# CS 216 Homework 2

Zachary DeStefano, 15247592

Due Date: April 25, 2014

# Problem 1

We will prove that $(f * g) * h = f * (g * h)$. First off, let $x = f * g$ and let $y = g * h$.

$$x(t) = (f * g)(t) = \sum_{s=-\infty}^{\infty} f(t - s)g(s)$$

$$y(t) = (g * h)(t) = \sum_{s=-\infty}^{\infty} g(s)h(t - s)$$

This means that

$$(x * h)(t) = \sum_{v=-\infty}^{\infty} x(v)h(t - v) = \sum_{v=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} f(v - s)g(s)h(t - v)$$

Similarly

$$(f * y)(t) = \sum_{v=-\infty}^{\infty} f(t - v)y(v) = \sum_{v=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} f(t - v)g(s)h(v - s)$$

After a change of variables (TODO: More detail), the two sides are equal.

# Problem 2

We will prove that $(f * g) * h \neq f * (g * h)$ if * is correlation. First off, let $x = f * g$ and let $y = g * h$.

$$x(t) = (f * g)(t) = \sum_{s=-\infty}^{\infty} f(t + s)g(s)$$

$$y(t) = (g * h)(t) = \sum_{s=-\infty}^{\infty} g(s)h(t + s)$$

This means that

$$(x * h)(t) = \sum_{v=-\infty}^{\infty} x(v)h(t + v) = \sum_{v=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} f(v + s)g(s)h(t + v)$$

Similarly

$$(f * y)(t) = \sum_{v=-\infty}^{\infty} f(t + v)y(v) = \sum_{v=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} f(t + v)g(s)h(v + s)$$

After a change of variables (TODO: More detail), the two sides are definitely not equal.

## Problem 3

Let $g_1(x) = g_2(y) = N(0, \sigma^2)$. Then

$$g_1(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$$

$$g_2(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2\sigma^2}}$$

This means that

$$g_1(x)g_2(y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

This allows us to say that $g(x, y) = g_1(x)g_2(y)$