CS 217 Homework 3

Zachary DeStefano, 15247592

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Problem 1

Part 1

If L is the direction of the light source, N is the normal to the surface, we know that the intensity of the light I is given by

$$I = |L||N|cos(\alpha)$$

where α is the angle between L and N.
That equation is maximized when $cos(\alpha) = 1$ so that $\alpha = 0$
This means that the normal is pointing in the direction of the light source.
Thus the light source is in the direction of (a, b).
To find the corresponding point on the sphere, we need to find c such that $a^2 + b^2 + c^2 = r^2$

Since we are projecting the sphere onto the x-y plane, we are going to see the positive side, thus we will use the positive solution to the above equation. Thus our point on the sphere is as follows:

$$(a, b, \sqrt{r^2 - (a^2 + b^2)})$$

That vector has magnitude r thus the unit vector for this point that points in the direction of the light source is as follows:

$$(\frac{a}{r}, \frac{b}{r}, \frac{\sqrt{r^2 - (a^2 + b^2)}}{r})$$

Part 2

Let L be the direction of the light from (a, b, c),
N be the normal to the surface at (a, b, c), and
E be the vector pointing from (a, b, c) to the viewer.

L will be the reflection vector of E from the surface with normal N since we have specular reflection
As illustrated below it will hold that

$$L + E = 2N(E \cdot N)$$

Thus we have the formula for L

$$L = 2N(E \cdot N) - E$$

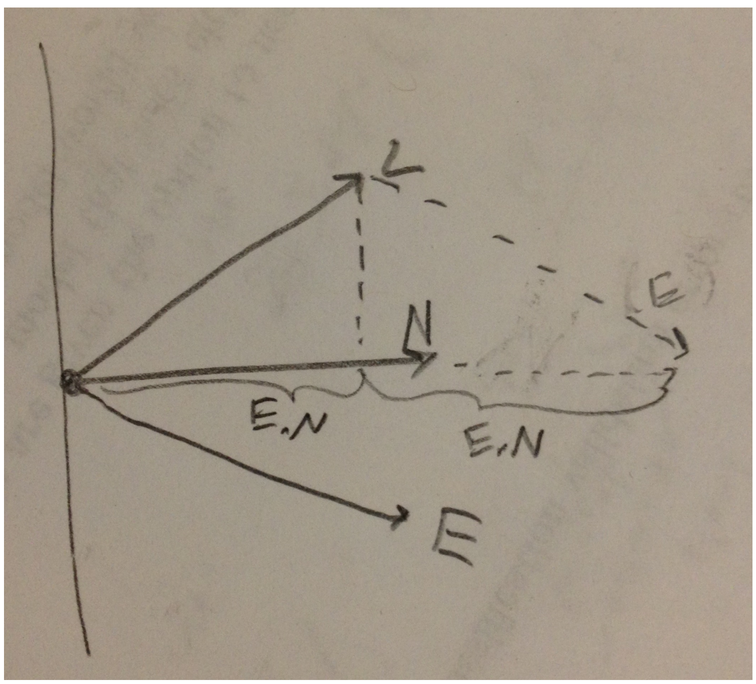


Figure 1: Illustration of normal vector N, light vector L, and viewing vector E and their relationships

E is parallel to the viewing direction thus
$E = (0, 0, 1)$
$N = \frac{1}{r}(a, b, c)$
We can thus say that $E \cdot N = \frac{c}{r}$, finally letting us say that

$$L = (\frac{2ac}{r^2}, \frac{2bc}{r^2}, \frac{2c^2}{r^2} - 1)$$

Now it holds that

$$||L||_2^2 = \frac{4a^2c^2 + 4b^2c^2 + 4c^4 - 4r^2c^2 + r^4}{r^4}$$

Factoring out $4c^2$ gives us

$$||L||_2^2 = \frac{4c^2(a^2 + b^2 + c^2 - r^2) + r^4}{r^4}$$

We know that $r^2 = a^2 + b^2 + c^2$ thus finally $||L||_2^2 = 1$
Proving that L is a unit vector and thus in its final form

Problem 2

For the images 1 to 11, here are the normal vectors in order at the bright spot, which would be the lighting direction if the sphere were diffuse. Each row indicates a vector in order (x, y, z)

```
N =

   -0.0672    0.1261    0.9897
   -0.0924   -0.0168    0.9956
   -0.2269   -0.0504    0.9726
   -0.2689   -0.1681    0.9484
   -0.2941   -0.0588    0.9540
   -0.2185    0.1513    0.9640
   -0.2269    0.0504    0.9726
   -0.1681    0.1092    0.9797
   -0.1681    0.0504    0.9845
   -0.0252    0.0672    0.9974
   -0.1849   -0.0672    0.9805
```

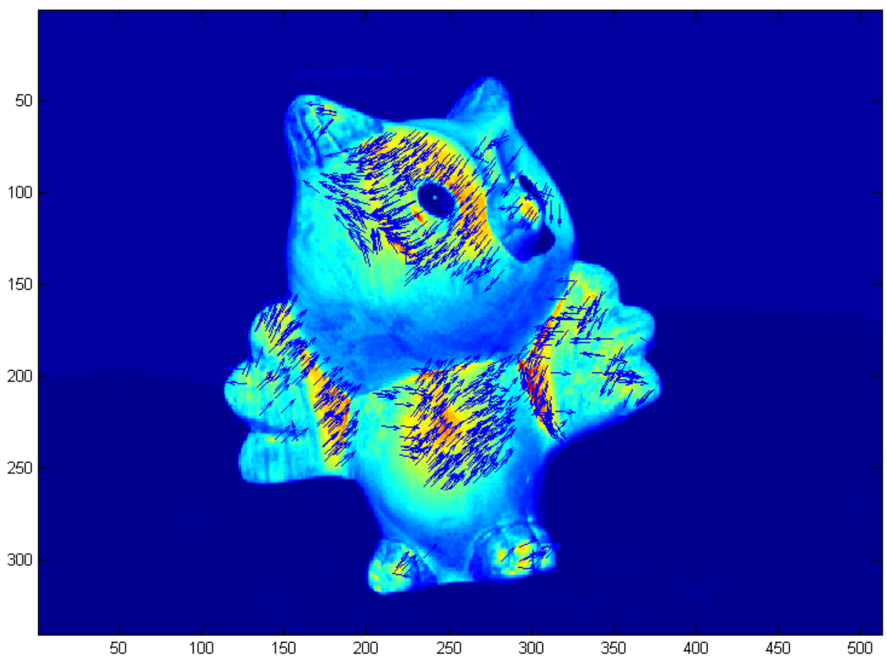
Here are the lighting directions L for images 1 to 11 with the sphere being chrome and specular. The format for specifying vectors is the same as above.

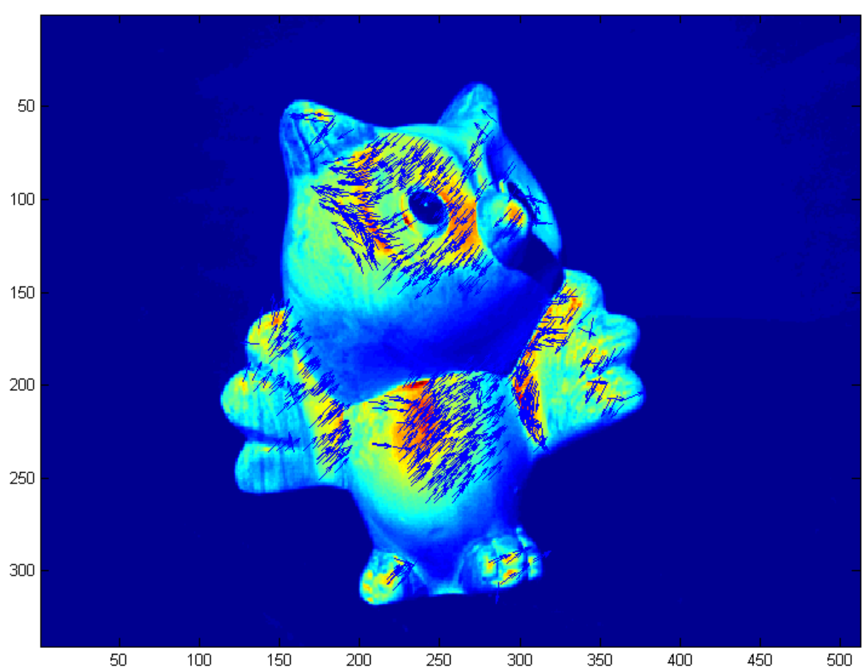
```
L =

   -0.1331    0.2495    0.9592
   -0.1841   -0.0335    0.9823
   -0.4414   -0.0981    0.8920
   -0.5101   -0.3188    0.7989
   -0.5612   -0.1122    0.8201
   -0.4213    0.2916    0.8588
   -0.4414    0.0981    0.8920
   -0.3293    0.2141    0.9196
   -0.3309    0.0993    0.9384
   -0.0503    0.1341    0.9897
   -0.3625   -0.1318    0.9226
```

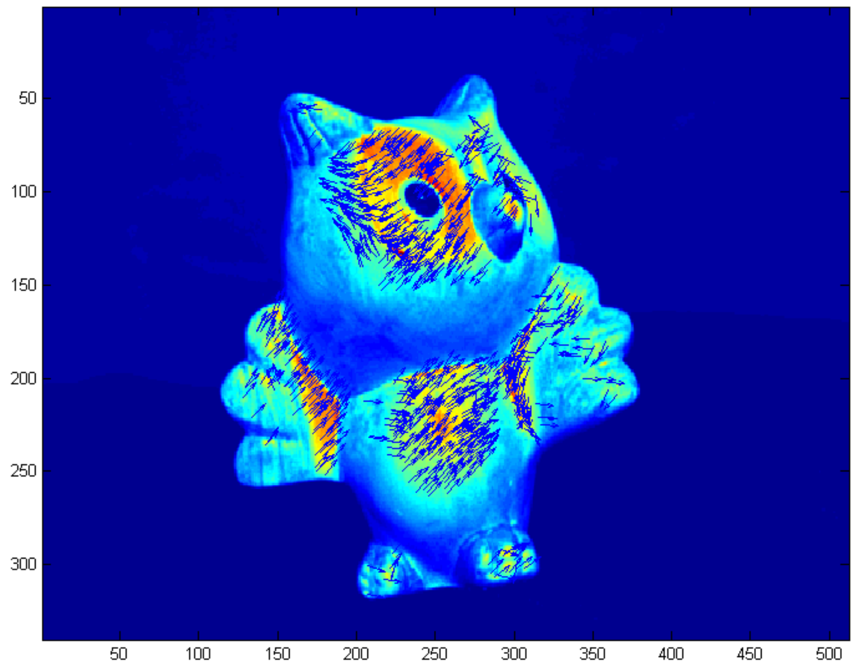
The code that I used to compute this is attached in prob2script.m

Problem 3

Here are a few of the needle diagrams with the owl images. The value I obtain for ρ_r is 0.5471. The code to compute it is attached in prob3script.m.



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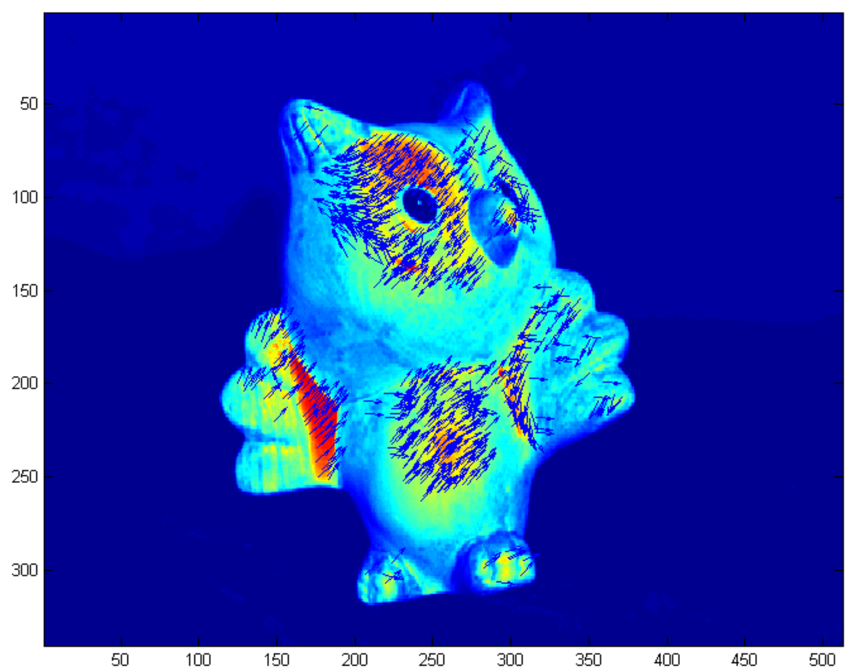
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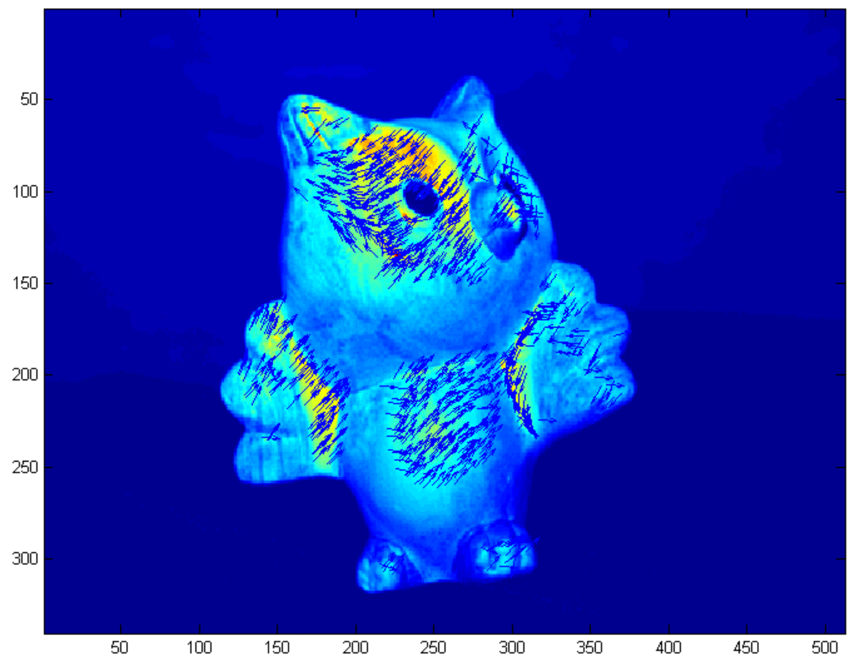
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Problem 4

I found the following average ρ values for this problem
$\rho_r = 0.5471$
$\rho_g = 0.6722$
$\rho_b = 0.6722$

The code to compute everything is in prob4script.m. It is mostly the same as the code from Problem 3, except that I vary the color channel selection. I also added code at the end to plot the ρ diagram.

Here are the needle diagrams for this problem. As can be observed, the normals point in roughly the same direction when a pixel has a normal in multiple channels.

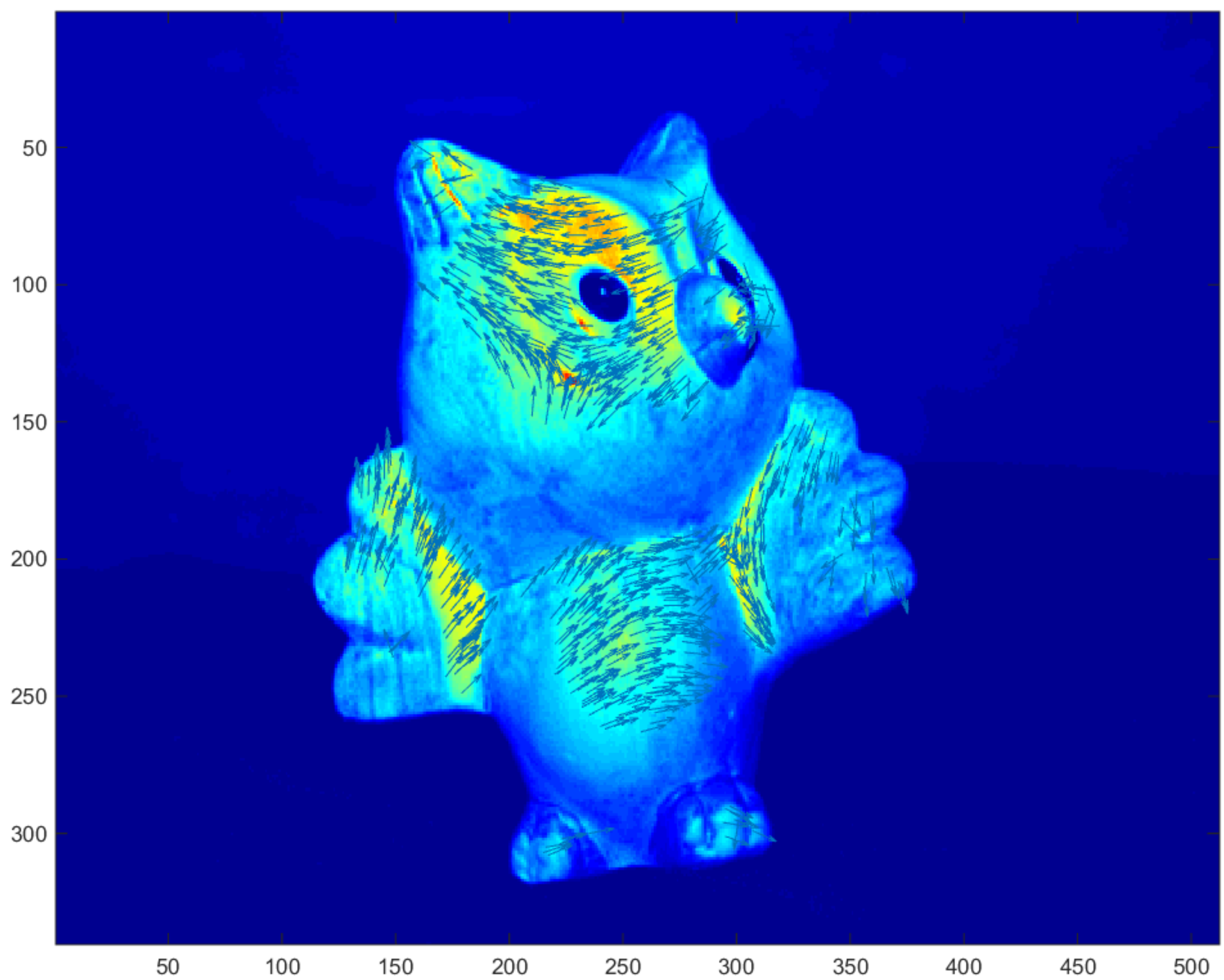

Figure 2: Needle Diagram for Red Channel

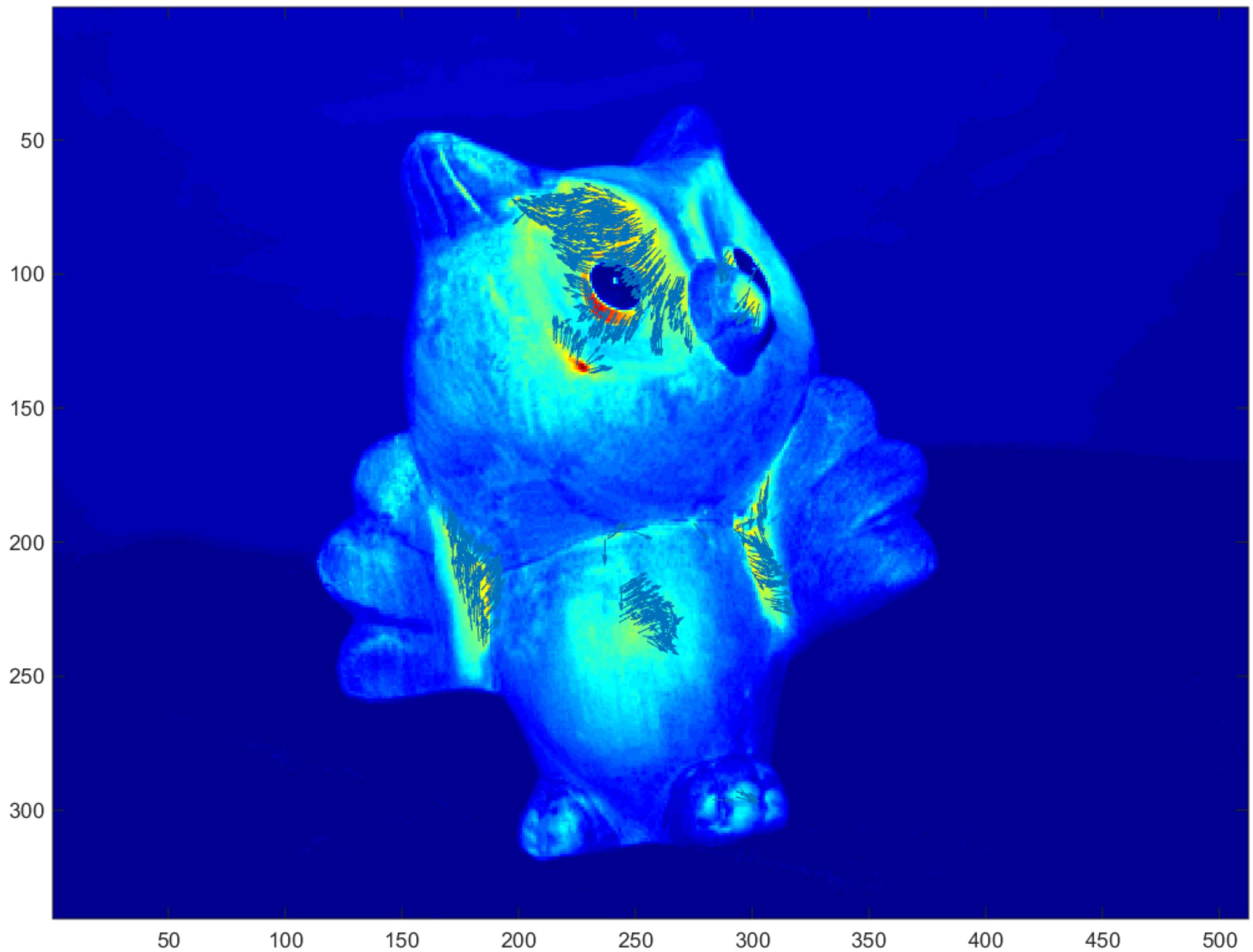

Figure 3: Needle Diagram for Green Channel

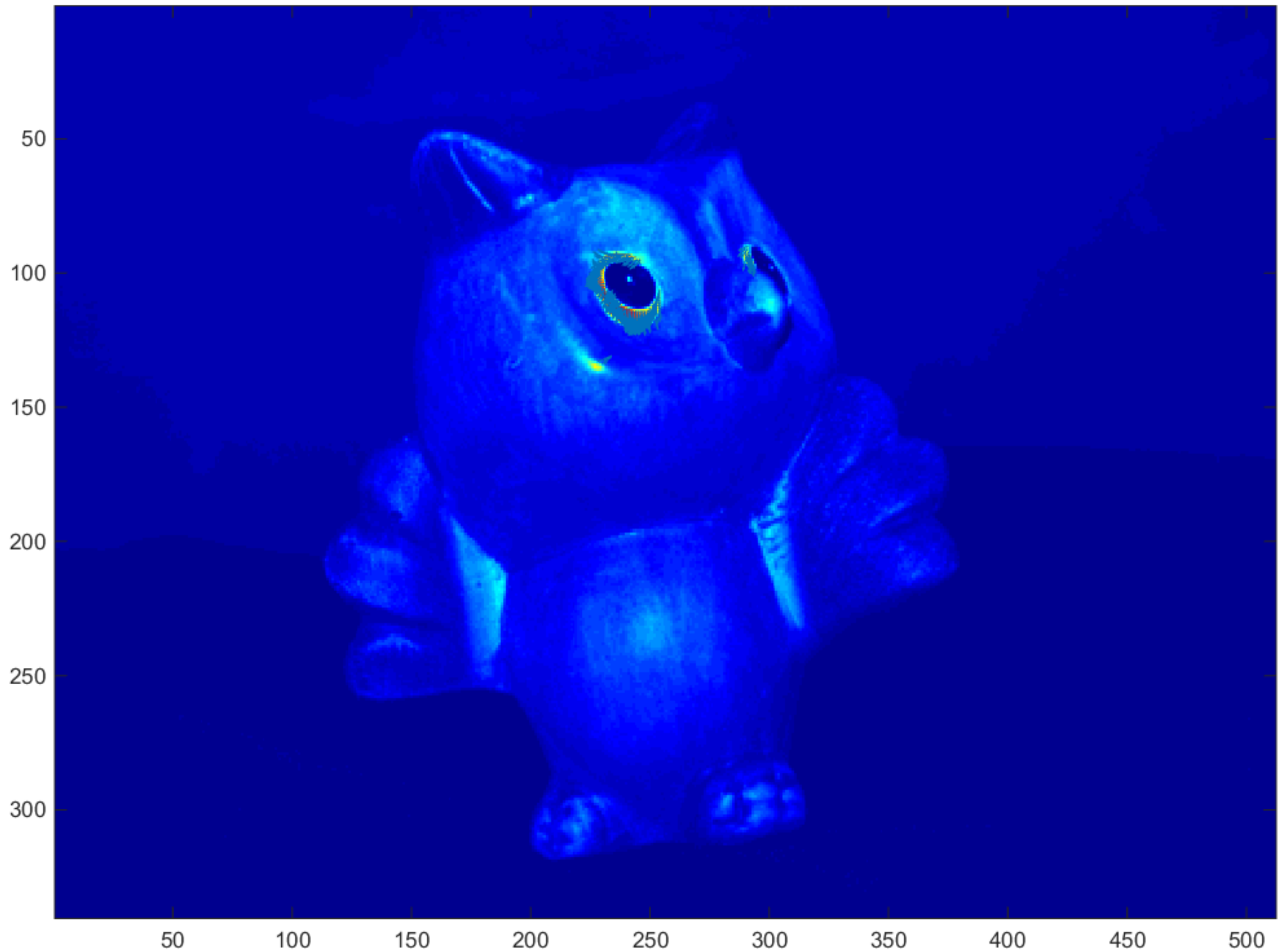

Figure 4: Needle Diagram for Blue Channel

Here are the figures for the ρ values at each pixel with each channel. I set $\rho = 0$ if it could not be computed at that pixel.

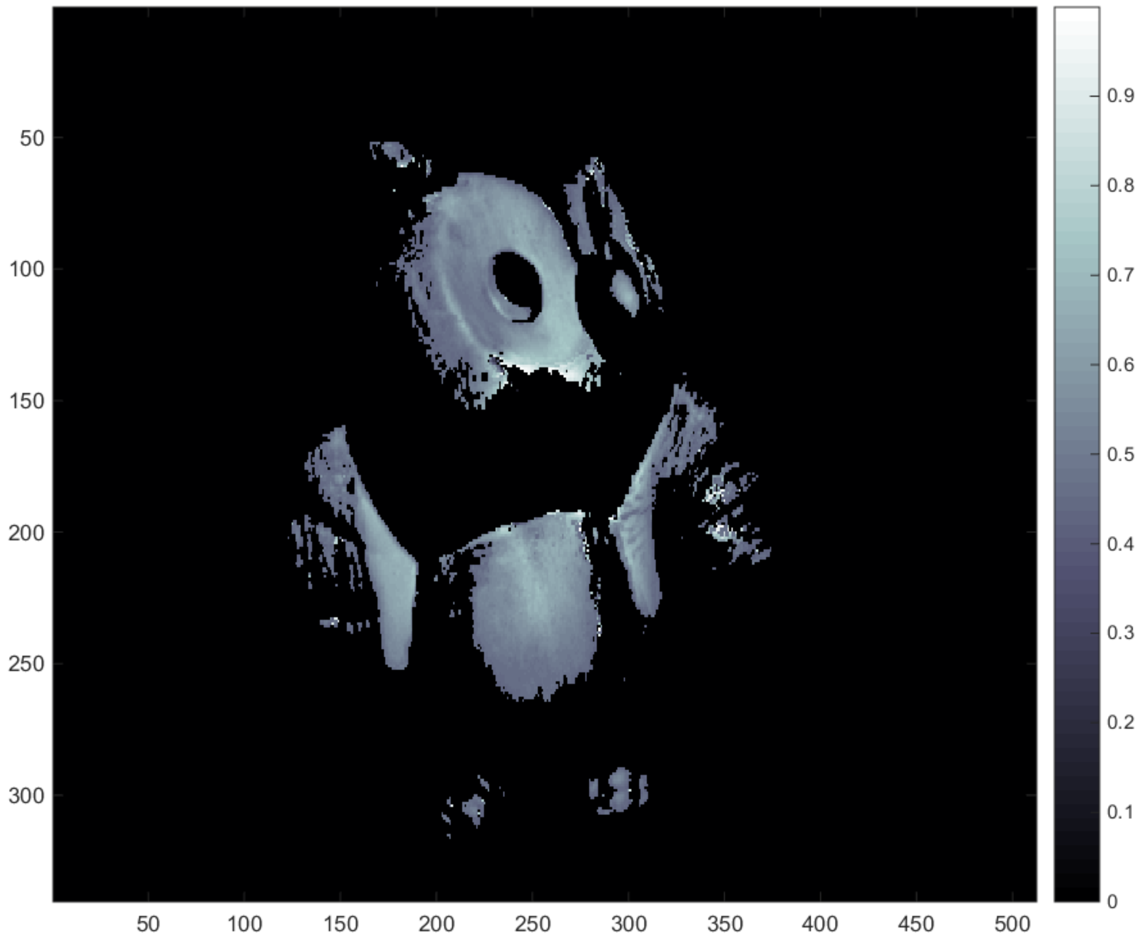

Figure 5: ρ Plot for Red Channel

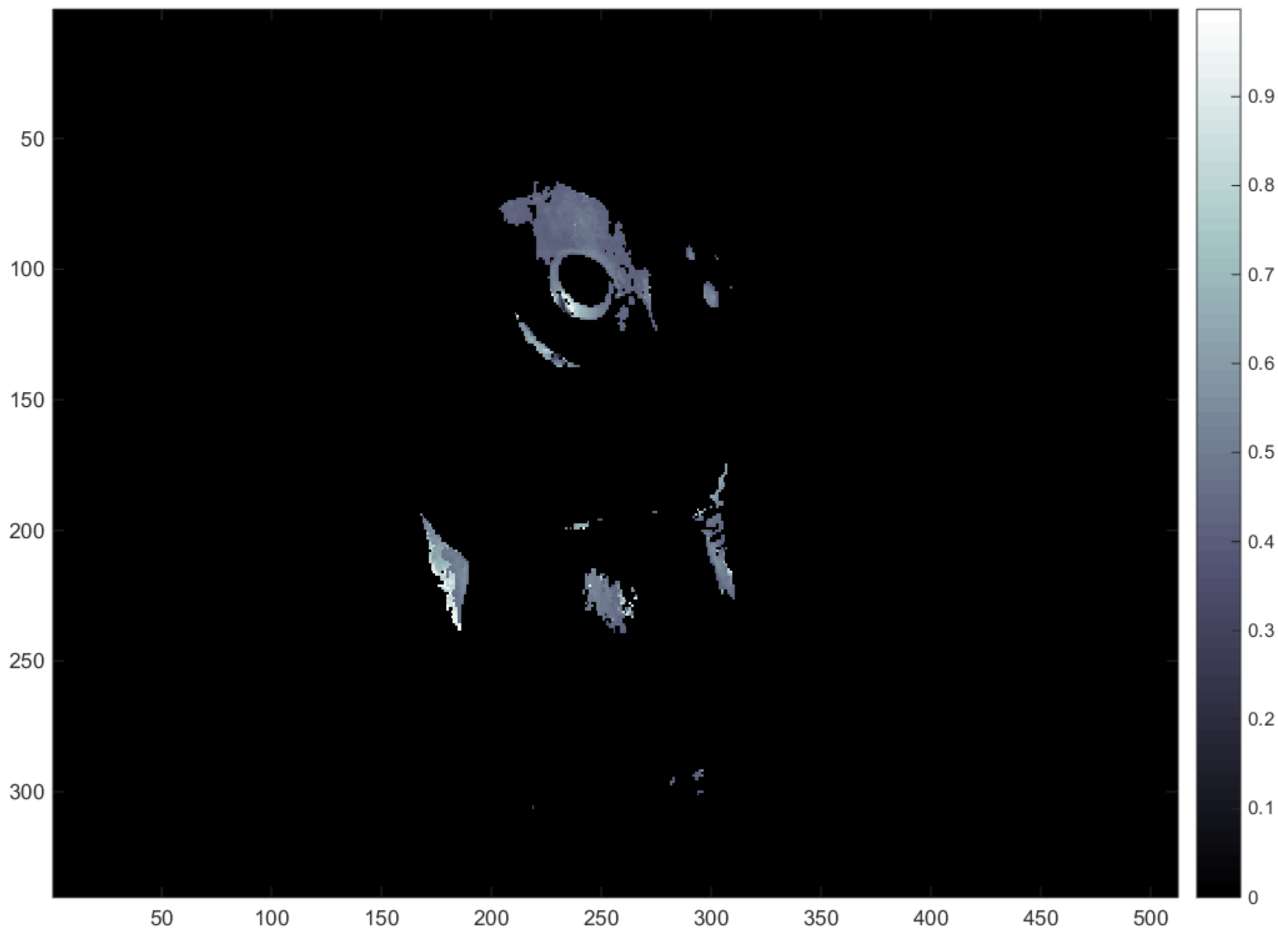

Figure 6: ρ Plot for Green Channel

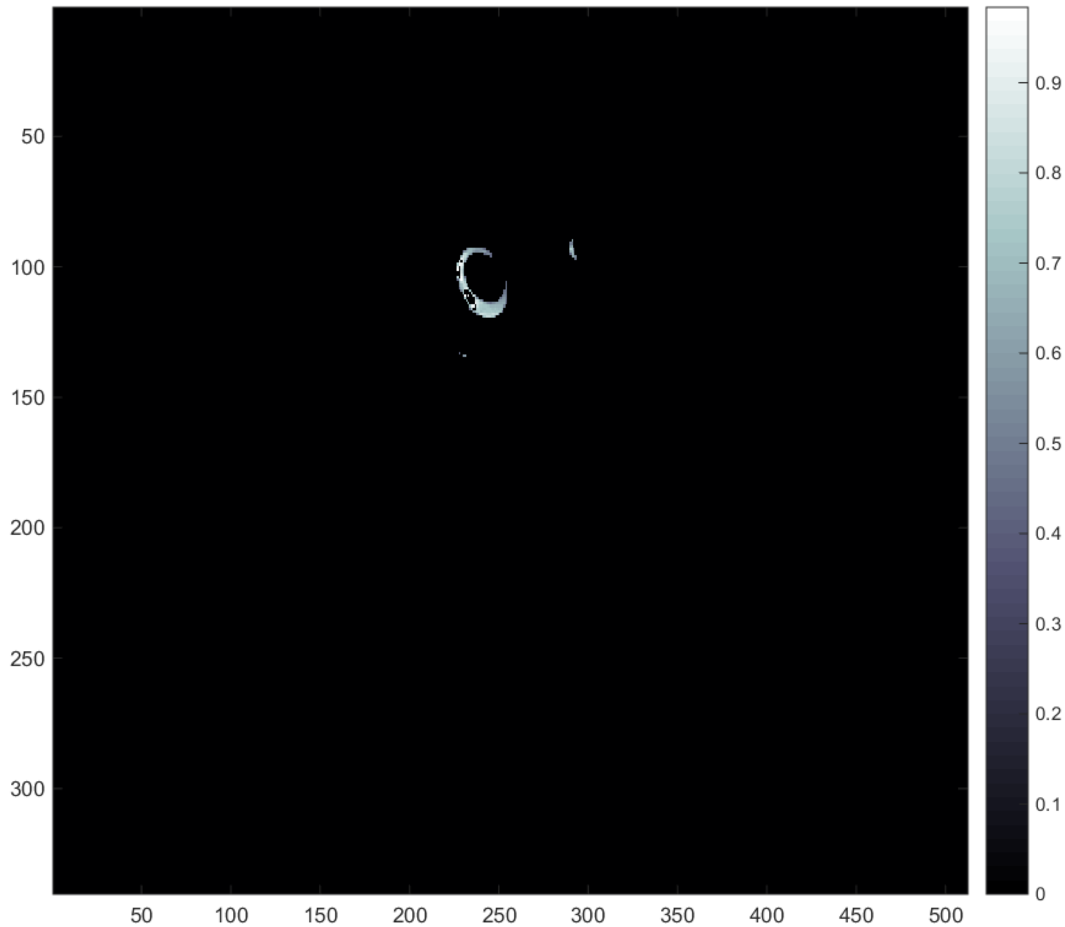

Figure 7: ρ Plot for Blue Channel