

Homework 5 – Due: 10/02/2024 11:59 pm

Problem 1. [30 points] Write a C++ program to estimate PI using the numerical integration method. Your program should ask the user to input the total number of rectangles n that are used to estimate the area of a quarter of the circle with radius $r = 1$.

Define $\text{PI} = 3.14159265$ and report your error when you use $n = 10, 10^2, 10^3$ and 10^4 rectangles.

Please submit your .cpp file as “yourLastName_hw5_prob1.cpp”.

Problem 2. [35 points] The Maclaurin series of $\sin(x)$ can be written as:

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

Here the factorial of a positive integer $n!$ is defined as the product of all positive integers less than or equal to n .

Write a C++ program that takes a double variable x and returns the approximation of $\sin(x)$ using the first k terms of the Maclaurin series. For example, when $k = 1$, we have $\sin(x) \approx x$. When $k = 2$, we have $\sin(x) \approx x - \frac{x^3}{3!}$.

The program should display the difference between your approximations and $\sin(x)$ in `<cmath>` on your screen for $k = 1, 2, 3, \dots, 10$.

Please report your results for the case $x = 0.5$ in the write-up.

Please submit your .cpp file as “yourLastName_hw5_prob2.cpp”.

Problem 3. [35 points] Write a C++ program to

- (1) Generate a 2D pattern with 101 rows and 101 columns using a nested loop as follows: At the location (i, j) , that is the i^{th} row and j^{th} column, print out ‘x’ if the distance between this location and the center of the grid (51, 51) is less than 30.0. Otherwise, print out ‘ ’.
- (2) The routine automatically saves the pattern to a text file “pattern.txt”.

Submit your .cpp file as “yourLastName_hw5_prob3.cpp”. No write-up is required, and you do not need to submit the output file generated by your C++ routine.

Bonus problem [+10 points] *Mystery data.* You received a piece of mystery data “dat_hw5_bonus.txt” that were corrupted in some way. All we know is that the data contain ASCII characters that represent an image. Please write a C++ program that takes “dat_hw5_bonus.txt” as input and generate an output file “repaired_data.txt” that contains the correct pattern. What do you see?

The bonus puzzle is just for fun, and you have to complete it independently. We will not discuss it during our office hours. If you figure it out, please submit your .cpp file and the recovered data “repaired_data.txt” through Canvas. Describe your work in the write-up to receive full credit.

What to submit:

There should be 4 files in your submission:

1. A write up (any type- .txt, .docx, .pdf are all fine) that contains your answers to all questions in problem 1-3.
2. The .cpp file for your problem 1. Please name this file as [YourLastName]_prob1.cpp.
3. The .cpp file for your problem 2. Please name this file as [YourLastName]_prob2.cpp.
4. The .cpp file for your problem 3. Please name this file as [YourLastName]_prob3.cpp.

Optional Short answers questions. The following questions will not be graded. You may use them for preparing your next week’s quiz.

(1) How many times does the following loop execute?

```
int i = 0;
while (i <= 5) {
    cout << "Hi." << endl;
    i++;
}
```

(2) What is the output of the following code block?

```
for (int i = 20; i >= 5; i -= 5){
    cout << i << endl;
}
```

(3) What is the output of the following code block?

```
int factorial = 1;
int n = 3;
for (int i = 1; i <= n; ++i) {
    factorial *= i;
}
cout<< "Factorial of " << n << " = " << factorial << endl;
```

(4) Identify and correct all errors in the following code that asks the user to input a positive integer n and compute its factorial $n!$. The factorial of n is defined as

$$n! = n \times (n-1) \times (n-2) \times \dots \times 1.$$

For example, $4! = 4 \times 3 \times 2 \times 1 = 24$.

```
#include <iostream>
using namespace std;

int main() {

    int result;
    int n;

    do {
        cout << "Please enter a positive integer n: " <<
endl;
        cin << n;

    } while (n > 0);

    for (int i == n, i > 0, i--) {

        result *= i;

    }

    cout << "n! is " << result << endl;

    return 0;

}
```