Considerations when using Precise Orbit Determination to Study Satellite Drag and Assess Atmospheric Density Models

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1 Summary

Purpose: This document details the specifics of how the GEODYN II software (referred to simply as GEODYN) can be used to assess atmospheric density models. The density models used in this study are 3 versions of the MSIS empirical models: MSISe86, MSISe00, and MSIS2, the DTM87 model, and the Jaachia71 model. The assessment was performed using ICESat-2 Precision Clock Error (PCE) data as the tracking data type– a trajectory output from a very well determined reduced dynamics run of GEODYN.

Section Outline: Section 2 details some necessary fundamentals regarding satellite drag and the drag coefficient. Section 3 provides some highlights of how the drag acceleration is implemented and considered in the GEODYN program. Section 4 combines our understanding to assess the currently available density models using this run configuration.

Intended Audience: The intended purpose for this document is to serve as an internal document that expresses our current understanding of atmospheric drag implementation within GEODYN as well as how this implementation can be used to assess the density models. The intended audience for this document is the curators of GEODYN in Code 61 as well as the leading members of the CCMC.

Conclusions of Results This document demonstrates a proof-of-concept for using GEODYN's POD software to assess ITM atmospheric density models. Our results show that for the ICESat-2 Satellite, in its altitude regime of 450 - 480 km and during the time period from November 9 to November 24 in 2018, the MSIS2 model provides a better POD estimate than the other empirical models we analyze.

2 Satellite Drag Background

2.1 Brief Introduction to Drag

The density of the upper atmosphere is one of the primary uncertainties when performing an orbit determination in low earth orbit (LEO). The effects of satellite drag and its uncertainties are directly tied to our ability to estimate the neutral density and its variations over time. At altitudes less than 1000 km, satellites experience a significant acceleration due to atmospheric drag. The atmosphere itself varies significantly, with factors such as changing solar flux and geomagnetic activity having huge impacts on the neutral density values at a satellite altitude, as well as increasing the uncertainty in how these values will change over time.

Precise orbit determination (POD) programs such as GEODYN must model the acceleration due to drag and often employ empirical Ionosphere-Thermosphere-Mesosphere (ITM) models such as MSIS, Jaachia, and DTM to estimate the density along the orbit of the spacecraft to then calculate the contribution of drag to the accelerations acting on the satellite. While empirical models are fast and excellent for climatological research, they often lack in their abilities to accurately forecast into the future. Physics-based models are a class of models that can be used to make up for these deficiencies. Physics models offer great potential for forecasting, but lack the accuracy of semi-empirical models and are much slower (*Hsu*, 2016), (*Sutton*, 2018). This latter point has been a restrictive factor keeping physics models from being widely used for POD since the need for fast computation and interpolation along the orbit of a satellite is a major limitation. Another important capability of physics-based models (maybe even more so than forecasting) is their ability to model altitudes and/or geomagnetic conditions outside of the range of conditions for which the empirical models were derived.

Software like GEODYN II (henceforth referred to simply as GEODYN) has been optimized for fields of research like Geodesy, where the focus is not on the atmospheric density itself and clever schemes can be employed to compensate for the errors in estimating the acceleration due to drag (and the force model in general). One such scheme that is important to understand for this research is the use of once per revolution empirically estimated general accelerations. These will be discussed in Section 3.

In this document, we strive to outline and describe the methods and considerations when using precise orbit determination to assess density models.

2.2 Atmospheric Drag in General

The density in the upper atmosphere varies greatly with solar and geomagnetic activity. The interaction between atmospheric particles and the spacecraft, and the physical frameworks employed to model these interactions, is a major factor to consider when modeling drag. This means that to properly model accelerations due to drag, one must consider the following:

- Density of the total neutral particles and the total charged particles, as well as the densities of the constituents making up these totals
- Accurate knowledge of the spacecraft's position
- Accurate knowledge of the spacecraft's geometry and structure (shape, size, and mass of all components)
- Accurate understanding of the spacecraft's attitude (its orientation relative to its environment)

Taking a step back to fundamentals, we can conceptually define how these pieces fit together and develop a physically meaningful framework for modelling atmospheric drag. The total drag force on a satellite surface is given by the force due to incident atmospheric particles impacting a surface combined with the force from scattered particles departing from the surface. An equation can be written out to represent this total drag force acting on a satellite as it orbits,

$$F_D = \frac{1}{2}\,\rho\, C_D\, A_{sat}\, V_{sat}^2 \tag{1}$$

A physical drag coefficient C_D is inserted to represent the scaling of this net force relative to the density of the atmosphere and the area of the surface of the spacecraft. F_D is total drag force on the satellite surface, ρ is the density of the atmosphere, C_D is the drag coefficient, A_{sat} is the cross-sectional area of the satellite that is normal to the air flow, and V_{sat} is the velocity of the satellite relative to the atmosphere.

Note that if we divide both sides of this equation by the mass of the satellite (m_{sat}), we get the expression for satellite drag acceleration. Again, the total drag force F_D physically includes the force from the particles hitting the satellite surface plus the force from the particles leaving the satellite surface. So C_D is included in this expression as a scaling factor between this total drag force and the product of the atmospheric density, the front-facing area of the satellite and the squared relative velocity of the satellite (*Bernstein*, 2020). With this understanding, it follows that the C_D, being the term representing the satellite's interaction with the atmosphere, will be dependent on composition of the atmosphere, a fact that is not actually considered in GEODYN's current force model.

2.3 Modeling the Drag Coefficient

Drag coefficients can be physically modeled to provide information about the force exerted on the satellite surface due to the ways that atmospheric particles interact with the surface. These interactions encompass the nature of how the particles scatter after impacting the satellite. This has to do with surface chemistry and surface physics. Recall that force is equal to the change in momentum. When there is more momentum exchange in these interactions based on how the particles scatter, the drag coefficient increases (*Bernstein*, 2020). **A larger drag coefficient means more drag force acting on the satellite.**

GEODYN does not include a physical drag coefficient model, but rather estimates the drag effect through either a corrected C_D or a general empirical acceleration in the in-track direction.

Analytically modeling the drag coefficient A way to handle the drag coefficient is to model it analytically for the time period in which one is interested. There are some assumptions that one must consider before doing so:

- simple convex panel model, i.e., no shadowing or multiple reflections
- various reflection models based on limited empirical evidence
- Etc.

The method for analytically calculating the C_D is omitted here as it is slightly beyond the scope of this document. One common method is Sentmen's method for analytically calculating the C_D (*Sentman*, 1961a), (*Sentman*, 1961b).

2.4 The connection between C_D and Density

- In practice, C_D represents the scaling factor between the variations in density and the drag acceleration.
- C_D can also be physically calculated through the interaction between the atmospheric particles and the surface of the satellite as it moves through the atmosphere.
- A large C_D indicates more drag.
- In the case of GEODYN, if we allow the C_D to be adjusted to make the required accelerations match the observations, actual changes in the atmospheric neutral density are effectively incorporated as adjustments in the C_D. In addition, if the density model is found to be over-/under-estimating the density, then the C_D will be adjusted to be larger/smaller in a non-physical way.

3 Satellite Drag in GEODYN

3.1 Overview of POD

The focus here is to identify the general flow of GEODYN to better understand how the force model fits into the whole scheme.

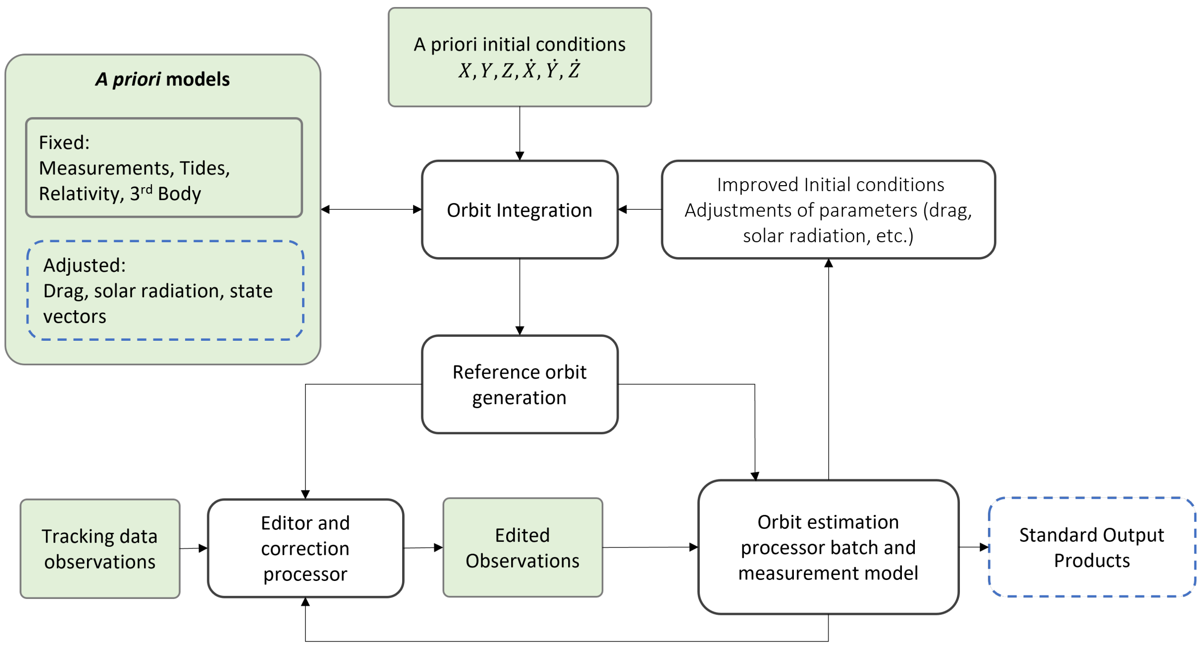

Figure 1: The above flowchart depicts the overall conceptual processes used by GEODYN to perform precise orbit determination. The intention of this figure is to show how the force model fits into the overall POD process.

- The basic problem that GEODYN addresses is to calculate, from a given set of observations of the spacecraft, a set of parameters specifying the trajectory of a spacecraft.
- The flow of GEODYN is quite complicated and also changes depending on the type of run one is doing. Above is a general depiction of a standard POD process (modified from (*Vetter*, 2007).
- GEODYN uses a batch least squares scheme that uses all observations simultaneously to estimate the parameter set. The particular method selected for GEODYN is a partitioned Bayesian least squares method.

3.2 Example Run Procedure

Let's consider a run of the ICESat2 satellite for a single day:

1. *FIRST ITERATION*
   (a) GEODYN is given some a priori initial conditions for the parameters that will be adjusted:
      i. X_0, Y_0, Z_0, $\dot{X}_0$, $\dot{Y}_0$, $\dot{Z}_0$
      ii. $C_D = 2.2$
   (b) The coordinates for this time step are passed through the Force Model to determine the accelerations and the force model partials in the variational/normal equations.
      i. A dynamic array is updated to store the updated C_D which was updated based on the atmospheric density and spacecraft total area.
   (c) The initial conditions are passed to the orbit propagator which uses Cowell's Method to numerically integrate the spacecraft orbit and the variational equations to the next time step.
   (d) At some point after the full orbit is complete, GEODYN uses a batch least squares scheme that uses all observations simultaneously to estimate the parameter set. The particular method is a partitioned Bayesian least squares method.
   (e) The corrections from the least squares procedure are used to update the parameters which are passed as inputs to the next iteration.
2. *SECOND ITERATION*
   (a) GEODYN uses the updated initial conditions from the previous iteration
      i. X, Y, Z, $\dot{X}$, $\dot{Y}$, $\dot{Z}$
      ii. $C_D =$ Updated
   (b) Repeat steps from First Iteration
3. *THIRD ITERATION - END*
   (a) Continue this procedure until the RMS of fit of the least squares reduction converges to a value less than 2%.

3.3 GEODYN's Computation of Drag Acceleration

We have outlined the procedures in GEODYN that are used to make up the dynamic force model. The subroutines we focus on here are the ones that have to do with the drag acceleration.

F.f90 The `F.f90` subroutine controls the force models through the calculation of the accelerations acting on the satellite and the subsequent computation of the variational partials.

1. Get empirically estimated accelerations (*Only if turned on*)
   - `GENAC3`, `RESON` (9 Parameter acceleration), `GENACC`,
2. Compute s/c attitude
   - `TOPATT`, `SPOTAT`, `GPSATT`, `ERSATT`, `MGSAT1`, `MOCATT`, `TDRSAT`, `GFOATT`, `TRMMAT`, `EUVEAT`, `ICEATT`, `CRYOATT`, `HY2ATT`, `SARLAT`, `GRLATT`
   - `EXTATT`
3. Compute drag acceleration
   - Call `DRAG`
4. Calculate remaining accelerations

DRAG.f90

1. Calculate acceleration due to drag
   - Obtain atmospheric density and partial derivative of density w.r.t. height
   - If the satellite is not cannonball use the variable area model `AA(KSDIND)` is the interpolated cross-section or ratio which is calculated in `EXTATT.f90`. This is plugged into `BWDRAG` along with the inputs described below
   - **`BWDRAG.f90`**– compute the total drag acceleration from each flat plate in the model and the associated partials with respect to plate area. C_D will be modified to reflect the actual projected satellite area as predicted by the variable area model (box-wing). This will affect both acceleration and partial values. The inputs the the `BWDRAG.f90` subroutine can be seen in Table 1.
2. (back in `DRAG.f90`) Calculate V-Matrix in True-of-Date time
   - Calculate Velocity part of V-matrix
   - Sum in position part of V-matrix coming from atmospheric velocity
   - Calculate position part of V-matrix coming from change in atmospheric density
     - First calculate `DHT/DX`
     - Calculate `(DACC/DDENS)*(DDENS/DHT)`
   - Finally sum in position part of v-matrix coming from change in atmospheric density
3. Get explicit partials if drag is adjusted
   - `XDRREF` is the relative velocity vector in true of reference
   - Calculate explicit partials
4. Return to `F.f90`

BWDRAG Inputs	**Description**
TDNRM1, TDNRM2, TDNRM3	Spacecraft flat plate normal vectors in true of date system (X, Y, and Z comp)
VEL	Spacecraft velocity with respect to the atmosphere
XDOT, YDOT, XDOT	Spacecraft velocity magnitude (X,Y, and Z component magnitudes)
SCAREA	Spacecraft cross sectional area as entered on the SATPAR card
CD	CD=(SAT AREA/SAT MASS)/ 2* VELOCITY*(CD+CDDOT*TIME+CDDDOT*TIME**2) $$C_D = \frac{A_s}{m_s}/\left(2vC_D + \dot{C_D}t + \ddot{C_D}t^2\right)$$
BWAREA	Flat plate areas
DXDD	True of integration acceleration due to drag
PXDDT	Partial of plate acceleration w.r.t plate area
JARADJ	Pointer to area storage location in PXDDT
IMAPAR	Mapping pointer from unadjusted to adjusted panel area
NFACE	Number of flat plates used ot models satellite

Table 1: Descriptions for the Inputs to the BWDRAG.f90 subroutine

3.4 Forces Acting on a Satellite

GEODYN models the forces acting on a satellite to a very high fidelity. These forces include: the geopotential, the luni-solar potentials, planetary potentials, solar radiation pressure, solid Earth potential, ocean tidal potential, atmospheric drag, and general accelerations. All of these forces depend on the relative satellite position with the final two forces (drag and general accelerations) being additionally dependent on relative satellite velocity (*Beall et al.*, 2015).

Advances in conservative force modeling have shifted the majority of error in POD to the non-conservative forces such as atmospheric drag, solar radiation pressure, and the effects of Earth's albedo (*McLaughlin et al.*, 2011).

3.5 Impact of Empirical Accelerations

GEODYN gets a very accurate model of the above forces and can optionally apply once per revolution (OPR) empirically-estimated accelerations to "soak up" any additional errors or poorly modeled variations in the accelerations acting on the satellite. This is done iteratively in the POD scheme by comparing the modeled forces (and the orbit estimate that is constructed using them) to the "true" data provided by various tracking data types. In-track empirical accelerations and C_D are not typically estimated together. Employing this methodology allows users of GEODYN to get very accurate orbits for a satellite, but is limited for assessing the errors in the density model. *This is because the errors in the density model are absorbed into the general accelerations along with any other uncertainties from the other forces.* The empirical accelerations *can* be looked at in the along-track (or more ideally, the in-track) direction, but these results should be cross referenced with identical runs in which the general empirical accelerations are turned off to see how much of the absorbed acceleration is due to drag.

(From Eric Sutton– "Another distinction worth noting is that, the empirical accelerations are simple sine/cosine curves, whereas an estimated drag model is based on a density model, possibly attitude, etc.")

3.6 Modeling Satellites as Boxwings vs Cannonballs

As shown in the Section 3.3, if the spacecraft in question is not a spherical shape, then GEODYN utilizes a box-wing drag procedure. This technique treats the cross sectional area of the satellite that is normal to the flow of the atmosphere as individual panels. The acceleration due to drag and the partials are calculated for each individual plate then summed. The C_D is updated to reflect the total projected area.

The cannonball method is much simpler as the cross-sectional area does not change with respect to orientation of the satellite. This means no attitude information is needed from the user and the calculation of the acceleration, the partials, and the updated C_D is from the single area value given by the user on the `SATPAR` card.

3.7 GEODYN's adjustment of C_D is complicated in

I have not found the *exact* subroutine/location where the C_D is adjusted as of yet.

For now I know that it is stored in a dynamic array which is passed through many subroutines. It is tracked and indexed as part of the `PARMVC` variable.

4 Assessing the Density Models

4.1 Updates we have made

- Added MSISe00 to the GEODYN source code
- Added MSIS2 to the GEODYN source code
- Added corrections to the DRHODZ calculation in all MSIS routines

4.2 Methods Employed to Assess the Density Models

1. **Residual summary** (RMS of fit and mean residuals)
   - The RMS of fit of these residuals represents the consistency of the orbit estimation to the observations. A decrease in the RMS of fit indicates that the forces on the spacecraft that are modeled are a better representation of the actual forces on the spacecraft, and thus a decrease in error from that density model or modeling technique.
2. **Observation residuals** of tracking data(Component-XYZ)
   - When using PCE as a tracking datatype the observation residuals represent a direct comparison of the *true orbit* (PCE) to the *estimated* orbit (POD run).
3. **Adjustment of time dependent** C_D + ratio of C_D to a priori
   - These runs allow the drag coefficient to be estimated and adjusted every 9 hours during the POD and the prediction. The comparison of adjusted drag coefficients compares the relative strength of the drag force on the spacecraft for each density model.
   - This provides a valuable insight into the performance of each density model because C_D acts as a scaling factor between the density from the density model and the estimated acceleration due to drag.
   - The ratio of the adjusted C_D to the a priori C_D gives a single value representing how much the C_D is being adjusted to account for errors in the density model relative to the observations from the PCE data.
4. **Direct comparison of model density** along determined orbit of the satellite + Model density scaled with C_D ratio scaling factor
   - We can directly compare the density outputs from the models to see how they differ.

- We can use the C_D ratio (Adjusted/a priori) as a scaling factor to multiply against the modeled density estimates. These new values for density will represent "toward truth" values for the modeled densities.

5. **In-track component residuals** (NTW system)

   - As stated above, when using PCE as a tracking datatype the observation residuals represent a direct comparison of the *true orbit* (PCE) to the *estimated orbit* (POD run). If we transform our data to the NTW satellite coordinate system, the T component (in-track) is parallel to the velocity vector and will contain any information related to changes in the spacecraft's orbit due to drag.

6. **Arc overlaps of in-track component residuals** (NTW system)

   - During overlaps, the prediction of the state will be slightly different from adjacent arcs which include the same time span. Small differences in overlaps imply precision in the state estimate. The improvement of the drag model should be most notable in the in-track component of the position and velocity vectors.

7. Empirical Accelerations

   - Since none of the force models are perfect, once per revolution (OPR) empirical accelerations can be estimated using GEODYN in the along-track and cross-track directions. These accelerations are typically made up of non-conservative forces. By estimating these accelerations, the orbit accuracy is increased significantly.
   - The magnitude of the empirical accelerations will decrease if the force models are improved so they can also be used as a test of relative force model accuracy.
   - Some of the acceleration due to drag may be absorbed by estimating these general accelerations.

4.3 Satellite Coordinate Systems: NTW and RSW

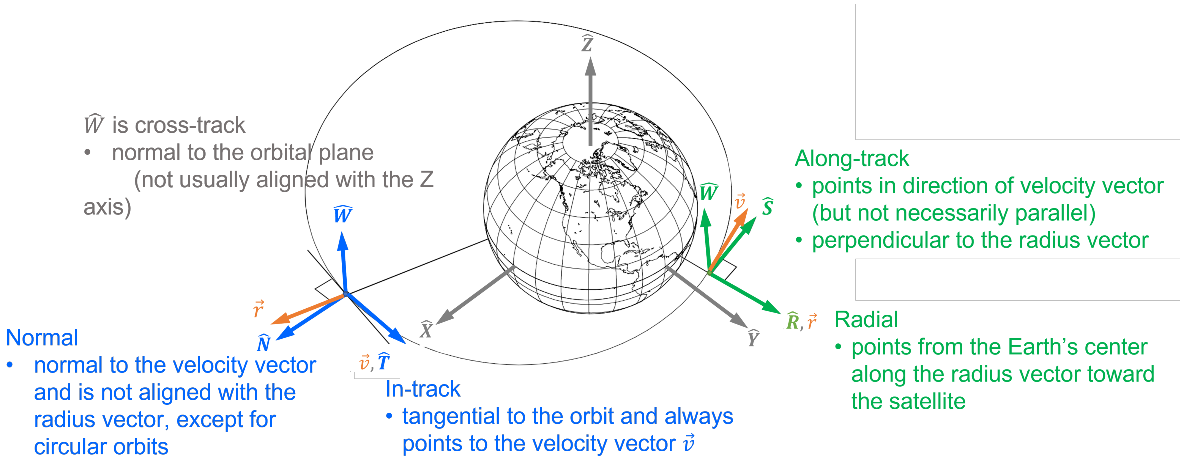

Figure 2: The above is a schematic showing the RSW and NTW satellite coordinate systems and details regarding their components. The NTW system's in-track component is parallel to the velocity vector, making it an effective tool for assessing relative effects due to atmospheric drag. This schematic is sourced and modified from (*David*, 2013).

- Both systems lie in the orbit plane
- The third axis $\hat{W}$ is normal to the orbit plane
- $\vec{r}$ is the position vector
- $\vec{v}$ is the velocity vector
- In-track variations are not the same as along-track variations in the RSW system. In-track variations are in the direction of the velocity, whereas along-track variations are simply along the velocity vector.
- We use **NTW to analyze drag effects** on the orbit because drag always affects the relative velocity vector.
- The NTW coordinate system has the following unit vectors and transformation.

$$\hat{T} = \frac{\vec{v}}{\mid v \mid} \qquad \hat{W} = \frac{\vec{r} \times \vec{v}}{\mid \vec{r} \times \vec{v} \mid} \qquad \hat{N} = \hat{T} \times \hat{W}$$

$$\vec{r}_{XYZ} = \left[\hat{N} \vdots \hat{T} \vdots \hat{W} \right] \vec{r}_{NTW}$$

- The RSW coordinate system has the following unit vectors and transformation.

$$\hat{T} = \frac{\vec{r}}{\mid r \mid} \qquad \hat{W} = \frac{\vec{r} \times \vec{v}}{\mid \vec{r} \times \vec{v} \mid} \qquad \hat{N} = \hat{W} \times \hat{R}$$

$$\vec{r}_{XYZ} = \left[\hat{R} \vdots \hat{S} \vdots \hat{W} \right] \vec{r}_{RSW}$$

4.4 Assessment Results and Discussions

These results presented in this section are for the ICESat-2 satellite tracked with PCE data for a 15 day period. Each arc is a single day arc consisting of 3 hours padding on each end and an additional day of prediction (so 54 hours total).

4.4.1 Run Specifications

Arc lengths	Single day arcs (3H padding on ends + 24H of prediction = 54H total)
Dates	2018/11/9 - 2018/11/23
Estimated Parameters	**Initial Conditions:** $X, Y, Z, \dot{X},\ \dot{Y},\ \dot{Z}$
	Time dependent drag options with constraints: • CD update every 9 hours for the whole 54 hour arc – Standard deviation of C_D: 10 • CONDRG: Constraints for time dependent drag – Weight assigned to adjacent periods: 0.50000 – Correlation time 28800.
Tracking Data type	PCE trajectory data (output from reduced dynamics run) PCE reference system: J2000 *Reduced Dynamic Run- empirical accelerations* • Along-track and cross-track 1 cycle per revolution general accelerations estimated approximately every 0.25 of an orbit revolution with nearest neighbor covariance constraint (*Luthcke et al.*, 2003) (*Thomas et al.*, 2021).
ICESat-2 Attitude	Telemetered spacecraft body-fixed reference frame to inertial reference frame quaternions Telemetered solar array drive angles (for non-conservative force modeling) (*Thomas et al.*, 2021)

Integration step size	10 seconds
Empirical accelerations (ACCEL9 card)	OFF
Using PCE tracking data	Remove reference to all GPS sats
Print trajectory to ORBFIL	• Mean of Year 2000 • Single satellite • Unit of ORBFIL: Unit 131 • Time interval between successive trajectory outputs: 60 seconds
Print residuals to IIEOUT *(No RESID file created)*	Last iteration only
Reference System	• Max number of arc iterations: 19 • Min number of arc iterations: 3 • Max # of arc iters during global iters after the first: 3 • Coordinate reference system: Mean of J2000

4.4.2 Residual summary (RMS of fit, mean residuals)

The table below shows the RMS of fit of the residuals for the PCE trajectory run of ICESat-2. We show results for the 5 density models currently available to `Pygeodyn`. These values were gathered across 15 single day arcs from November 9 to November 24 in 2018.

The table below shows the average of the RMS of fit across all 15 arcs. This result shows the MSIS 2 model as having the lowest value (and therefore best overall fit of the residuals) across all models.

Density Model	Mean Residual (cm)	Mean RMS of Fit
`MSISe86`	2.253	1.875
`MSISe00`	1.093	1.756
`MSIS2`	-1.040	1.377
`DTM87`	-2.293	3.892
`Jaachia71`	-1.213	2.517

Table 2: Residuals Summary Across All Arcs

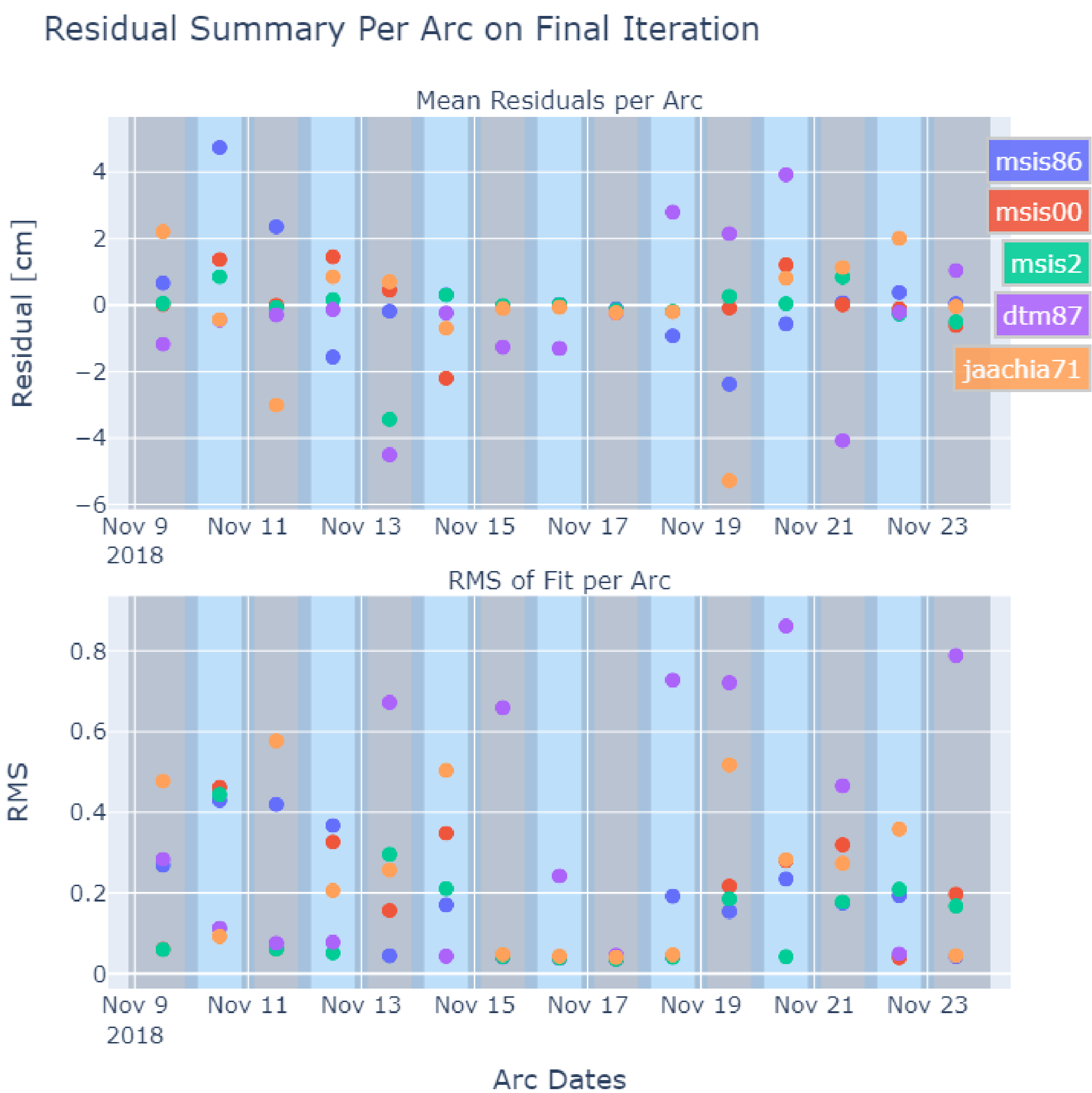

Figure 3: Residual summaries for each density model across 15 single day arcs of the ICESat-2 PCE Run. The top panel shows the mean of the residuals for the final iteration of each arc. The bottom panel shows the mean RMS of fit of the residuals for the final iteration of each arc.

Figure 3 shows the residual summaries for the final iteration across all 15 arcs. The top panel of Figure 3 shows the mean residual summary for each arc, while the bottom panel shows the mean RMS of Fit for each arc. This plot shows that MSIS2 demonstrates a lower RMS of fit across most arcs during this time period.

4.4.3 Tracking data observation residuals (Component-XYZ)

Figure 4 shows the Observation residuals in the XYZ inertial frame. This plot is harder to read since the XYZ coordinate system does not offer a component that contains information on the impact of the force model due to the drag acceleration. Even with this, we can see that some models definitely perform better than the others. DTM87 and Jaachia71 have worse overall residuals than the 3 MSIS models. For a more accurate look at these residuals, we will need to refer to the NTW coordinate system (Shown in Section 4.4.6).

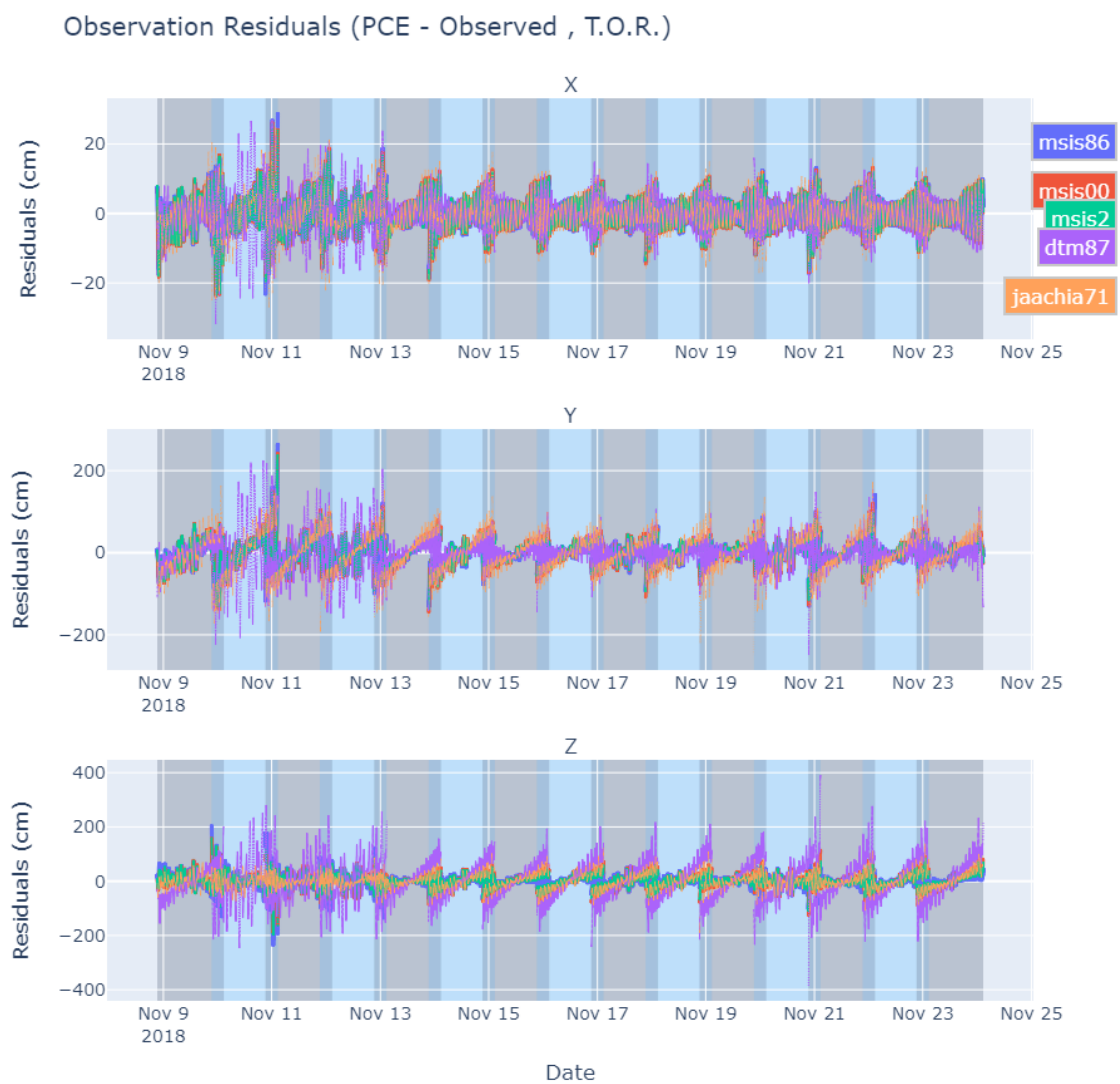

Figure 4: Observation residuals in the XYZ coordinate system for each density model across 15 single day arcs of the ICESat-2 PCE run. The top panel shows the X component, the middle panel shows the Y component, and the bottom panel shows the Z component.

4.4.4 Adjustment of time dependent drag + ratio of C_D priori

Figure 5 shows the adjusted time dependent drag coefficients. This GEODYN run is set up to adjust the drag coefficients with the DRAG card every 9 hours of the 54 hour arc. The top plot shows the output of this C_D adjustment for each density model. The a priori value for the drag coefficient of ICESat-2 was determined to be 2.2 (*Thomas et al.*, 2021). As discussed in Section 2.2, the drag coefficient in GEODYN will serve as a scaling factor between the variations in density (or the errors in the density models) and the acceleration due to drag. Therefore these C_D adjustments are compensating for errors in the density model. The further the C_D is from the original a priori guess, the more erroneous the density model. These results suggest that all of the empirical models employed here are giving much lower estimates for density than determined by the PCE data, with MSIS2 providing the most accurate values since it has the least adjustment.

The bottom panel in Figure 5 is showing the ratio of the adjusted C_D to the a priori value. These time series can be further used as scaling factors to multiply across the estimated model density time series to get “towards true” density estimates from the models. This is done in the next section (Section 4.4.5).

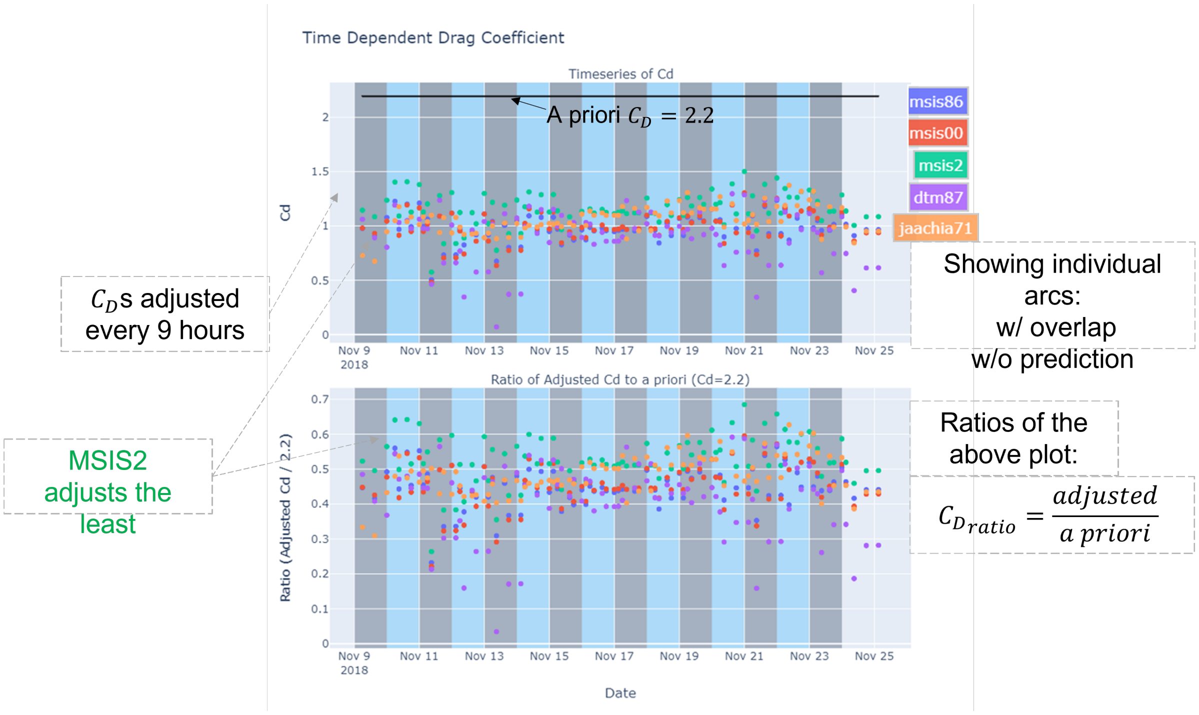

Figure 5: Depicted here is comparison across all five density models of the time dependent adjustment of the drag coefficient for the ICESat-2 PCE run of GEODYN. The C_D is set to adjust every 9 hours for the entirety of the 54 hour arc (orbit determination and prediction phase). The top panel shows the a priori estimation for C_D at 2.2 (black line) and GEODYN's adjustment of the C_D for each density model to account for errors in the estimated neutral density. In the case of all density models, the C_D is adjusted to be lower, indicating an underestimation of the density relative to what the PCE data suggests. The bottom panel shows the ratio of the adjusted C_Ds to the a priori estimate ($\frac{C_{D,\text{model}}}{C_{D,0}}$). The values that make up the scatter plots in the bottom panel can subsequently be used as scaling factors to the density.

4.4.5 Scaled model density along determined orbit of the satellite

Figure 6 shows the comparison of the model density estimates along the determined orbit of the ICESat-2 satellite (top panels) as well as the model density multiplied by the C_D ratio scaling factor from the above section. The density can be looked at in this way to see the direct comparison between the density models as well as the values of density that are determined to be "towards true" after multiplying them by the scaling factor. This ratio is considered a correction factor to the density when used in this way.

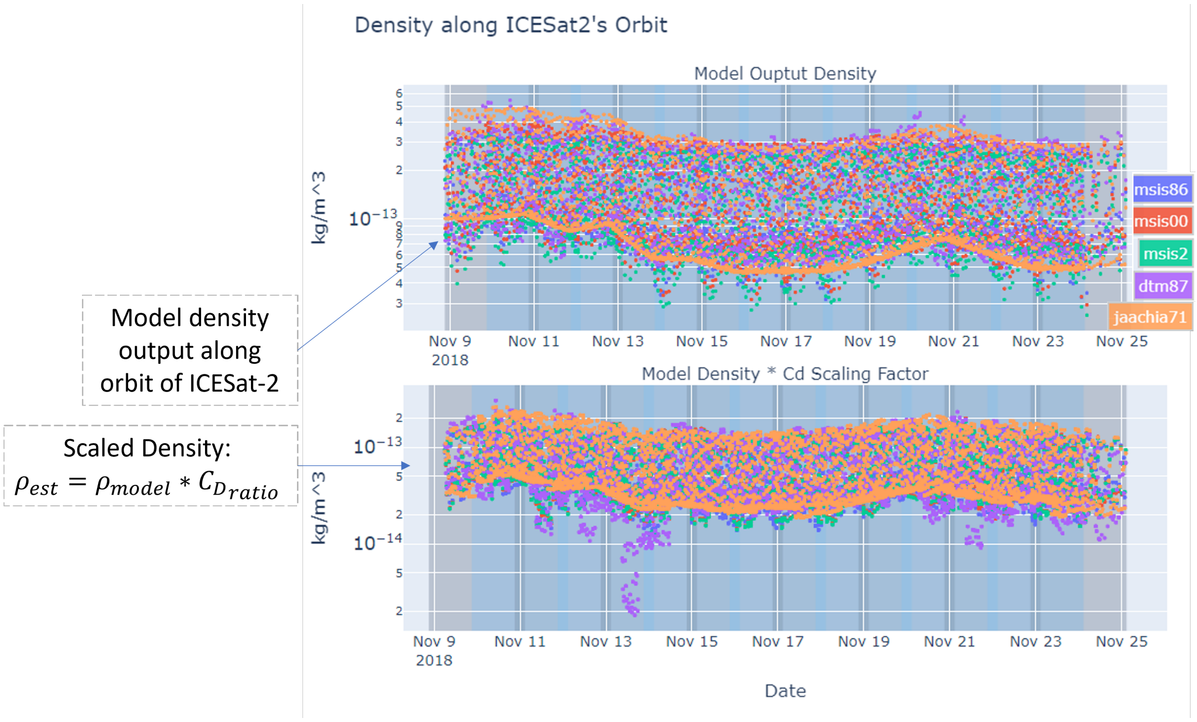

Figure 6: The top panel shows a direct comparison of the density model's estimated neutral density along the orbit of the ICESat-2 satellite. The bottom panel takes the above densities and multiplies them by the C_D ratios from Figure 5. These scaled densities represent "toward truth" values of the estimate densities as determined by the GEODYN PCE run.

4.4.6 In-track residuals (NTW system)

As discussed in Section 4.3, when doing satellite drag work it is useful to use the in-track component of the NTW coordinate system to compare the relative effects of satellite drag between different density models. These plots show a direct comparison of the in-track component residuals between the five available density models. These residuals were calculated in the post-processing analysis using the PCE data and the output from the ORBFIL (trajectory data) for each run. We convert from the XYZ inertial coordinate system to the NTW satellite coordinate system using the processes outlined in Section 4.3, then calculate the residuals for the in-track component T as,

$$T_{\text{resid}} = T_{\text{PCE}} - T_{\text{ORBFIL}}.$$

Figure 7 shows these in-track residuals in its bottom panel for the 15 arcs we selected for this report. For simplicity, we only plot every other arc, instead of all adjacent arcs. The grey section of each arc represents the time when GEODYN is performing an orbit determination, while the blue section is the time when GEODYN is performing a prediction. The top panel of Figure 7 shows the C_D ratio along the time period as a gauge for how GEODYN is reacting to the given neutral density from each model. Figure 7 shows that MSIS2 has the lowest residuals of the density models used, indicating that it experiences the most realistic drag according to the PCE data. Figure 8 is the same plot as Figure 7, but zoomed in on only the first arc and shows these results more clearly.

The final plot, Figure 9, shows the arc overlap period between two adjacent arcs. The plot is zoomed into the portion of the arc where the first arc is ending before the prediction begins and the second arc is beginning. It can be seen that the two arcs are almost perfectly out of phase, a feature that is not yet well understood and needs to be assessed further.

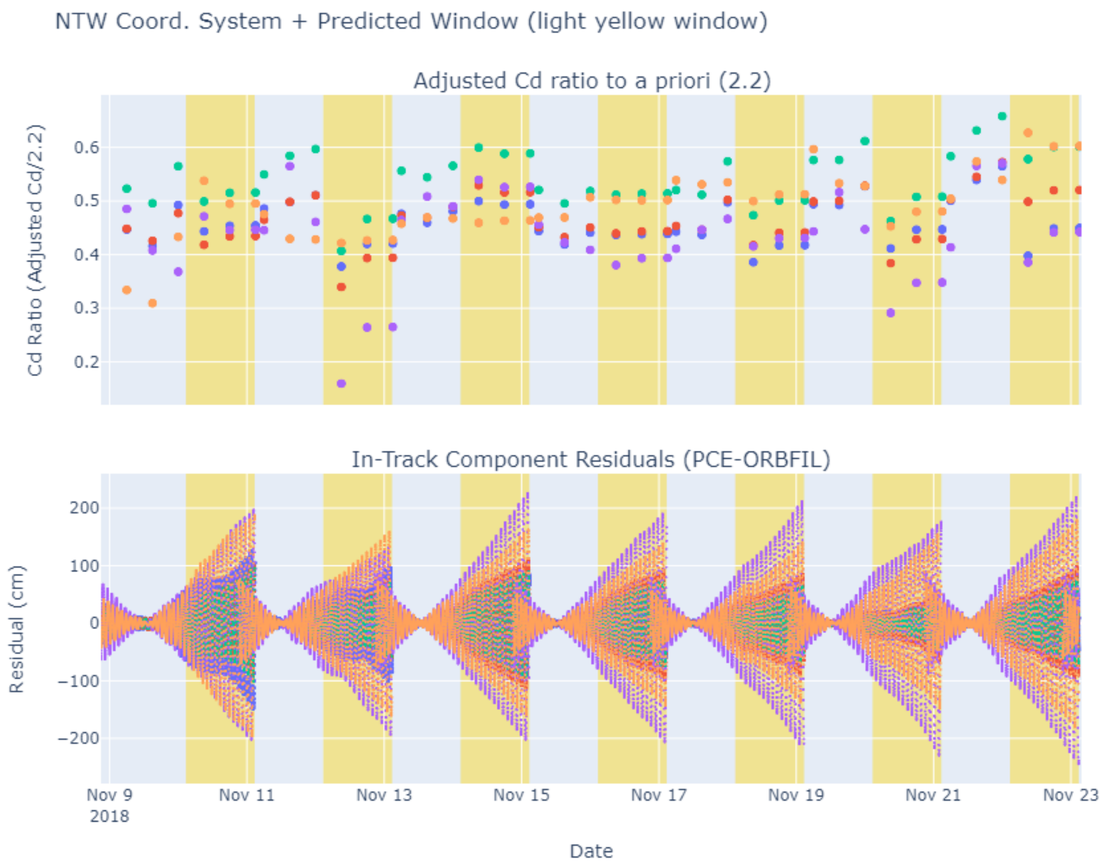

Figure 7: Comparison of the residuals of the in-track component for the ICESat2 PCE run of GEODYN for the five available density models. For visual simplicity, we only plot every other arc instead of all adjacent arcs. The grey section of each arc represents the time when GEODYN is performing an orbit determination, while the blue section is the time when GEODYN is performing a prediction. The top panel shows the C_D ratio along the time period as a gauge for how GEODYN is reacting to the given neutral density from each model.

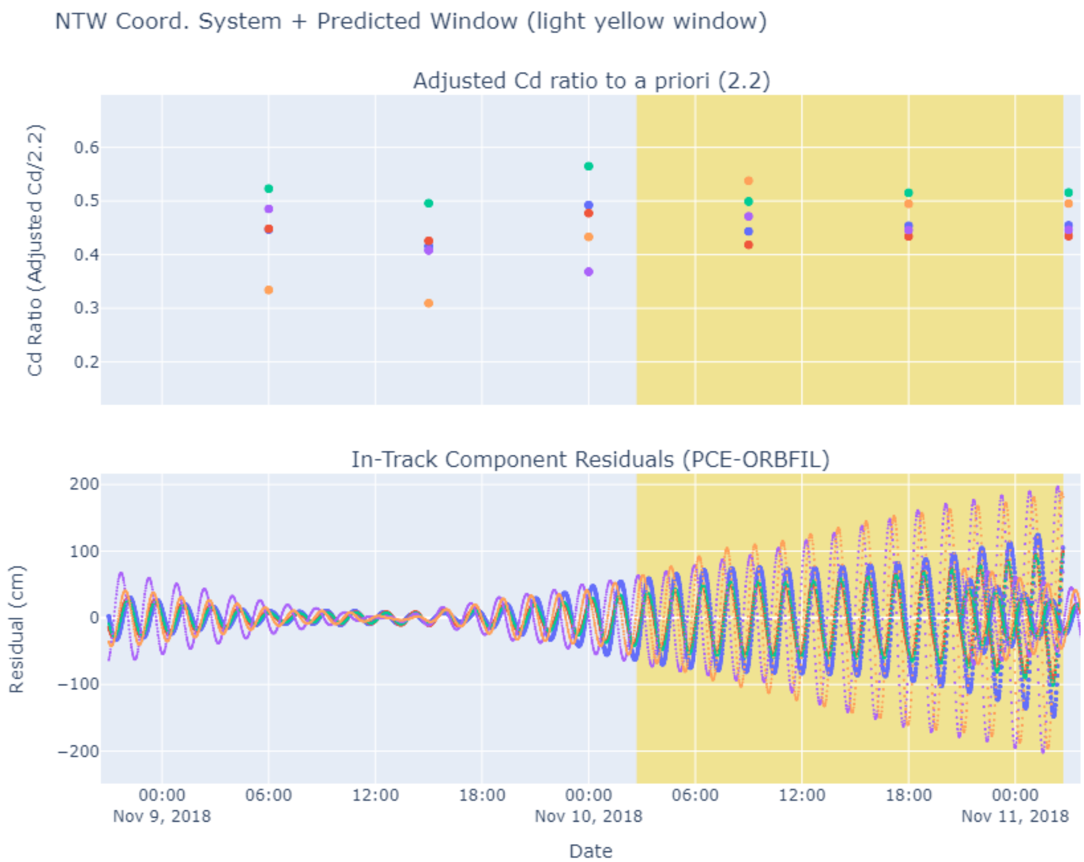

Figure 8: This is the same plot as in Figure 8, but zoomed in on one arc to better show the details of each density model's residuals.

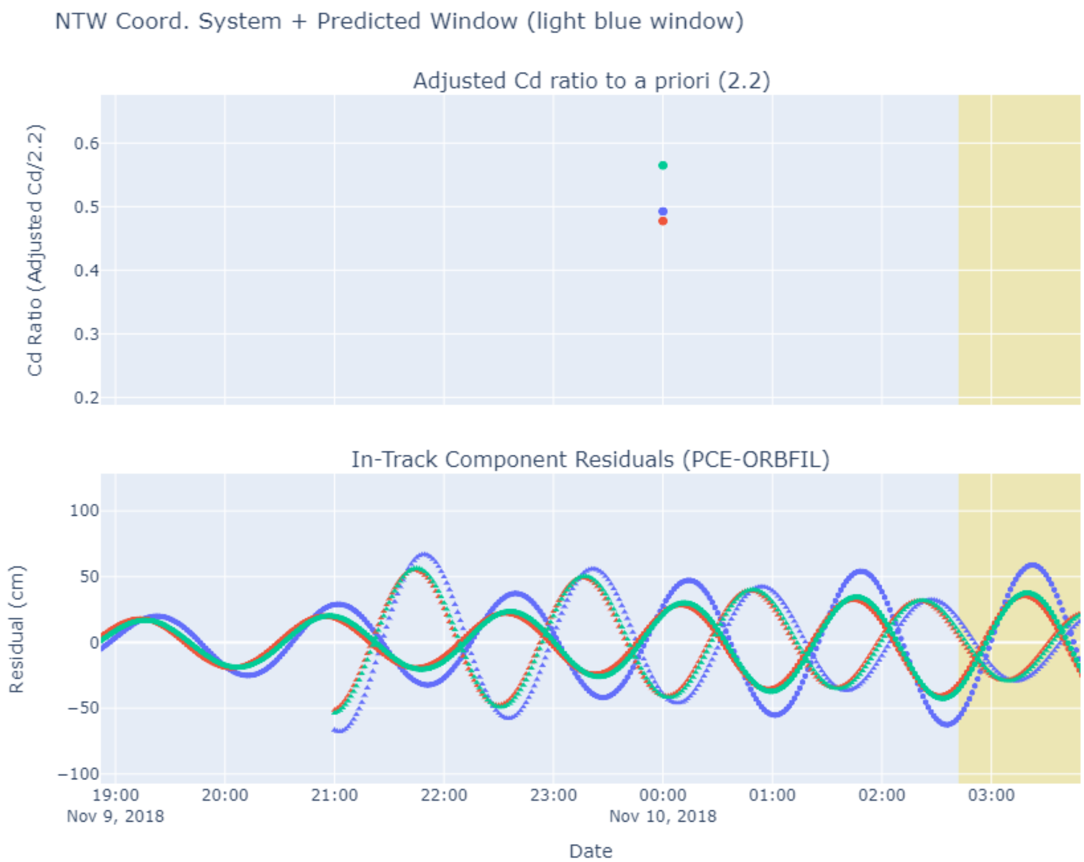

Figure 9: In-track residuals for two adjacent arcs containing significant overlap. The first arc is marked with circles while the second arc is marked with triangles. The two arc's residuals are seen to be out of phase, the reason for which is not yet understood.

5 Extra– Questions

1. So the OPR empirical accelerations are calculated in the along-track and radial direction. I see in many places that "Improvements in atmospheric drag modeling will occur predominantly in the along-track direction (direction along the satellite's orbit)" but this is not actually accurate if the orbit is non-circular. One would need to use the in-track component from the NSW system. So would one therefore need to be calculating and absorbing errors from in-track within GEODYN or is it sufficient to transform to the NSW system and look at in-track in post?
2. Is there anything wrong with increasing the orbit integration step size? Will this reduce the OD?
3. What is the difference between the variational equations and the normal equations?
4. Why does changing the time between C_D adjustments have such a big impact on the resulting adjustments?
5. For the Arc Overlaps, why are the overlaps so out of phase?
6. Nuances in the Setup of each arc:
   (a) The arc length of the run must be considered
   (b) The number of times you adjust the C_D and the cadence of such an adjustment
   (c) The step size impacts the OD and the adjusted parameters

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