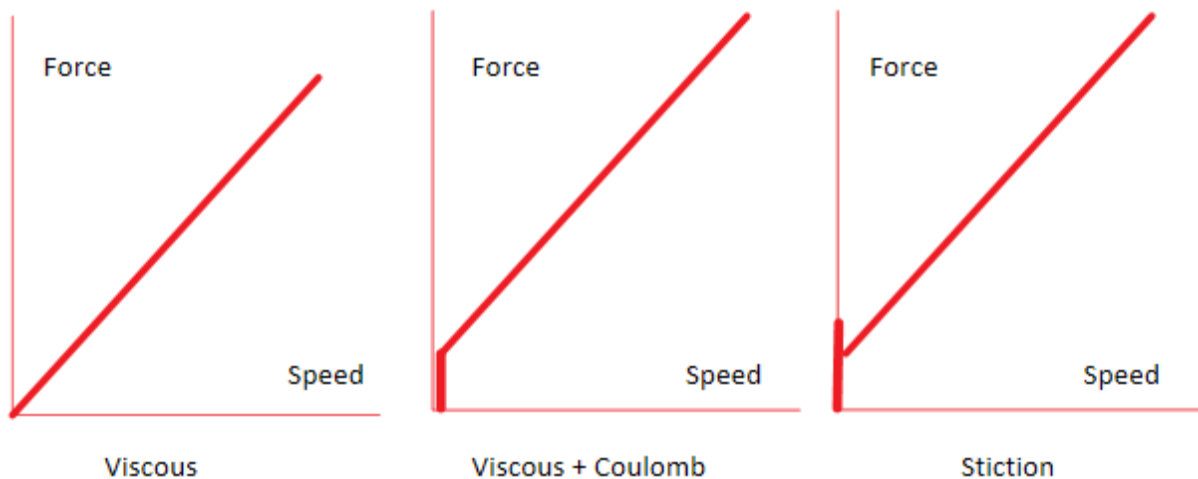
First assume that energy is conserved in the motor in which case  $K_b = K_t$  and only one needs to be estimated. From the steady-state electrical equation:

$$V_{ss} = Ri_{ss} + K_b \omega_{ss}$$

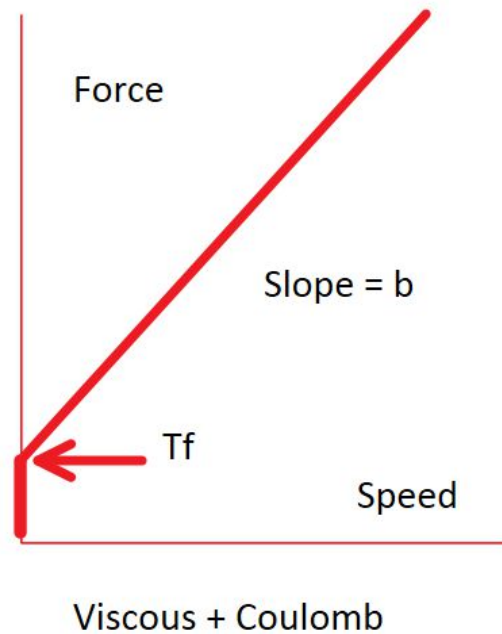
If resistance is known, speed is measured and current and voltage are measured these equations can be used to calculate  $K_b$  which, by assumption equal to  $K_t$ . For the Lego NXT motors, since they contain a gear train and have significant losses assume  $K_t$  is about 65% of  $K_b$ . This value can be obtained by an experiment where a known load is applied, and the current is measured.

Viscous and Coulomb Friction Estimation:

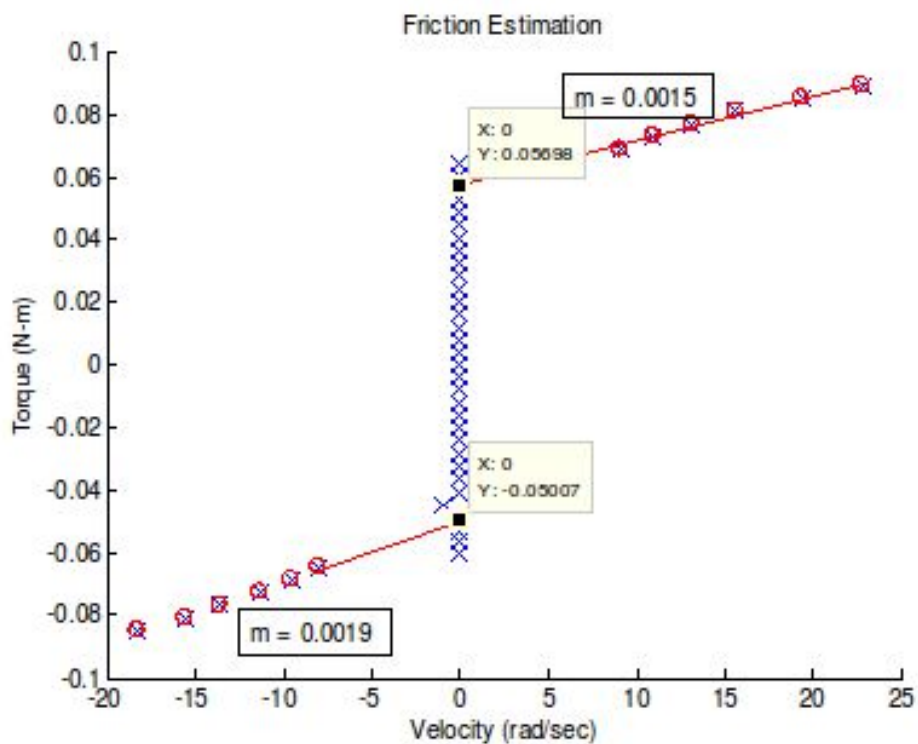
There are typically 3 different components of friction: viscous, coulomb, and static. This leads to three different types of friction models:



For a motor curves can be obtained by plotting steady state speed and measured torque  $K_t i$ .



An experimental example is:



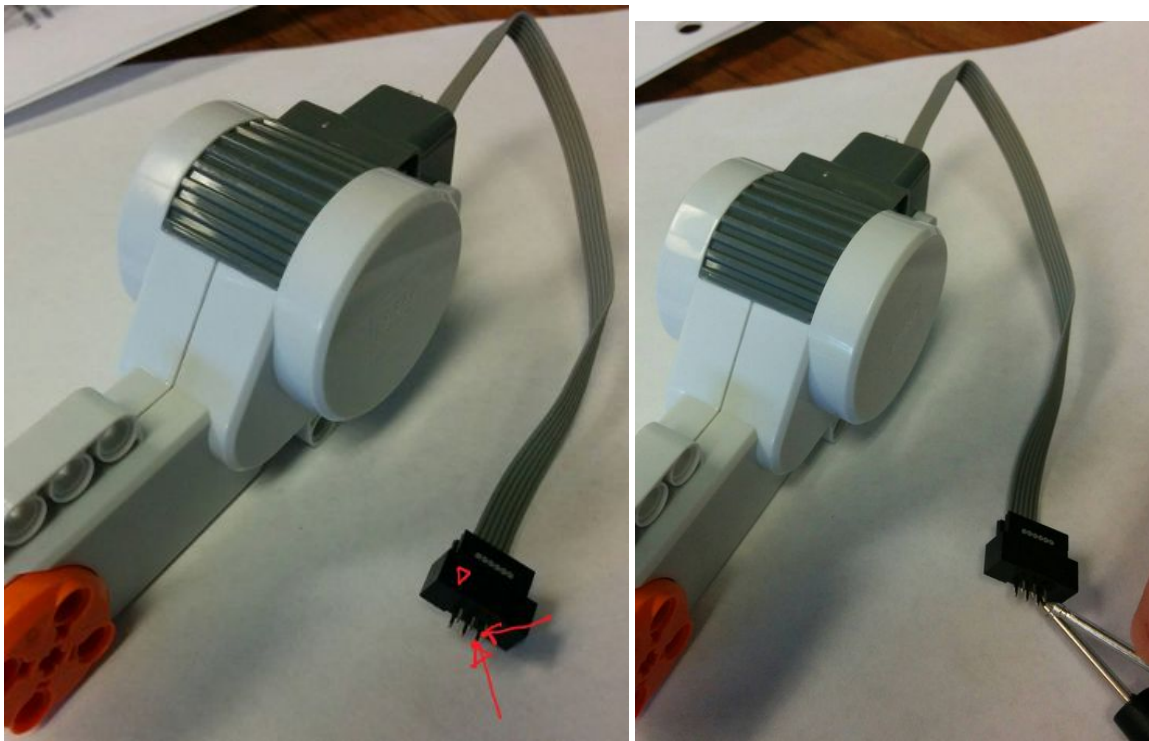
This will require steady state measurements of voltage and current at a couple different speeds. Once this graph is obtained then the viscous damping coefficient  $b$  is the slope of the line, and the constant coulomb friction is the y-intercept.

## System Measurements

### Resistance and Voltage:

Resistance and voltage can be measured by using a multimeter and measuring between the two motor leads.

If an adapter cable is available you can disconnect the motor and use the adapters to measure the resistance. For example if a header socket is provided it is the two end terminals:



- **If** no tools or cables are provided you can make the measurement while the motor is connected.  
**First make sure your board is completely powered off – it is not connected to the USB cable, and**

**there are NO batteries installed.** To do this find where you have access to the motor leads; shown below:

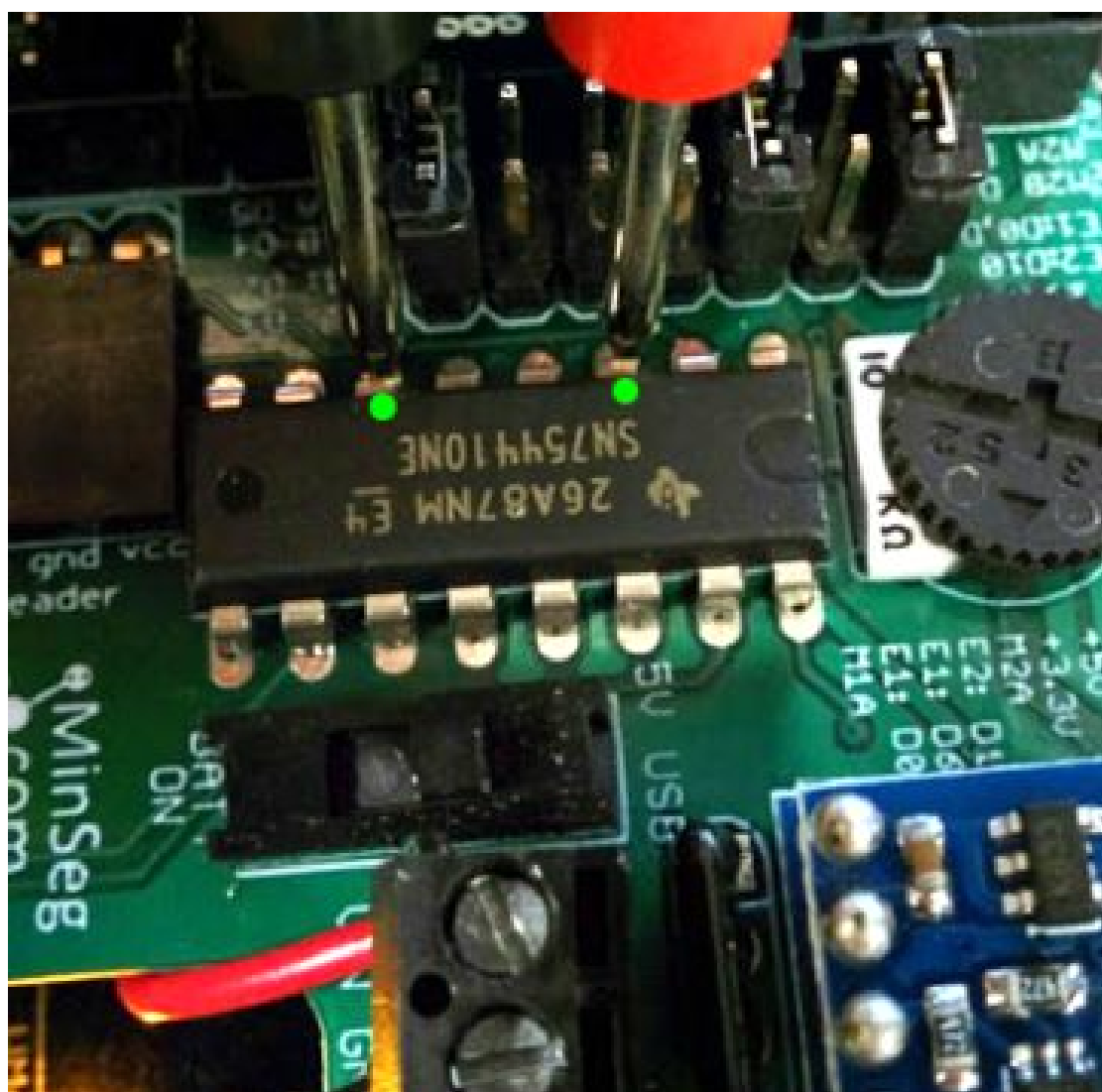




Figure 1: Motor wire connections on M1V4 board – resistance and voltage measurement from pins 3 and 6 of the motor driver.



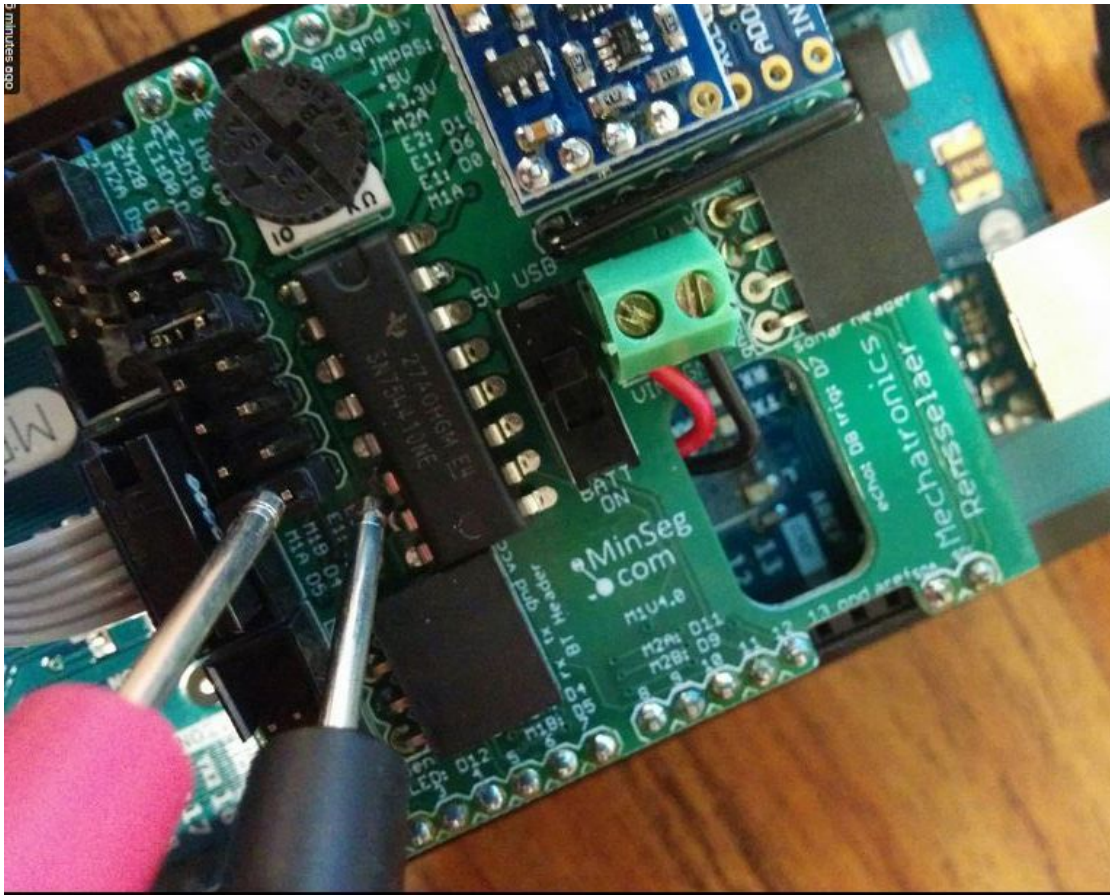


Figure 2: Alternate location for resistance or voltage/resistance measurements

You need to measure the resistance between the two motor leads; M1 and M2. You can do this from the last jumper (connected to M1 – one of the motor leads, also connected to the 3<sup>rd</sup> motor driver chip) and the 6<sup>th</sup> pin on the driver chip. Alternately you can simply measure between pin 3 and pin 6 on the driver chip:

With the powered completely off (usb disconnected) and the motor connected this will allow measurement of the resistance.

Record the value of the average motor resistance (turn the motor by hand between measurements):

R\_avg = \_\_\_\_\_

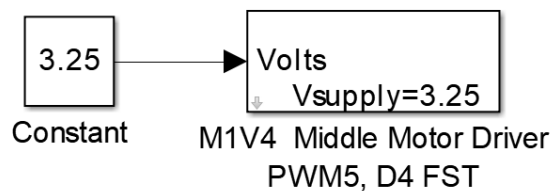


### Driver Supply Voltage (direct measurement):

The Motor driver block computes the appropriate PWM value based on the desired voltage. To do this it has to scale the actual available voltage at the output of the actual driver, to the correct PWM value.

The value of  $V_{supply}$  in this block needs to be the maximum voltage available to your device, in this case the motor. To find this value turn the driver completely ON, and measure the voltage at the output pins of the driver. This value is the maximum voltage you can obtain from the output of the driver  $V_{supply}$ .

To find this, run the following Simulink diagram:



Since the  $V_{supply}$  is 3.25 and the input is 3.25 the PWM duty cycle will be 100% (fully on). With the motor connected and freely spinning measure the voltage between the motor leads (pins 3 and 6 on the driver - Figure 5: Voltage Measurement location for M1V4).

$V_{supply}$ =\_\_\_\_\_

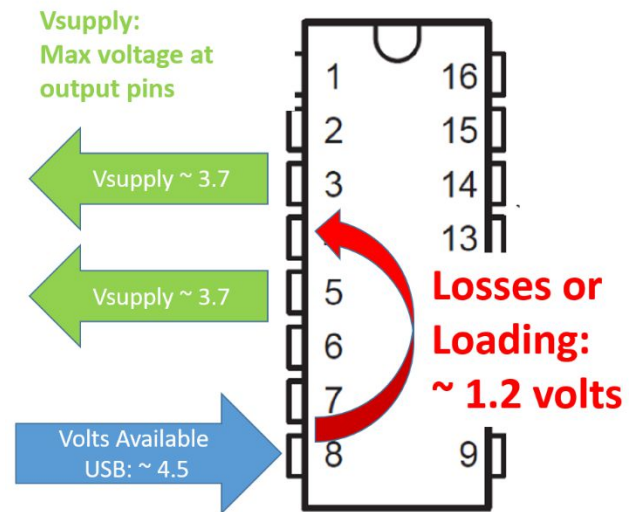
This value is the driver voltage supply available to the motor when it is fully ON. This value needs to be specified in the motor driver block so it can correctly scale the voltage to the correct PWM value. **If your specified  $V_{supply}$  in the driver block is different than the measured value, change the value in the driver block to what you measured.**

### Driver Supply Voltage (indirect approximation):

If the supply voltage was estimated directly in the previous step you do not have to do this.

The SN754410 loses approximately 1.2 volts from what it has available; the DRV8833 driver the loss is about 0.6 volts. If you know what the voltage to the driver chip you can estimate its maximum output voltage. For example if USB is used, 4.5 volts, then the max output expected for the SN754410 driver would be approximately  $4.5 - 1.2 = 3.7$  volts. You can estimate the available volts by measuring pin 8, which should be about 4.5 volts for USB.

## SN754410 Driver Losses



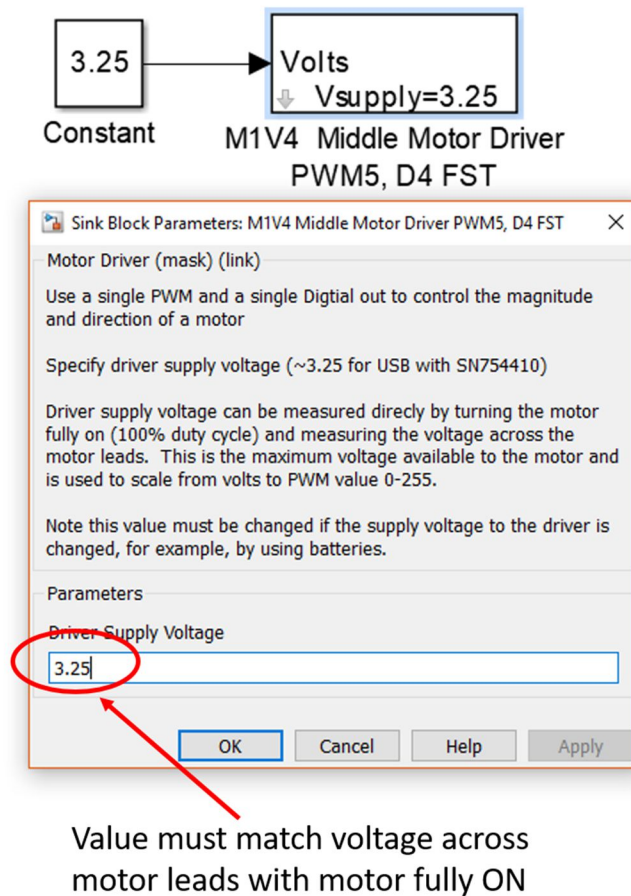


Figure 3: Specification of Driver supply voltage Vsupply for motor driver block

### Speed, Voltage and current:

The current and voltage need to be measured at different speeds. The Simulink diagram below can be used in external mode at 0.03 seconds to record the steady state velocity. Make sure "send single Port0" is commented out when using external mode. This same diagram can be used later to capture the step response in normal mode.

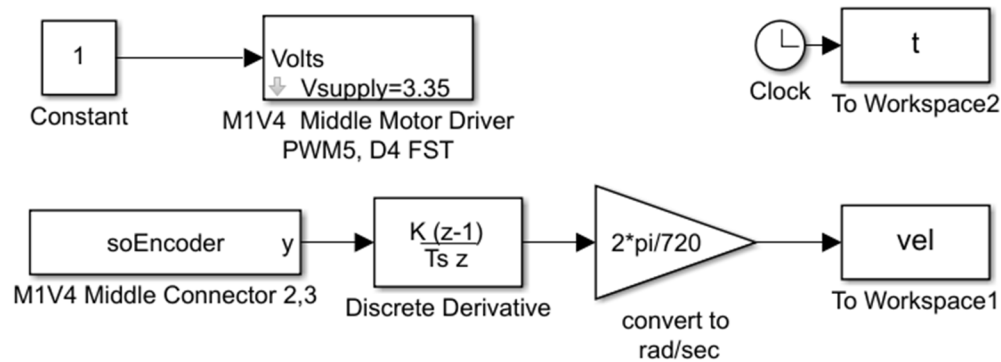


Figure 4: Simulink diagram for speed measurements

#### Voltage Measurement:

When the motor is spinning at steady state the voltage can be measured between the motor leads the same way the resistance was measured. from the last jumper that connects to pin #6 on the driver chip: **There will be current flowing so be VERY careful where you place the multimeter leads.** Alternatively, you can measure between pin #6 and #3 on the driver chip.

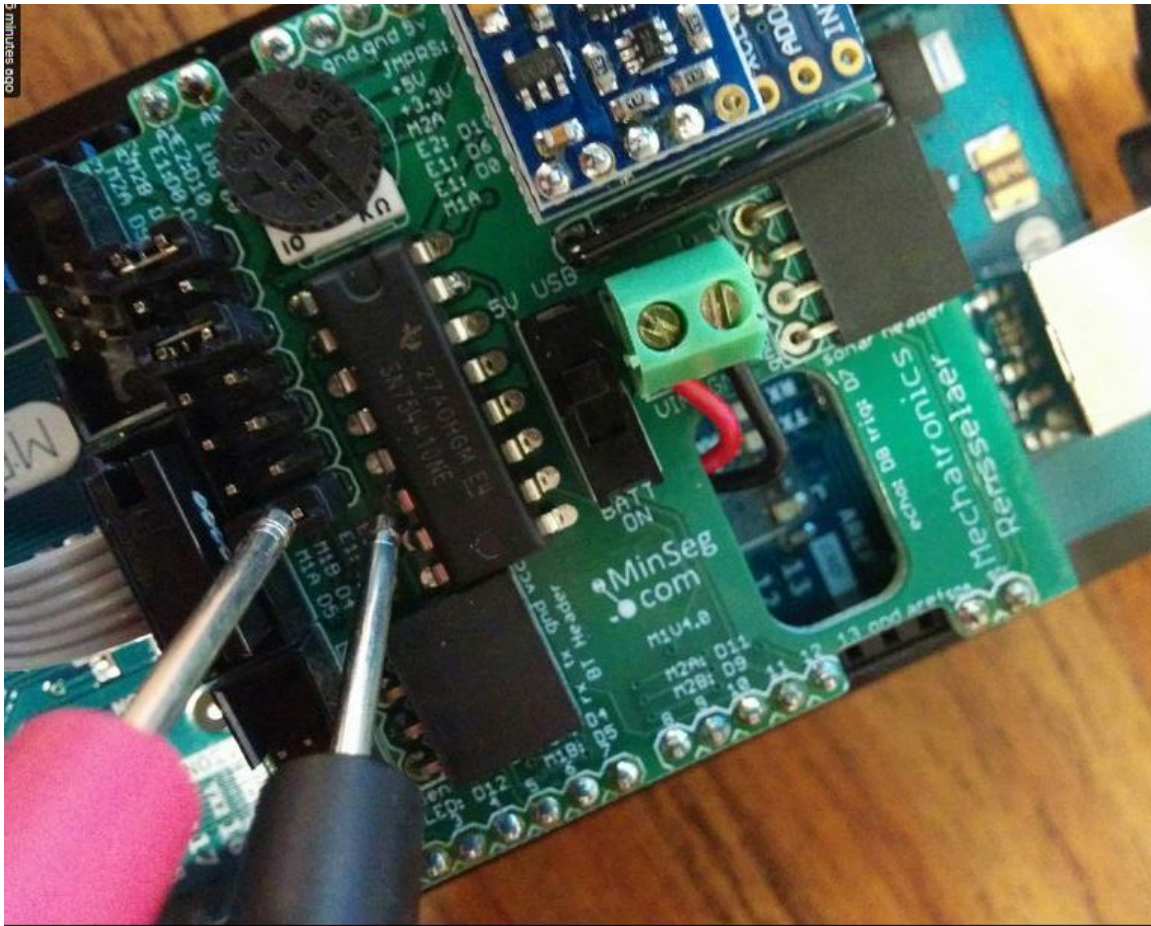


Figure 5: Voltage Measurement location for M1V4

#### Current Measurement:

- First ensure the motor is spinning at a constant velocity
- Remove the last jumper (it will stop spinning)
- Then put the multimeter in current mode and connect the two leads that were connected with the jumper to the multimeter. The motor will start to spin as the multimeter completes the circuit and you can measure the current. **There will be current flowing so be VERY careful where you place the multimeter leads so you do not short them.**

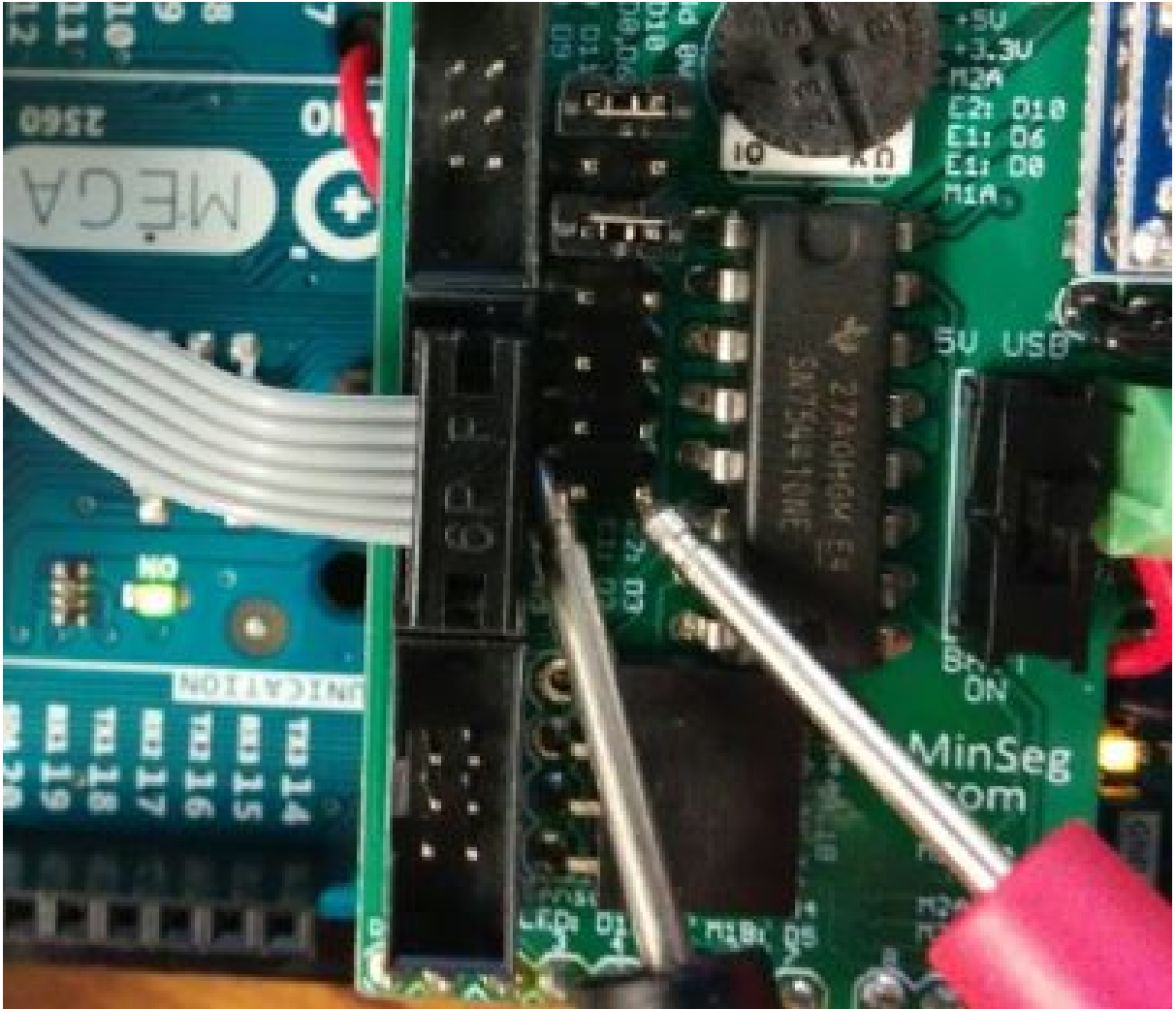


Figure 6: Current measurement with jumper removed.

#### Data Collection:

- Run the motor at least 3 different steady state speeds, and record the speed data (1.5, 2, and 3 volts would be good values to start with)
  - At the steady state speeds measure the velocity and current. This may require multiple tests. Record voltage and current at each speed.

$V_{1\text{specified}} = \underline{\hspace{2cm}}, V_{1\text{measured}} = \underline{\hspace{2cm}}, \omega_1 = \underline{\hspace{2cm}}, I_1 = \underline{\hspace{2cm}}$

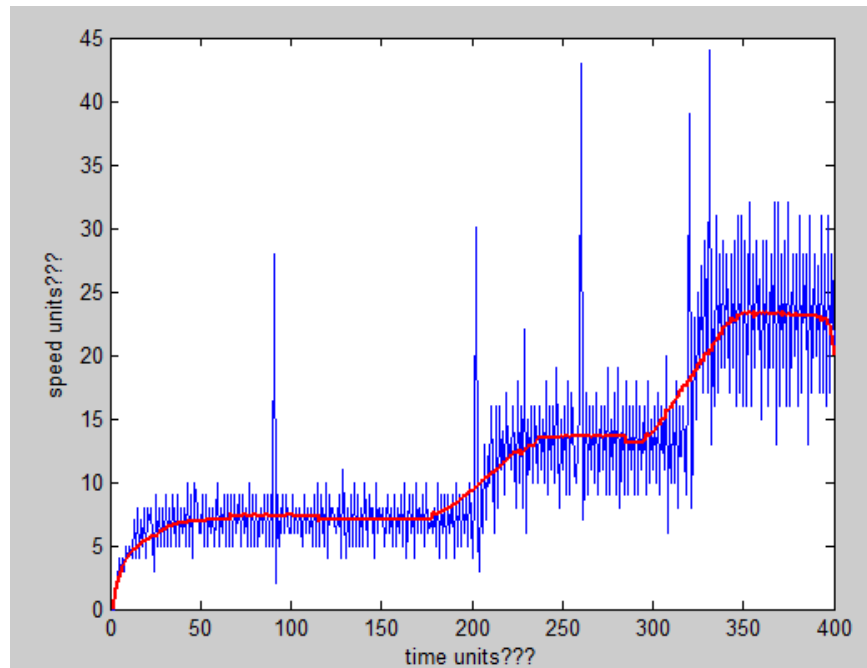
$V_{2\text{specified}} = \underline{\hspace{2cm}}, V_{2\text{measured}} = \underline{\hspace{2cm}}, \omega_2 = \underline{\hspace{2cm}}, I_2 = \underline{\hspace{2cm}}$

$V_{3\text{specified}} = \underline{\hspace{2cm}}, V_{3\text{measured}} = \underline{\hspace{2cm}}, \omega_3 = \underline{\hspace{2cm}}, I_3 = \underline{\hspace{2cm}}$

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Depending on your MATLAB version and USB port, you may need to smooth the velocity data to help determine the steady state speed if external mode is used:



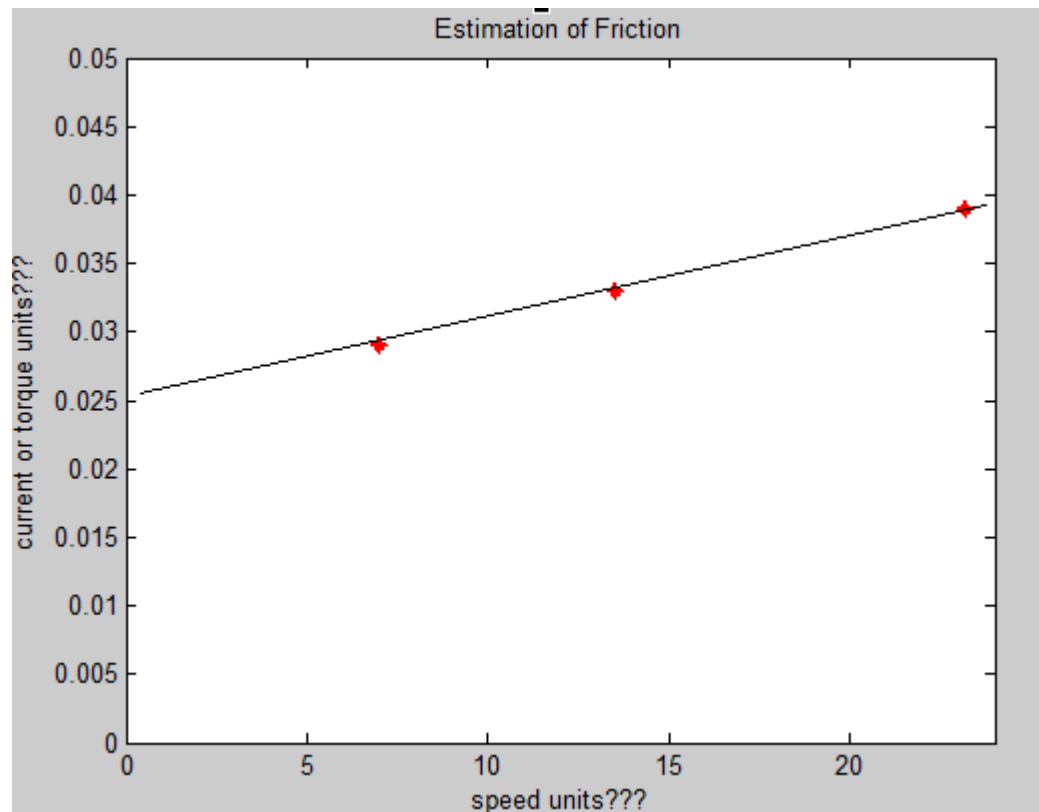
## Parameter Identification

- Use the collected data and the steady state equation to determine  $K_b$  from measured speed, current and voltage at the 3 different speeds:

- $V_{ss} = Ri_{ss} + K_b\omega_{ss}$

$K_b =$  \_\_\_\_\_ (specify units)

- Use the data to plot the speed vs. torque curve (with the correct units) to determine the coulomb friction value, and the damping coefficient. An example graph is shown below (units and data are not correct):



### Questions:

- Create a plot of current (amps, y-axis) versus speed (rad/sec. x-axis). Clearly label the axes and provide a title for the graph



- Do the measured voltages match the voltage set in the Simulink diagram when you use the multimeter to measure the voltage?
- What are your experimental values for (with units!!):
  - $R, K_b, b, T_f, K_t$ ?

## Part 2: Motor Transient Response

From the mechanical and electrical DC motor equations the 2<sup>nd</sup> order transfer function for a DC motor is:

$$G(s) = \frac{\omega(s)}{V(s)} = \frac{K_t}{LJs^2 + (JR + Lb)s + (Rb + K_tK_b)}$$

If the inductance is “small” it can be neglected ( $L=0$ ) and the first order transfer function is obtained:

$$G(s) = \frac{\omega(s)}{V(s)} = \frac{K_t}{JR s + (Rb + K_tK_b)}$$

Where the motor parameters are:

- $K_t$  – torque constant
- $K_b$  – back emf constant
- $b$  – viscous damping coefficient
- $R$  – armature resistance
- $J$  – armature inductance (or combined motor and load inertia if load is attached)
- $L$  – inductance

The equation in time constant form is:

$$G(s) = \frac{\omega(s)}{V(s)} = \frac{K_t/(Rb + K_tK_b)}{JR/(Rb + K_tK_b)s + 1} = \frac{K}{\tau s + 1}$$

- **Time Constant:**  $\tau = \frac{JR}{Rb + K_tK_b}$
- **Steady State Gain:**  $K = \frac{K_t}{Rb + K_tK_b}$

From this equation the time constant and steady state gain can be identified. Calculate  $K$  from the parameters you identified (include units):

- **Steady State Gain:**  $K = \frac{K_t}{Rb + K_tK_b} = \underline{\hspace{2cm}}$  (Units  $\underline{\hspace{2cm}}$ )

### Questions:

- What is your calculated values for the steady state gain  $K$ ?

### Parameter Estimation from step response

Obtain the step response of the motor. An example step response is shown below (you may already have this data from a previous lab). (earlier versions of MATLAB would need to obtain this data with serial mode, later versions (2020a) only use Monitor and Tune (external) mode).

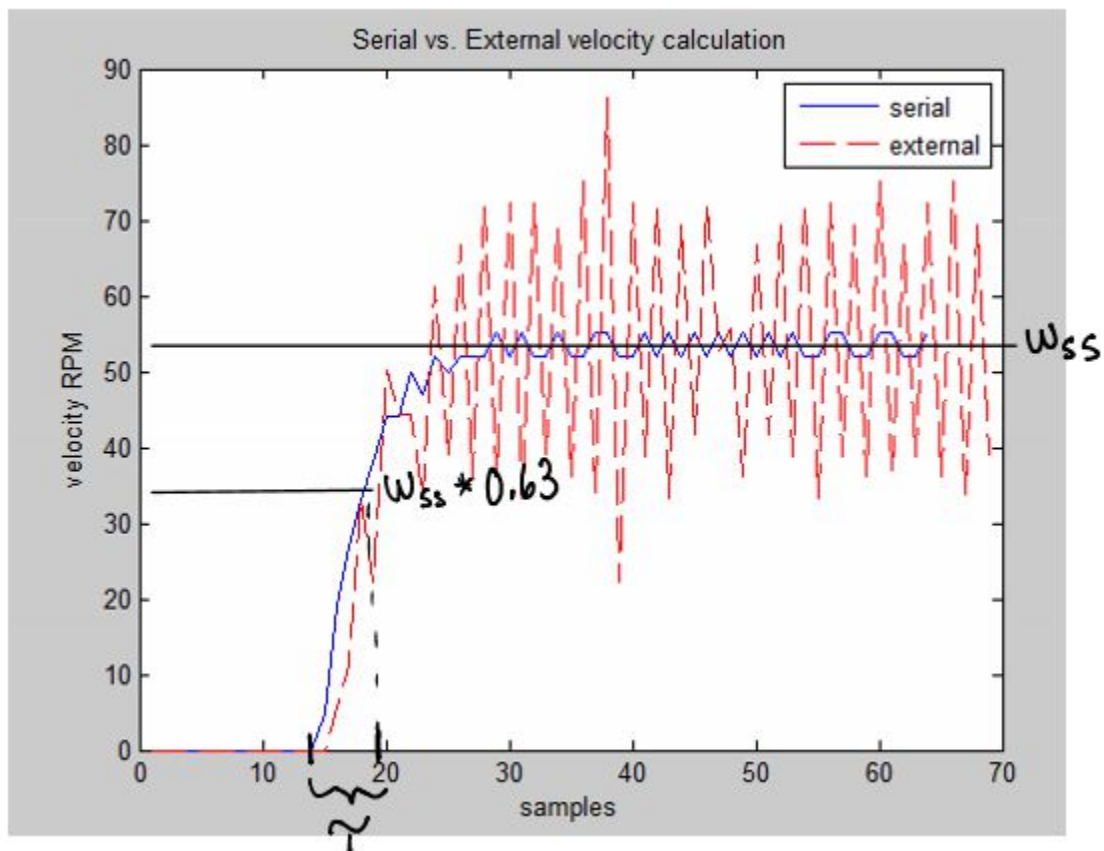


Figure 7: Sample Step response data obtained with 0.03 second sample time, 3 volt step

Take note of what voltage you used to generate this step response  $V_{in}$  (this should be the actual measured voltage):

$V_{in} =$  \_\_\_\_\_

Calculate the experimental steady state gain ( the step must be from zero speed as in the graph above):

$$K_{exp} = \omega_{ss}/V_{in} = \underline{\hspace{2cm}}$$

Where  $\omega_{ss}$  is the steady-state value from the step response graph, and  $V_{in}$  is the voltage used to generate the step response.

Next experimentally compute  $\tau$  by finding how long it takes for the step response to reach 63% of its steady state value:

$$\tau_{exp} = \underline{\hspace{2cm}}$$

The only unknown in the formula for the time constant  $\tau$  is the inertia. Use the formula for the step response to calculate the inertia J:

$$J = \underline{\hspace{2cm}} \text{ (be sure to include units)}$$

### Questions:

- What are your experimental values for the time constant and steady state gain  $\tau, K$ ?
- What is the inertia identified from the time constant?
- What is the percent error from the calculated K versus the  $K_{exp}$  value from the step response?
- $K_t$  can be determined by applying known loads and measuring the current. For this motor it is determined that  $K_t$  is approximately 65% of  $K_b$ . Why?
- If all the linear motor parameters are determined from separate experiments, the computed time constant for the resulting linear model is approximately  $\tau = .042$ . How does your value for the time constant compare to this - why?