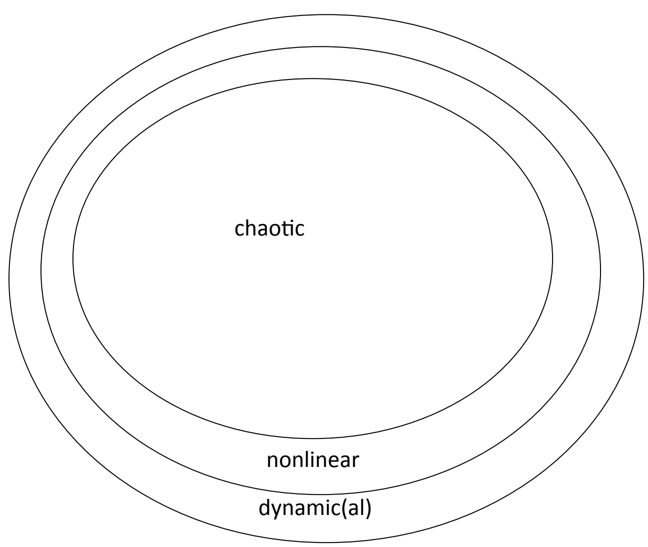
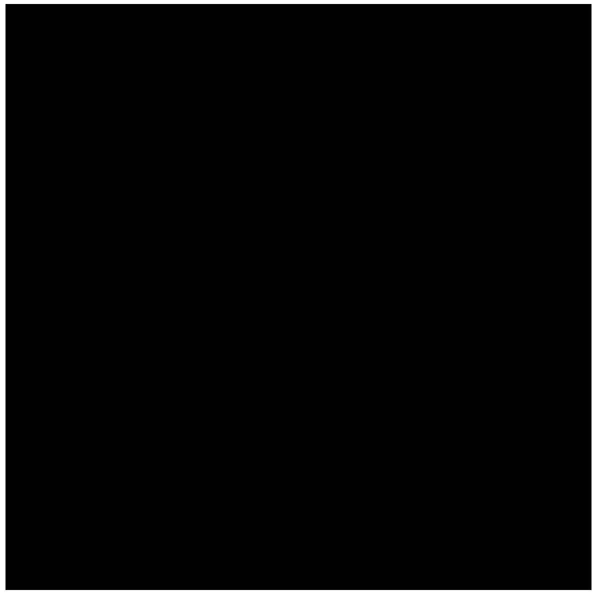
Chaos

Complex behavior, arising in a deterministic nonlinear dynamic system, which exhibits two special properties:

- sensitive dependence on initial conditions
- characteristic structure…

Systems that exhibit chaos are ubiquitous; many of them are also simple, well-known, and "well-understood"





www.nasa.gov/mission_pages/cassini/multimedia/pia06243.html

Numerical Evidence That the Motion of Pluto Is Chaotic

Gerald Jay Sussman and Jack Wisdom

The Digital Orrery has been used to perform an integration of the motion of the outer planets for 845 million years. This integration indicates that the long-term motion of the planet Pluto is chaotic. Nearby trajectories diverge exponentially with an *e*-folding time of only about 20 million years.

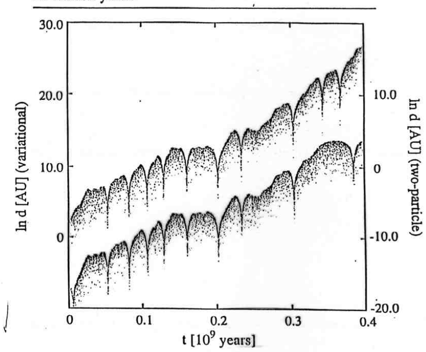


Figure 3: The exponential divergence of nearby trajectories is indicated by the average linear growth of the logarithms of the distance measures as a function of time. In the upper trace we see the growth of the variational distance around a reference trajectory. In the lower trace we see how two Plutos diverge with time. The distance saturates near 80AU when the Plutos are on opposite sides of the Sun. The variational method of studying neighboring trajectories does not have the problem of saturation. Note that the two methods are in excellent agreement until the two-trajectory method has nearly saturated.

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Deterministic Nonperiodic Flow[1]

EDWARD N. LORENZ

Massachusetts Institute of Technology

(Manuscript received 18 November 1962, in revised form 7 January 1963)

ABSTRACT

Finite systems of deterministic ordinary nonlinear differential equations may be designed to represent forced dissipative hydrodynamic flow. Solutions of these equations can be identified with trajectories in phase space. For those systems with bounded solutions, it is found that nonperiodic solutions are ordinarily unstable with respect to small modifications, so that slightly differing initial states can evolve into considerably different states. Systems with bounded solutions are shown to possess bounded numerical solutions.

A simple system representing cellular convection is solved numerically. All of the solutions are found to be unstable, and almost all of them are nonperiodic.

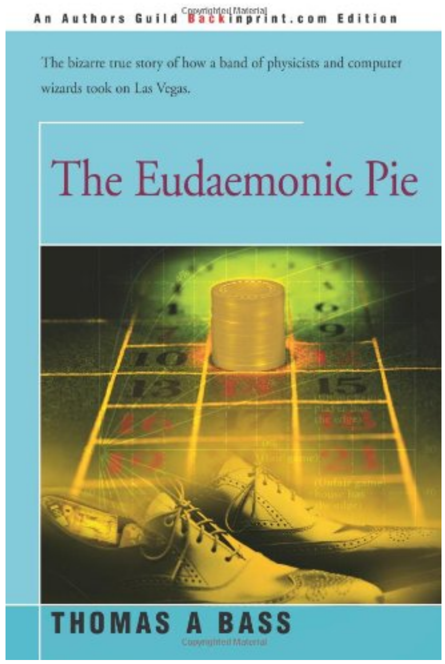
The feasibility of very-long-range weather prediction is examined in the light of these results.

PERIOD THREE IMPLIES CHAOS

TIEN-YIEN LI AND JAMES A. YORKE

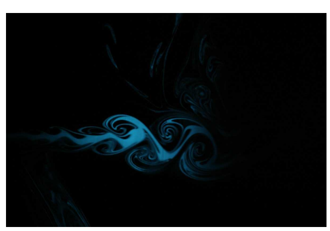
1. Introduction. The way phenomena or processes evolve or change in time is often described by differential equations or difference equations. One of the simplest mathematical situations occurs when the phenomenon can be described by a single number as, for example, when the number of children susceptible to some disease at the beginning of a school year can be estimated purely as a function of the number for the previous year. That is, when the number x_{n+1} at the beginning of the $n + 1$st year (or time period) can be written

$$x_{n+1} = F(x_n), \tag{1.1}$$



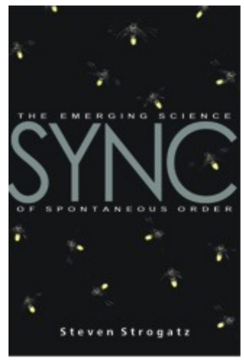
Where nonlinear dynamics turns up

- Flows (of fluids, heat, …)
  - Eddy in creek
  - Weather
  - Vortices around marine invertebrates
  - Air/fuel flow in combustion chambers



Where nonlinear dynamics turns up

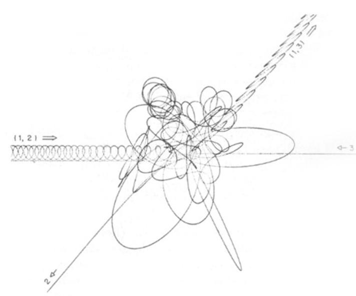
- Driven nonlinear oscillators
  - Pendula
  - Hearts
  - Fireflies



- and lots of other electronic, chemical, & biological systems

Where nonlinear dynamics turns up

- Classical mechanics
  - three-body problem
  - paired black holes
  - pulsar emission
  - ….



Hut & Bahcall *Ap.J.* ***268***:*319*

- Protein folding
- Population biology
- And many, many other fields (including yours)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