Prolog Programming Assignment #2: State Space Problem Solving

Learning Abstract

Prolog Programming Assignment #2 is made up of 9 tasks. The 9 tasks of the assignment involve the Towers of Hanoi problem and trying to solve it Prolog, using unit testing and predicates. Task 1-2 involve setting up the program I prolog and understanding the material before beginning on task 3. Task 3-8 help you get to the complete solution of the problem. In task 9 we are asked to show all of our source code.

Task 1: One Move Predicate and a Unit Test

State Space Operator Code:

```
m12([Tower1Before,Tower2Before,Tower3],[Tower1After,Tower2After,Tower3]) :-
Tower1Before = [H|T],
Tower1After = T,
Tower2Before = L,
Tower2After = [H|L].
```

Unit Test Code:

```
test__m12 :-
write('Testing: move_m12\n'),
TowersBefore = [[t,s,m,l,h],[],[]],
trace('','TowersBefore',TowersBefore),
m12(TowersBefore,TowersAfter),
trace('','TowersAfter',TowersAfter).
```

Unit Test Demo:

```
1 ?- consult('toh.pro').
true.
2 ?- test__m12.
Testing: move_m12
TowersBefore = [[t,s,m,l,h],[],[]]
TowersAfter = [[s,m,l,h],[t],[]]
true.
```

Task 4: The Remaining Five Move Predicates and a Unit Tests State Space Operator Code:

```
m12([Tower1Before,Tower2Before,Tower3],[Tower1After,Tower2After,Tower3]) :-
    Tower1Before = [H|T],
    Tower1After = T,
   Tower2Before = L,
    Tower2After = [H|L].
m13([Tower1Before,Tower2,Tower3Before],[Tower1After,Tower2,Tower3After]) :-
    Tower1Before = [H|T],
    Tower1After = T,
    Tower3Before = L,
    Tower3After = [H|L].
m21([Tower1Before,Tower2Before,Tower3],[Tower1After,Tower2After,Tower3]) :-
    Tower2Before = [H|T],
    Tower2After = T,
    Tower1Before = L,
    Tower1After = [H|L].
m23([Tower1,Tower2Before,Tower3Before],[Tower1,Tower2After,Tower3After]) :-
    Tower2Before = [H|T],
    Tower2After = T,
    Tower3Before = L
    Tower3After = [H|L].
m31([Tower1Before,Tower2,Tower3Before],[Tower1After,Tower2,Tower3After]) :-
    Tower3Before = [H|T],
    Tower3After = T,
    Tower1Before = L,
    Tower1After = [H|L].
m32([Tower1,Tower2Before,Tower3Before],[Tower1,Tower2After,Tower3After]) :-
    Tower3Before = [H|T],
    Tower3After = T,
   Tower2Before = L,
   Tower2After = [H|L].
```

Unit Test Code:

```
test__m12 :-
          write('Testing: move_m12\n'),
          TowersBefore = [[t,s,m,l,h],[],[]],
          trace('','TowersBefore',TowersBefore),
          m12(TowersBefore, TowersAfter),
          trace('','TowersAfter',TowersAfter).
          write('Testing: move_m13\n'),
          TowersBefore = [[t,s,m,l,h],[],[]],
          trace('','TowersBefore',TowersBefore),
          m13(TowersBefore, TowersAfter),
          trace('','TowersAfter',TowersAfter).
104
          write('Testing: move m21\n'),
          TowersBefore = [[m,1,h],[t,s],[]],
          trace('', 'TowersBefore', TowersBefore),
          m21(TowersBefore, TowersAfter),
          trace('','TowersAfter',TowersAfter).
110
111
112
          write('Testing: move_m23\n'),
113
114
          TowersBefore = [[l,h],[t,s],[m]],
          trace('', 'TowersBefore', TowersBefore),
115
          m23(TowersBefore, TowersAfter),
116
117
          trace('','TowersAfter',TowersAfter).
118
119
          write('Testing: move m31\n'),
120
121
          TowersBefore = [[m,l,h],[],[t,s]],
          trace('', 'TowersBefore', TowersBefore),
122
          m31(TowersBefore, TowersAfter),
123
124
          trace('', 'TowersAfter', TowersAfter).
125
126
      test m32 :-
127
          write('Testing: move m32\n'),
          TowersBefore = [[l,h],[m],[t,s]],
128
129
          trace('', 'TowersBefore', TowersBefore),
130
          m32(TowersBefore, TowersAfter),
          trace('','TowersAfter',TowersAfter).
```

Unit Test Demo:

```
Testing: move m12
TowersBefore = [[t,s,m,l,h],[],[]]
TowersAfter = [[s,m,1,h],[t],[]]
true.
3 ?- test m13.
Testing: move m13
TowersBefore = [[t,s,m,l,h],[],[]]
TowersAfter = [[s,m,l,h],[],[t]]
true.
4 ?- test m21.
Testing: move_m21
TowersBefore = [[m,l,h],[t,s],[]]
TowersAfter = [[t,m,l,h],[s],[]]
true.
5 ?- test__m23.
Testing: move_m23
TowersBefore = [[1,h],[t,s],[m]]
TowersAfter = [[1,h],[s],[t,m]]
true.
6 ?- test__m31.
Testing: move_m31
TowersBefore = [[m,l,h],[],[t,s]]
TowersAfter = [[t,m,1,h],[],[s]]
true.
7 ?- test_m32.
Testing: move_m32
TowersBefore = [[1,h],[m],[t,s]]
TowersAfter = [[l,h],[t,m],[s]]
true.
```

Task 5: Valid State Predicate and Unit Test

Predicate:

```
147
      valid_state([A|[B|[C]]]) :- state(A), state(B), state(C).
148
          state([]).
          state([t]).
150
          state([s]).
          state([m]).
          state([1]).
          state([h]).
          state([t,s]).
          state([t,s,m]).
156
          state([t,s,m,1]).
          state([t,s,1]).
158
          state([t,s,l,h]).
159
          state([t,s,h]).
          state([t,s,m,h]).
          state([t,m]).
          state([t,m,1]).
          state([t,m,l,h]).
          state([t,m,h]).
          state([t,1,h]).
          state([t,1]).
167
          state([t,h]).
          state([t,s,m,1,h]).
          state([s]).
170
          state([s,m]).
171
          state([s,m,1]).
172
          state([s,m,1,h]).
          state([s,1]).
173
174
          state([s,l,h]).
          state([s,h]).
175
176
          state([s,m,h]).
          state([m]).
178
          state([m,1]).
179
          state([m,1,h]).
          state([m,h]).
          state([1]).
          state([1,h]).
          state([h]).
```

Unit Test Code:

```
test valid state :-
          write('Testing: valid_state\n'),
134
135
          test__vs([[1,t,s,m,h],[],[]]),
136
          test_vs([[t,s,m,l,h],[],[]]),
          test_vs([[],[h,t,s,m],[1]]),
137
          test__vs([[],[t,s,m,h],[1]]),
138
139
          test_vs([[],[h],[l,m,s,t]]),
          test_vs([[],[h],[t,s,m,1]]).
          test vs(S):-
142
          valid state(S),
          write(S), write(' is valid.'), nl.
          test_vs(S) :-
          write(S), write(' is invalid.'), nl.
```

Unit Test Demo:

```
1 ?- consult('toh.pro').
true.
2 ?- test__valid_state.
Testing: valid_state
[[1,t,s,m,h],[],[]] is invalid.
[[t,s,m,l,h],[],[]] is valid.
[[],[h,t,s,m],[1]] is invalid.
[[],[t,s,m,h],[1]] is valid.
[[],[h],[l,m,s,t]] is invalid.
[[],[h],[t,s,m,l]] is valid.
true [
```

<u>Task 6: Defining the write_sequence predicate</u> Predicate:

```
write_sequence([]).
      write_sequence([H|T]) :-
          sequence(H,A),
190
          write(A),nl,write_sequence(T).
     sequence(m12,This) :-
          This = 'Transfer a disk from tower 1 to tower 2.'.
      sequence(m13,This) :-
        This = 'Transfer a disk from tower 1 to tower 3.'.
     sequence(m21,This) :-
          This = 'Transfer a disk from tower 2 to tower 1.'.
     sequence(m23,This) :-
         This = 'Transfer a disk from tower 2 to tower 3.'.
      sequence(m31,This) :-
          This = 'Transfer a disk from tower 3 to tower 1.'.
      sequence(m32,This) :-
          This = 'Transfer a disk from tower 3 to tower 2.'.
```

Unit Test Code:

```
test_write_sequence :-
write('First test of write_sequence ...'), nl,
write_sequence([m31,m12,m13,m21]),
write('Second test of write_sequence ...'), nl,
write_sequence([m13,m12,m32,m13,m21,m23,m13]).
```

Unit Test Demo:

```
1 ?- consult('toh.pro').
true.
2 ?- test write sequence.
First test of write sequence ...
Transfer a disk from tower 3 to tower 1.
Transfer a disk from tower 1 to tower 2.
Transfer a disk from tower 1 to tower 3.
Transfer a disk from tower 2 to tower 1.
Second test of write sequence ...
Transfer a disk from tower 1 to tower 3.
Transfer a disk from tower 1 to tower 2.
Transfer a disk from tower 3 to tower 2.
Transfer a disk from tower 1 to tower 3.
Transfer a disk from tower 2 to tower 1.
Transfer a disk from tower 2 to tower 3.
Transfer a disk from tower 1 to tower 3.
```

Task 7: Run the program to solve the 3 disk problem Intermediate with English Output:

Microsoft Windows [Version 10.0.22000.613] (c) Microsoft Corporation. All rights reserved. C:\Users\zchbo>swipl Welcome to SWI-Prolog (threaded, 64 bits, version 8.4.2) SWI-Prolog comes with ABSOLUTELY NO WARRANTY. This is free software. Please run ?- license. for legal details. For online help and background, visit https://www.swi-prolog.org For built-in help, use ?- help(Topic). or ?- apropos(Word). 1 ?- cd true. 2 ?- consult('C:/Users/zchbo/Desktop/344 Programming Languages/Prolog/toh.pro'). true. 3 ?- solve. PathSoFar = [[[s,m,l],[],[]]]Move = m12NextState = [[m,l],[s],[]]PathSoFar = [[[s,m,l],[],[]],[[m,l],[s],[]]]Move = m12NextState = [[l],[m,s],[]]Move = m13NextState = [[l],[s],[m]]PathSoFar = [[[s,m,l],[],[]],[[m,l],[s],[]],[[l],[s],[m]]]Move = m12NextState = [[],[l,s],[m]]Move = m13NextState = [[],[s],[l,m]]Move = m21NextState = [[s,l],[],[m]]PathSoFar = [[[s,m,l],[],[]],[[m,l],[s],[]],[[l],[s],[m]],[[s,l],[],[m]]] Move = m12NextState = [[l],[s],[m]]Move = m13NextState = [[l],[],[s,m]]PathSoFar = [[[s,m,l],[],[]],[[m,l],[s],[]],[[l],[s],[m]],[[s,l],[],[m]],[[l],[],[s,m]]]NextState = [[],[l],[s,m]] PathSoFar = [[[s,m,l],[],[]],[[m,l],[s],[]],[[l],[s],[m]],[[s,l],[],[m]],[[l],[],[s,m]]]Move = m21

Move = m23

```
NextState = [[1],[],[s,m]]
Move = m23
NextState = [[],[],[l,s,m]]
Move = m31
NextState = [[s],[l],[m]]
PathSoFar = [[[s,m,l],[],[]],[[m,l],[s],[]],[[l],[s],[m]],[[s,l],[],[m]],[[l],[s,m]],[[s],[l],[m]]]
Move = m12
NextState = [[],[s,l],[m]]
PathSoFar = [[[s,m,l],[],[[m,l],[s],[]],[[l],[s],[m]],[[s,l],[],[m]],[[l],[s,m]],[[],[s,m]],[[s],[l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[[s,l],[m]],[[s,l],[[s,l],[m]],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[s,l],[[
Move = m21
NextState = [[s],[l],[m]]
Move = m23
NextState = [[],[l],[s,m]]
Move = m31
NextState = [[m],[s,l],[]]
PathSoFar = [[[s,m,l],[],[],[[m,l],[s],[],[[l],[s],[m]],[[s,l],[],[m],[[s,m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[m]],[[s,l],[s,l],[m]],[[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,l],[s,
Move = m12
NextState = [[],[m,s,l],[]]
Move = m13
NextState = [[],[s,l],[m]]
Move = m21
NextState = [[s,m],[l],[]]
Move = m12
NextState = [[m],[s,l],[]]
Move = m13
NextState = [[m],[l],[s]]
Move = m12
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m],[l],[s]]
Move = m23
NextState = [[],[l],[m,s]]
Move = m31
NextState = [[s],[m,l],[]]
Move = m12
NextState = [[],[s,m,l],[]]
PathSoFar =
,[[],[s,m,l],[]]]
Move = m21
NextState = [[s],[m,l],[]]
```

Move = m21

```
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
PathSoFar =
Move = m12
NextState = [[],[s,m,l],[]]
PathSoFar =
,[[],[s,m,l],[]]]
Move = m21
NextState = [[s],[m,l],[]]
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
Move = m32
NextState = [[],[s,m,l],[]]
PathSoFar =
Move = m21
NextState = [[s],[m,l],[]]
PathSoFar =
],[[s],[m,l],[]]]
Move = m12
NextState = [[],[s,m,l],[]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
PathSoFar =
],[[s],[m,l],[]]]
Move = m12
NextState = [[],[s,m,l],[]]
Move = m13
NextState = [[],[m,l],[s]]
```

NextState = [[],[s,m,l],[]]

```
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
Move = m23
NextState = [[],[m,l],[s]]
PathSoFar =
Move = m21
NextState = [[m],[l],[s]]
Move = m23
NextState = [[],[l],[m,s]]
Move = m31
NextState = [[s],[m,l],[]]
PathSoFar =
Move = m12
NextState = [[],[s,m,l],[]]
PathSoFar =
,[[],[s,m,l],[]]]
Move = m21
NextState = [[s],[m,l],[]]
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
PathSoFar =
Move = m12
NextState = [[],[s,m,l],[]]
PathSoFar =
,[[],[s,m,l],[]]]
Move = m21
NextState = [[s],[m,l],[]]
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
Move = m32
```

NextState = [[s],[m,l],[]]

PathSoFar = Move = m21NextState = [[s],[m,l],[]]PathSoFar =],[[s],[m,l],[]]] Move = m12NextState = [[],[s,m,l],[]]Move = m13NextState = [[],[m,l],[s]]Move = m21NextState = [[m,s],[l],[]]Move = m23NextState = [[s],[l],[m]] PathSoFar =],[[s],[m,l],[]]] Move = m12NextState = [[],[s,m,l],[]]Move = m13NextState = [[],[m,l],[s]]Move = m21NextState = [[m,s],[l],[]]Move = m23NextState = [[s],[l],[m]]Move = m23NextState = [[],[m,l],[s]]Move = m13NextState = [[],[l],[m,s]] Move = m21NextState = [[l,m],[],[s]]Move = m23NextState = [[m],[],[l,s]]Move = m31NextState = [[s,m],[l],[]] Move = m32NextState = [[m],[s,l],[]]Move = m12NextState = [[],[m,l],[s]]PathSoFar = Move = m21NextState = [[m],[l],[s]] Move = m23NextState = [[],[l],[m,s]] Move = m31

NextState = [[m,s],[l],[]]

```
PathSoFar =
Move = m12
NextState = [[],[s,m,l],[]]
PathSoFar =
,[[],[s,m,l],[]]]
Move = m21
NextState = [[s],[m,l],[]]
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
PathSoFar =
Move = m12
NextState = [[],[s,m,l],[]]
PathSoFar =
,[[],[s,m,l],[]]]
Move = m21
NextState = [[s],[m,l],[]]
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
Move = m32
NextState = [[],[s,m,l],[]]
PathSoFar =
Move = m21
NextState = [[s],[m,l],[]]
PathSoFar =
],[[s],[m,l],[]]]
Move = m12
NextState = [[],[s,m,l],[]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
```

Move = m23NextState = [[s],[l],[m]] PathSoFar =],[[s],[m,l],[]]] Move = m12NextState = [[],[s,m,l],[]]Move = m13NextState = [[],[m,l],[s]]Move = m21NextState = [[m,s],[l],[]]Move = m23NextState = [[s],[l],[m]] Move = m23NextState = [[],[m,l],[s]]Move = m21NextState = [[m],[l],[s]]Move = m23NextState = [[],[l],[m,s]] Move = m31NextState = [[s],[m,l],[]]PathSoFar = Move = m12NextState = [[],[s,m,l],[]]PathSoFar = ,[[],[s,m,l],[]]] Move = m21NextState = [[s],[m,l],[]]Move = m23NextState = [[],[m,l],[s]]Move = m13NextState = [[],[m,l],[s]]Move = m21NextState = [[m,s],[l],[]]Move = m23NextState = [[s],[l],[m]]PathSoFar = Move = m12NextState = [[],[s,m,l],[]]PathSoFar = [[[s,m,l],[[s,l],[[s,m]],[[s,l],[[s,l],[[s,l],[]],[[s,l],[]],[[s,m]],[[s,l],[],[[s,m]],[[s,l],[]],[[s,m]],[[s,m]],[[s,m]],[[s,m,l],[s,m,l],[s,m,l],[s,m,l],[s,m,l],[s,m,l],[s,m,l],[s,m,l],[s,m,l],[s,m,l],[s,m,l],[s,m,l],[s,m,,[[],[s,m,l],[]]] Move = m21NextState = [[s],[m,l],[]]

NextState = [[m],[s,l],[]]

Move = m23NextState = [[],[m,l],[s]]Move = m13NextState = [[],[m,l],[s]]Move = m21NextState = [[m,s],[l],[]]Move = m23NextState = [[s],[l],[m]]Move = m32NextState = [[],[s,m,l],[]] PathSoFar = Move = m21NextState = [[s],[m,l],[]]PathSoFar =],[[s],[m,l],[]]] Move = m12NextState = [[],[s,m,l],[]]Move = m13NextState = [[],[m,l],[s]] Move = m21NextState = [[m,s],[l],[]]Move = m23NextState = [[s],[l],[m]]PathSoFar =],[[s],[m,l],[]]] Move = m12NextState = [[],[s,m,l],[]]Move = m13NextState = [[],[m,l],[s]]Move = m21NextState = [[m,s],[l],[]]Move = m23NextState = [[s],[l],[m]]Move = m23NextState = [[],[m,l],[s]]Move = m13NextState = [[],[l],[m,s]] Move = m21NextState = [[l,m],[],[s]]Move = m23NextState = [[m],[],[l,s]]Move = m31NextState = [[s,m],[l],[]] Move = m32

NextState = [[],[s,m,l],[]]

```
Move = m12
NextState = [[],[m,l],[s]]
PathSoFar =
NextState = [[m],[l],[s]]
Move = m23
NextState = [[],[l],[m,s]]
Move = m31
NextState = [[s],[m,l],[]]
PathSoFar =
Move = m12
NextState = [[],[s,m,l],[]]
PathSoFar =
,[[],[s,m,l],[]]]
Move = m21
NextState = [[s],[m,l],[]]
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
PathSoFar =
Move = m12
NextState = [[],[s,m,l],[]]
PathSoFar =
,[[],[s,m,l],[]]]
Move = m21
NextState = [[s],[m,l],[]]
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
Move = m32
```

NextState = [[],[m,l],[s]]

```
PathSoFar =
Move = m21
NextState = [[s],[m,l],[]]
PathSoFar =
],[[s],[m,l],[]]]
Move = m12
NextState = [[],[s,m,l],[]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
PathSoFar =
],[[s],[m,l],[]]]
Move = m12
NextState = [[],[s,m,l],[]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
Move = m23
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m],[l],[s]]
Move = m23
NextState = [[],[l],[m,s]]
Move = m31
NextState = [[s],[m,l],[]]
Move = m12
NextState = [[],[s,m,l],[]]
PathSoFar =
,[[],[s,m,l],[]]]
Move = m21
NextState = [[s],[m,l],[]]
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
```

NextState = [[s],[l],[m]]

```
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
PathSoFar =
Move = m12
NextState = [[],[s,m,l],[]]
PathSoFar =
,[[],[s,m,l],[]]]
Move = m21
NextState = [[s],[m,l],[]]
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
Move = m32
NextState = [[],[s,m,l],[]]
PathSoFar =
Move = m21
NextState = [[s],[m,l],[]]
],[[s],[m,l],[]]]
Move = m12
NextState = [[],[s,m,l],[]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
PathSoFar =
],[[s],[m,l],[]]]
Move = m12
NextState = [[],[s,m,l],[]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
```

NextState = [[s],[m,l],[]]

```
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[l],[m,s]]
Move = m21
NextState = [[l,m],[],[s]]
Move = m23
NextState = [[m],[],[l,s]]
Move = m31
NextState = [[s,m],[l],[]]
Move = m32
NextState = [[m],[s,l],[]]
Move = m12
NextState = [[],[m,l],[s]]
PathSoFar =
Move = m21
NextState = [[m],[l],[s]]
Move = m23
NextState = [[],[l],[m,s]]
Move = m31
NextState = [[s],[m,l],[]]
PathSoFar =
Move = m12
NextState = [[],[s,m,l],[]]
PathSoFar =
,[[],[s,m,l],[]]]
Move = m21
NextState = [[s],[m,l],[]]
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
Move = m12
NextState = [[],[s,m,l],[]]
PathSoFar =
,[[],[s,m,l],[]]]
Move = m21
```

```
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
Move = m32
NextState = [[],[s,m,l],[]]
PathSoFar =
Move = m21
NextState = [[s],[m,l],[]]
PathSoFar =
],[[s],[m,l],[]]]
Move = m12
NextState = [[],[s,m,l],[]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
PathSoFar =
],[[s],[m,l],[]]]
Move = m12
NextState = [[],[s,m,l],[]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
Move = m23
NextState = [[],[m,l],[s]]
PathSoFar =
Move = m21
NextState = [[m],[l],[s]]
Move = m23
NextState = [[],[l],[m,s]]
Move = m31
NextState = [[s],[m,l],[]]
PathSoFar =
Move = m12
```

```
NextState = [[],[s,m,l],[]]
PathSoFar =
,[[],[s,m,l],[]]]
Move = m21
NextState = [[s],[m,l],[]]
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
PathSoFar =
Move = m12
NextState = [[],[s,m,l],[]]
PathSoFar =
,[[],[s,m,l],[]]]
Move = m21
NextState = [[s],[m,l],[]]
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
Move = m32
NextState = [[],[s,m,l],[]]
PathSoFar =
Move = m21
NextState = [[s],[m,l],[]]
],[[s],[m,l],[]]]
Move = m12
NextState = [[],[s,m,l],[]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
```

NextState = [[m,s],[l],[]]

PathSoFar =],[[s],[m,l],[]]] Move = m12NextState = [[],[s,m,l],[]]Move = m13NextState = [[],[m,l],[s]]Move = m21NextState = [[m,s],[l],[]]Move = m23NextState = [[s],[l],[m]]Move = m23NextState = [[],[m,l],[s]]Move = m13NextState = [[],[l],[m,s]]Move = m21NextState = [[l,m],[],[s]]Move = m23NextState = [[m],[],[l,s]]Move = m31NextState = [[s,m],[l],[]]Move = m32NextState = [[m],[s,l],[]]Move = m12NextState = [[],[m,l],[s]]Move = m21NextState = [[m],[l],[s]]Move = m23NextState = [[],[l],[m,s]] Move = m31NextState = [[s],[m,l],[]]PathSoFar = Move = m12NextState = [[],[s,m,l],[]]PathSoFar = ,[[],[s,m,l],[]]] Move = m21NextState = [[s],[m,l],[]]Move = m23NextState = [[],[m,l],[s]] Move = m13NextState = [[],[m,l],[s]]Move = m21

NextState = [[],[m,l],[s]]

Move = m23NextState = [[s],[l],[m]] PathSoFar = Move = m12NextState = [[],[s,m,l],[]] PathSoFar = ,[[],[s,m,l],[]]] Move = m21NextState = [[s],[m,l],[]]Move = m23NextState = [[],[m,l],[s]]Move = m13NextState = [[],[m,l],[s]]Move = m21NextState = [[m,s],[l],[]]Move = m23NextState = [[s],[l],[m]] Move = m32NextState = [[],[s,m,l],[]]PathSoFar = Move = m21NextState = [[s],[m,l],[]]PathSoFar =],[[s],[m,l],[]]] Move = m12NextState = [[],[s,m,l],[]]Move = m13NextState = [[],[m,l],[s]]Move = m21NextState = [[m,s],[l],[]] Move = m23NextState = [[s],[l],[m]]PathSoFar =],[[s],[m,l],[]]] Move = m12NextState = [[],[s,m,l],[]] Move = m13NextState = [[],[m,l],[s]]Move = m21NextState = [[m,s],[l],[]]Move = m23NextState = [[s],[l],[m]] Move = m23

NextState = [[s],[m,l],[]]

PathSoFar = Move = m21NextState = [[m],[l],[s]]Move = m23NextState = [[],[l],[m,s]] Move = m31NextState = [[s],[m,l],[]] PathSoFar = Move = m12NextState = [[],[s,m,l],[]]PathSoFar = ,[[],[s,m,l],[]]] Move = m21NextState = [[s],[m,l],[]]Move = m23NextState = [[],[m,l],[s]]Move = m13NextState = [[],[m,l],[s]]Move = m21NextState = [[m,s],[l],[]]Move = m23NextState = [[s],[l],[m]] PathSoFar = Move = m12NextState = [[],[s,m,l],[]]PathSoFar = ,[[],[s,m,l],[]]] Move = m21 NextState = [[s],[m,l],[]]Move = m23NextState = [[],[m,l],[s]]Move = m13NextState = [[],[m,l],[s]]Move = m21NextState = [[m,s],[l],[]]Move = m23NextState = [[s],[l],[m]] Move = m32NextState = [[],[s,m,l],[]]PathSoFar = 11 Move = m21

NextState = [[],[s,m,l],[]]

PathSoFar =],[[s],[m,l],[]]] Move = m12NextState = [[],[s,m,l],[]]Move = m13NextState = [[],[m,l],[s]]Move = m21NextState = [[m,s],[l],[]]Move = m23NextState = [[s],[l],[m]] PathSoFar =],[[s],[m,l],[]]] Move = m12NextState = [[],[s,m,l],[]] Move = m13NextState = [[],[m,l],[s]]Move = m21NextState = [[m,s],[l],[]]Move = m23NextState = [[s],[l],[m]]Move = m23NextState = [[],[m,l],[s]]Move = m13NextState = [[],[l],[m,s]]Move = m21NextState = [[l,m],[],[s]] Move = m23NextState = [[m],[],[l,s]]Move = m31NextState = [[s,m],[l],[]] Move = m32NextState = [[m],[s,l],[]]Move = m12NextState = [[],[m,l],[s]]PathSoFar = Move = m21NextState = [[m],[l],[s]]Move = m23NextState = [[],[l],[m,s]] Move = m31NextState = [[s],[m,l],[]]PathSoFar = Move = m12

PathSoFar = ,[[],[s,m,l],[]]] Move = m21NextState = [[s],[m,l],[]]Move = m23NextState = [[],[m,l],[s]]Move = m13NextState = [[],[m,l],[s]]Move = m21NextState = [[m,s],[l],[]]Move = m23NextState = [[s],[l],[m]] PathSoFar = Move = m12NextState = [[],[s,m,l],[]]PathSoFar = ,[[],[s,m,l],[]]] Move = m21NextState = [[s],[m,l],[]] Move = m23NextState = [[],[m,l],[s]]Move = m13NextState = [[],[m,l],[s]]Move = m21NextState = [[m,s],[l],[]]Move = m23NextState = [[s],[l],[m]] Move = m32NextState = [[],[s,m,l],[]]PathSoFar = Move = m21NextState = [[s],[m,l],[]]PathSoFar =],[[s],[m,l],[]]] Move = m12NextState = [[],[s,m,l],[]] Move = m13NextState = [[],[m,l],[s]]Move = m21NextState = [[m,s],[l],[]]Move = m23NextState = [[s],[l],[m]]],[[s],[m,l],[]]]

Move = m21

```
Move = m12
NextState = [[],[s,m,l],[]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
Move = m23
NextState = [[],[m,l],[s]]
PathSoFar =
Move = m21
NextState = [[m],[l],[s]]
Move = m23
NextState = [[],[l],[m,s]]
Move = m31
NextState = [[s],[m,l],[]]
Move = m12
NextState = [[],[s,m,l],[]]
PathSoFar =
,[[],[s,m,l],[]]]
Move = m21
NextState = [[s],[m,l],[]]
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
Move = m12
NextState = [[],[s,m,l],[]]
PathSoFar =
,[[],[s,m,l],[]]]
Move = m21
NextState = [[s], [m,l], []]
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[m,l],[s]]
```

Move = m21

```
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
Move = m32
NextState = [[],[s,m,l],[]]
PathSoFar =
Move = m21
NextState = [[s],[m,l],[]]
PathSoFar =
],[[s],[m,l],[]]]
Move = m12
NextState = [[],[s,m,l],[]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
PathSoFar =
],[[s],[m,l],[]]]
Move = m12
NextState = [[],[s,m,l],[]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[l],[m,s]]
Move = m21
NextState = [[l,m],[],[s]]
Move = m23
NextState = [[m],[],[l,s]]
Move = m31
NextState = [[s,m],[l],[]]
Move = m32
NextState = [[m],[s,l],[]]
Move = m12
NextState = [[],[m,l],[s]]
```

```
NextState = [[m],[l],[s]]
Move = m23
NextState = [[],[l],[m,s]]
Move = m31
NextState = [[s],[m,l],[]]
PathSoFar =
Move = m12
NextState = [[],[s,m,l],[]]
PathSoFar =
,[[],[s,m,l],[]]]
Move = m21
NextState = [[s],[m,l],[]]
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
PathSoFar =
Move = m12
NextState = [[],[s,m,l],[]]
PathSoFar =
,[[],[s,m,l],[]]]
Move = m21
NextState = [[s],[m,l],[]]
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
Move = m32
NextState = [[],[s,m,l],[]]
PathSoFar =
Move = m21
NextState = [[s],[m,l],[]]
],[[s],[m,l],[]]]
Move = m12
```

NextState = [[],[s,m,l],[]]

```
NextState = [[],[s,m,l],[]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
],[[s],[m,l],[]]]
Move = m12
NextState = [[],[s,m,l],[]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
Move = m23
NextState = [[],[m,l],[s]]
PathSoFar =
Move = m21
NextState = [[m],[l],[s]]
Move = m23
NextState = [[],[l],[m,s]]
Move = m31
NextState = [[s],[m,l],[]]
Move = m12
NextState = [[],[s,m,l],[]]
PathSoFar =
,[[],[s,m,l],[]]]
Move = m21
NextState = [[s],[m,l],[]]
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
Move = m12
```

NextState = [[m],[],[l,s]]

PathSoFar = ,[[],[s,m,l],[]]] Move = m21NextState = [[s],[m,l],[]]Move = m23NextState = [[],[m,l],[s]]Move = m13NextState = [[],[m,l],[s]]Move = m21NextState = [[m,s],[l],[]]Move = m23NextState = [[s],[l],[m]]Move = m32NextState = [[],[s,m,l],[]]PathSoFar = Move = m21NextState = [[s],[m,l],[]]PathSoFar =],[[s],[m,l],[]]] Move = m12NextState = [[],[s,m,l],[]]Move = m13NextState = [[],[m,l],[s]] Move = m21NextState = [[m,s],[l],[]]Move = m23NextState = [[s],[l],[m]] PathSoFar =],[[s],[m,l],[]]] Move = m12NextState = [[],[s,m,l],[]]Move = m13NextState = [[],[m,l],[s]]Move = m21NextState = [[m,s],[l],[]]Move = m23NextState = [[s],[l],[m]]Move = m23NextState = [[],[m,l],[s]]Move = m13NextState = [[],[l],[m,s]]Move = m21NextState = [[l,m],[],[s]]Move = m23

NextState = [[s],[l],[m]]

```
Move = m31
NextState = [[s,m],[l],[]]
Move = m32
NextState = [[m],[s,l],[]]
Move = m12
NextState = [[],[m,l],[s]]
PathSoFar =
Move = m21
NextState = [[m],[l],[s]]
Move = m23
NextState = [[],[l],[m,s]]
Move = m31
NextState = [[s],[m,l],[]]
PathSoFar =
Move = m12
NextState = [[],[s,m,l],[]]
PathSoFar =
,[[],[s,m,l],[]]]
Move = m21
NextState = [[s],[m,l],[]]
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
PathSoFar =
Move = m12
NextState = [[],[s,m,l],[]]
PathSoFar =
[[[s,m,l],[[s,l],[[s,m]],[[s,l],[[s,m]],[[s,l],[[s,l],[[s,m]],[[s,l],[[s,m]],[[s,l],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m,l],[[s,m]],[[s,m,l],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]],[[s,m]
,[[],[s,m,l],[]]]
Move = m21
NextState = [[s],[m,l],[]]
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
```

```
Move = m32
NextState = [[],[s,m,l],[]]
PathSoFar =
Move = m21
NextState = [[s],[m,l],[]]
PathSoFar =
],[[s],[m,l],[]]]
Move = m12
NextState = [[],[s,m,l],[]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
PathSoFar =
],[[s],[m,l],[]]]
Move = m12
NextState = [[],[s,m,l],[]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
Move = m23
NextState = [[],[m,l],[s]]
PathSoFar =
Move = m21
NextState = [[m],[l],[s]]
Move = m23
NextState = [[],[l],[m,s]]
Move = m31
NextState = [[s],[m,l],[]]
PathSoFar =
Move = m12
NextState = [[],[s,m,l],[]]
PathSoFar =
,[[],[s,m,l],[]]]
Move = m21
NextState = [[s],[m,l],[]]
Move = m23
NextState = [[],[m,l],[s]]
```

```
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
NextState = [[],[s,m,l],[]]
PathSoFar =
,[[],[s,m,l],[]]]
Move = m21
NextState = [[s], [m,l], []]
Move = m23
NextState = [[],[m,l],[s]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
Move = m32
NextState = [[],[s,m,l],[]]
PathSoFar =
Move = m21
NextState = [[s],[m,l],[]]
PathSoFar =
],[[s],[m,l],[]]]
Move = m12
NextState = [[],[s,m,l],[]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
Move = m23
NextState = [[s],[l],[m]]
PathSoFar =
],[[s],[m,l],[]]]
Move = m12
NextState = [[],[s,m,l],[]]
Move = m13
NextState = [[],[m,l],[s]]
Move = m21
NextState = [[m,s],[l],[]]
```

Transfer a disk from tower 1 to tower 3.

Move = m23NextState = [[s],[l],[m]] Move = m23NextState = [[],[m,l],[s]]Move = m13NextState = [[],[l],[m,s]]Move = m21NextState = [[l,m],[],[s]]Move = m23NextState = [[m],[],[l,s]]Move = m31NextState = [[s,m],[l],[]]Move = m32NextState = [[m],[s,l],[]]Move = m21NextState = [[l,s,m],[],[]]Move = m23NextState = [[s,m],[],[l]]Move = m12NextState = [[m],[s],[l]] PathSoFar = Move = m12NextState = [[],[m,s],[l]] Move = m13NextState = [[],[s],[m,l]]Move = m21NextState = [[s],[],[m,l]]PathSoFar = [[s,m],[],[[m,[s,m]],[[n,[s,m]],[[n,[s,m]],[[n,[s,m]],[[n,[s,m]],[[n,[s,m]],[[n,[s,m]],[[n,[s,n]],[[s],[],[m,l]]] Move = m12NextState = [[],[s],[m,l]] Move = m13NextState = [[],[],[s,m,l]]PathSoFar = ,[[s],[],[m,l]],[[],[],[s,m,l]]] SolutionSoFar = [m12, m13, m21, m13, m12, m31, m12, m31, m21, m23, m12, m13, m21, m13]Solution ... Transfer a disk from tower 1 to tower 2. Transfer a disk from tower 1 to tower 3. Transfer a disk from tower 2 to tower 1.

Transfer a disk from tower 1 to tower 2.

Transfer a disk from tower 3 to tower 1.

Transfer a disk from tower 1 to tower 2.

Transfer a disk from tower 3 to tower 1.

Transfer a disk from tower 2 to tower 1.

Transfer a disk from tower 2 to tower 3.

Transfer a disk from tower 1 to tower 2.

Transfer a disk from tower 1 to tower 3.

Transfer a disk from tower 1 to tower 3.

Transfer a disk from tower 1 to tower 3.

Transfer a disk from tower 1 to tower 3.

true

Just English Output:

Solution ...

Transfer a disk from tower 1 to tower 2.

Transfer a disk from tower 1 to tower 3.

Transfer a disk from tower 2 to tower 1.

Transfer a disk from tower 1 to tower 3.

Transfer a disk from tower 1 to tower 2.

Transfer a disk from tower 3 to tower 1.

Transfer a disk from tower 3 to tower 1.

Transfer a disk from tower 3 to tower 1.

Transfer a disk from tower 2 to tower 1.

Transfer a disk from tower 2 to tower 3.

Transfer a disk from tower 1 to tower 3.

Transfer a disk from tower 1 to tower 3.

Transfer a disk from tower 1 to tower 3.

Transfer a disk from tower 1 to tower 3.

true

Questions:

- 1. What was the length of your program's solution to the three disk problem? 14 Steps.
- 2. What is the length of the shortest solution to the three disk problem? 7 steps.
- 3. How do you account for the discrepancy?

The program is using trial and error. It does not plan it's next move, instead it makes a move, and then another and so on until it is solved.

Task 8: Run the program to solve the 4 disk problem Demo:

```
Solution ...
Transfer a disk from tower 1 to tower 2.
Transfer a disk from tower 1 to tower 3.
Transfer a disk from tower 2 to tower 1.
Transfer a disk from tower 1 to tower 3.
Transfer a disk from tower 1 to tower 2.
Transfer a disk from tower 3 to tower 1.
Transfer a disk from tower 1 to tower 2.
Transfer a disk from tower 3 to tower 1.
Transfer a disk from tower 1 to tower 2.
Transfer a disk from tower 1 to tower 3.
Transfer a disk from tower 2 to tower 1.
Transfer a disk from tower 1 to tower 3.
Transfer a disk from tower 2 to tower 1.
Transfer a disk from tower 3 to tower 1.
Transfer a disk from tower 1 to tower 2.
Transfer a disk from tower 1 to tower 3.
Transfer a disk from tower 2 to tower 1.
Transfer a disk from tower 1 to tower 3.
true .
```

Questions:

- 1. What was the length of your program's solution to the four disk problem? 18 moves.
- 2. What is the length of the shortest solution to the four disk problem? 15 moves.

Task 9: Review your code and archive it

Inspector.pro

```
% --- in two ways, and pause for the programmer to check out the
check(Label,Name,Value) :-
    write(Label),
    write(Name),write(' = '),
    write(Value),nl,
    read(_).
    checkr(Label,Name,Value) :-
    write(Label),
    write(Name), write(' = '),
    reverse(Value, RValue),
    write(RValue),nl,
    trace(Label,Name,Value) :-
    write(Label),
    write(Name), write(' = '),
    write(Value),nl.
    tracer(Label,Name,Value) :-
    write(Label),
    write(Name),write(' = '),
    reverse(Value, RValue),
    write(RValue), nl.
    show(Name, Value) :-
    write(Name), write(' = '),
    write(Value), nl.
    showr(Name, Value) :-
    write(Name),write(' = '),
    reverse(Value, RValue),
    write(RValue),nl.
```

Toh.pro

```
% --- File: towers of hanoi.pro
 % --- Line: Program to solve the Towers of Hanoi problem
 :- consult('inspector.pro').
 make_move(TowersBeforeMove,TowersAfterMove,m12) :-
 m12(TowersBeforeMove, TowersAfterMove).
 make_move(TowersBeforeMove,TowersAfterMove,m13) :-
 m13(TowersBeforeMove, TowersAfterMove).
 make_move(TowersBeforeMove,TowersAfterMove,m21) :-
 m21(TowersBeforeMove, TowersAfterMove).
 make_move(TowersBeforeMove,TowersAfterMove,m23) :-
 m23(TowersBeforeMove, TowersAfterMove).
 make move(TowersBeforeMove,TowersAfterMove,m31) :-
 m31(TowersBeforeMove, TowersAfterMove).
 make_move(TowersBeforeMove,TowersAfterMove,m32) :-
 m32(TowersBeforeMove, TowersAfterMove).
 % --- from the start state to the goal state.
 extend_path([[[s,m,l,h],[],[]]],[],Solution),
 write_solution(Solution).
 extend_path(PathSoFar,SolutionSoFar,Solution) :-
 PathSoFar = [[[],[],[s,m,1,h]]|_],
 showr('PathSoFar',PathSoFar),
 showr('SolutionSoFar',SolutionSoFar),
 Solution = SolutionSoFar.
 extend_path(PathSoFar,SolutionSoFar,Solution) :-
 PathSoFar = [CurrentState|_],
 showr('PathSoFar',PathSoFar),
 make_move(CurrentState, NextState, Move),
 show('Move',Move),
 show('NextState',NextState),
 not(member(NextState,PathSoFar)),
 valid_state(NextState),
 Path = [NextState | PathSoFar],
 Soln = [Move|SolutionSoFar],
 extend_path(Path,Soln,Solution).
write solution(S) :-
```

```
nl, write('Solution ...'), nl, nl,
reverse(S,R),
write_sequence(R),nl.
% --- Unit test programs
m12([Tower1Before,Tower2Before,Tower3],[Tower1After,Tower2After,Tower3]) :-
    Tower1Before = [H|T],
    Tower1After = T,
    Tower2Before = L,
    Tower2After = [H|L].
m13([Tower1Before,Tower2,Tower3Before],[Tower1After,Tower2,Tower3After]) :-
    Tower1Before = [H|T],
    Tower1After = T,
    Tower3Before = L,
    Tower3After = [H|L].
m21([Tower1Before,Tower2Before,Tower3],[Tower1After,Tower2After,Tower3]) :-
    Tower2Before = [H|T],
    Tower2After = T,
    Tower1Before = L,
    Tower1After = [H|L].
m23([Tower1,Tower2Before,Tower3Before],[Tower1,Tower2After,Tower3After]) :-
    Tower2Before = [H|T],
    Tower2After = T,
    Tower3Before = L,
    Tower3After = [H|L].
m31([Tower1Before,Tower2,Tower3Before],[Tower1After,Tower2,Tower3After]) :-
    Tower3Before = [H|T],
    Tower3After = T,
    Tower1Before = L,
    Tower1After = [H|L].
m32([Tower1,Tower2Before,Tower3Before],[Tower1,Tower2After,Tower3After]) :-
    Tower3Before = [H|T],
    Tower3After = T,
    Tower2Before = L,
    Tower2After = [H|L].
test m12 :-
    write('Testing: move_m12\n'),
    TowersBefore = [[t,s,m,l,h],[],[]],
    trace('', 'TowersBefore', TowersBefore),
    m12(TowersBefore, TowersAfter),
```

```
trace('','TowersAfter',TowersAfter).
          write('Testing: move_m13\n'),
          TowersBefore = [[t,s,m,l,h],[],[]],
          trace('', 'TowersBefore', TowersBefore),
          m13(TowersBefore, TowersAfter),
          trace('','TowersAfter',TowersAfter).
104
          write('Testing: move_m21\n'),
          TowersBefore = [[m,l,h],[t,s],[]],
          trace('', 'TowersBefore', TowersBefore),
          m21(TowersBefore, TowersAfter),
          trace('', 'TowersAfter', TowersAfter).
110
111
112
113
          write('Testing: move_m23\n'),
114
          TowersBefore = [[1,h],[t,s],[m]],
115
          trace('', 'TowersBefore', TowersBefore),
116
          m23(TowersBefore, TowersAfter),
          trace('','TowersAfter',TowersAfter).
117
118
119
120
          write('Testing: move_m31\n'),
121
          TowersBefore = [[m,1,h],[],[t,s]],
122
          trace('', 'TowersBefore', TowersBefore),
          m31(TowersBefore, TowersAfter),
123
          trace('','TowersAfter',TowersAfter).
124
125
126
          write('Testing: move_m32\n'),
128
          TowersBefore = [[1,h],[m],[t,s]],
          trace('', 'TowersBefore', TowersBefore),
129
130
          m32(TowersBefore, TowersAfter),
131
          trace('','TowersAfter',TowersAfter).
132
133
      test valid state :-
134
          write('Testing: valid state\n'),
135
          test vs([[1,t,s,m,h],[],[]]),
          test vs([[t,s,m,l,h],[],[]]),
136
137
          test__vs([[],[h,t,s,m],[1]]),
138
          test__vs([[],[t,s,m,h],[1]]),
          test__vs([[],[h],[l,m,s,t]]),
          test__vs([[],[h],[t,s,m,1]]).
          test vs(S):-
          valid_state(S),
          write(S), write(' is valid.'), nl.
```

```
144
          test_vs(S):-
          write(S), write(' is invalid.'), nl.
146
      valid_state([A|[B|[C]]]) :- state(A), state(B), state(C).
          state([]).
          state([t]).
          state([s]).
          state([m]).
          state([1]).
          state([h]).
          state([t,s]).
          state([t,s,m]).
          state([t,s,m,1]).
          state([t,s,1]).
          state([t,s,l,h]).
          state([t,s,h]).
          state([t,s,m,h]).
          state([t,m]).
          state([t,m,1]).
          state([t,m,1,h]).
          state([t,m,h]).
          state([t,1,h]).
          state([t,1]).
166
          state([t,h]).
          state([t,s,m,l,h]).
          state([s]).
170
          state([s,m]).
          state([s,m,1]).
          state([s,m,l,h]).
172
173
          state([s,1]).
          state([s,l,h]).
          state([s,h]).
175
176
          state([s,m,h]).
          state([m]).
          state([m,1]).
178
179
          state([m,1,h]).
          state([m,h]).
          state([1]).
          state([1,h]).
          state([h]).
      write_sequence([]).
      write_sequence([H|T]) :-
          sequence(H,A),
```

```
191
192
      sequence(m12,This) :-
193
          This = 'Transfer a disk from tower 1 to tower 2.'.
194
      sequence(m13,This) :-
195
196
          This = 'Transfer a disk from tower 1 to tower 3.'.
197
      sequence(m21,This) :-
198
199
          This = 'Transfer a disk from tower 2 to tower 1.'.
200
201
      sequence(m23,This) :-
          This = 'Transfer a disk from tower 2 to tower 3.'.
202
203
204
      sequence(m31,This) :-
          This = 'Transfer a disk from tower 3 to tower 1.'.
205
206
      sequence(m32,This) :-
207
          This = 'Transfer a disk from tower 3 to tower 2.'.
208
209
210
211
212
      test write sequence :-
          write('First test of write sequence ...'), nl,
213
214
          write_sequence([m31,m12,m13,m21]),
215
          write('Second test of write_sequence ...'), nl,
216
          write_sequence([m13,m12,m32,m13,m21,m23,m13]).
```