# CS 217 Homework 3

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# Problem 1

## Part 1

If $L$ is the direction of the light source, $N$ is the normal to the surface, we know that the intensity of the light $I$ is given by

$$I = |L||N|cos(\alpha)$$

where $\alpha$ is the angle between $L$ and $N$.
That equation is maximized when $cos(\alpha) = 1$ so that $\alpha = 0$
This means that the normal is pointing in the direction of the light source.
Thus the light source is in the direction of $(a, b)$.
To find the corresponding point on the sphere, we need to find $c$ such that $a^2 + b^2 + c^2 = r^2$

Since we are projecting the sphere onto the x-y plane, we are going to see the positive side, thus we will use the positive solution to the above equation. Thus our point on the sphere is as follows:

$$(a, b, \sqrt{r^2 - (a^2 + b^2)})$$

That vector has magnitude $r$ thus the unit vector for this point that points in the direction of the light source is as follows:

$$(\frac{a}{r}, \frac{b}{r}, \frac{\sqrt{r^2 - (a^2 + b^2)}}{r})$$

**Part 2**

Let $L$ be the direction of the light from $(a, b, c)$,
$N$ be the normal to the surface at $(a, b, c)$, and
$E$ be the vector pointing from $(a, b, c)$ to the viewer.

$L$ will be the reflection vector of $E$ from the surface with normal $N$ since we have specular reflection
As illustrated below it will hold that

$$L + E = 2N(E \cdot N)$$

Thus we have the formula for $L$

$$L = 2N(E \cdot N) - E$$

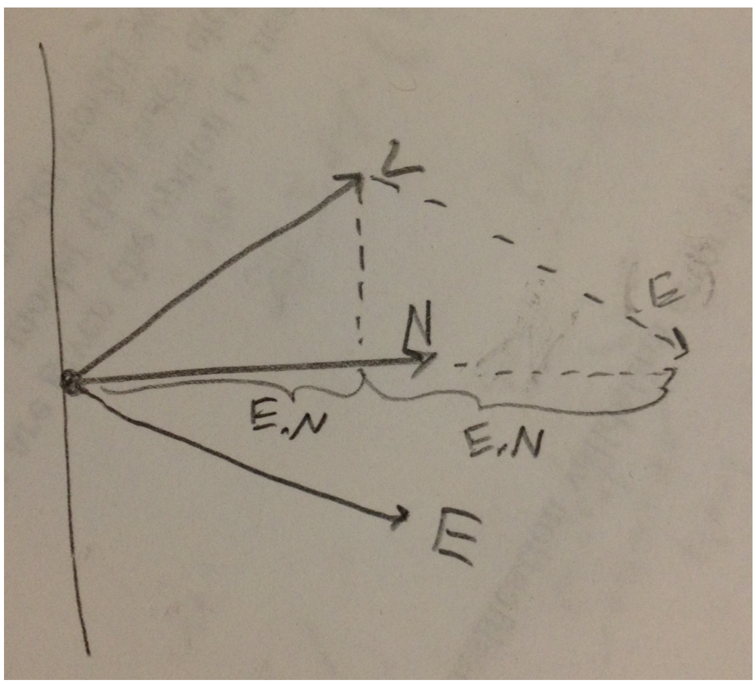



Figure 1: Illustration of normal vector N, light vector L, and viewing vector E and their relationships

E is parallel to the viewing direction thus
$E = (0, 0, 1)$
$N = \frac{1}{r}(a, b, c)$
We can thus say that $E \cdot N = \frac{c}{r}$, finally letting us say that

$$L = (\frac{2ac}{r^2}, \frac{2bc}{r^2}, \frac{2c^2}{r^2} - 1)$$

Now it holds that

$$||L||_2^2 = \frac{4a^2c^2 + 4b^2c^2 + 4c^4 - 4r^2c^2 + r^4}{r^4}$$

Factoring out $4c^2$ gives us

$$||L||_2^2 = \frac{4c^2(a^2 + b^2 + c^2 - r^2) + r^4}{r^4}$$

We know that $r^2 = a^2 + b^2 + c^2$ thus finally $||L||_2^2 = 1$
Proving that $L$ is a unit vector and thus in its final form

## Problem 2

For the images 1 to 11, here are the normal vectors in order at the bright spot, which would be the lighting direction if the sphere were diffuse. Each row indicates a vector in order $(x, y, z)$

```
N =

   -0.0672    0.1261    0.9897
   -0.0924   -0.0168    0.9956
   -0.2269   -0.0504    0.9726
   -0.2689   -0.1681    0.9484
   -0.2941   -0.0588    0.9540
   -0.2185    0.1513    0.9640
   -0.2269    0.0504    0.9726
   -0.1681    0.1092    0.9797
   -0.1681    0.0504    0.9845
   -0.0252    0.0672    0.9974
   -0.1849   -0.0672    0.9805
```

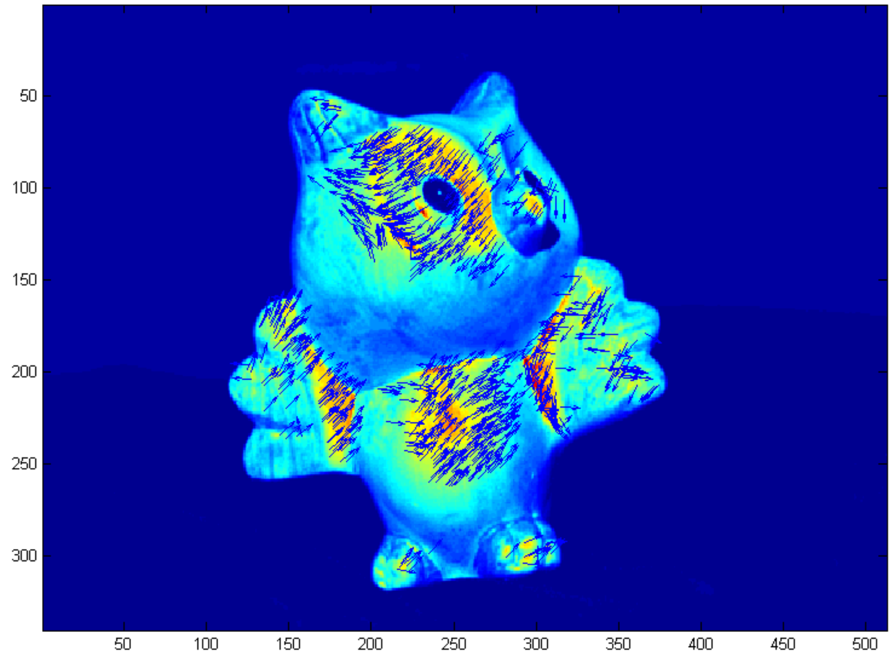
Here are the lighting directions $L$ for images 1 to 11 with the sphere being chrome and specular. The format for specifying vectors is the same as above.

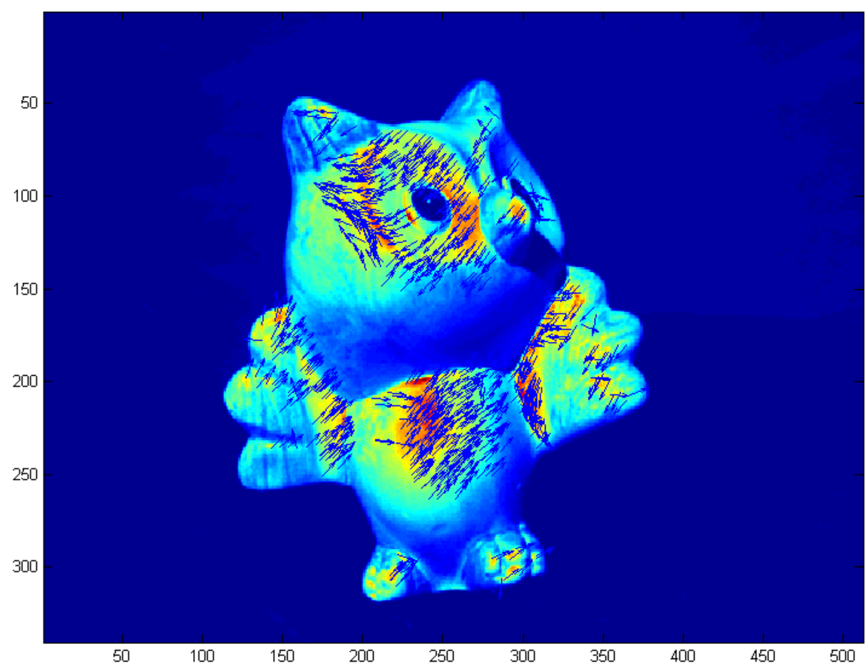
```
L =

   -0.1331    0.2495    0.9592
   -0.1841   -0.0335    0.9823
   -0.4414   -0.0981    0.8920
   -0.5101   -0.3188    0.7989
   -0.5612   -0.1122    0.8201
   -0.4213    0.2916    0.8588
   -0.4414    0.0981    0.8920
   -0.3293    0.2141    0.9196
   -0.3309    0.0993    0.9384
   -0.0503    0.1341    0.9897
   -0.3625   -0.1318    0.9226
```

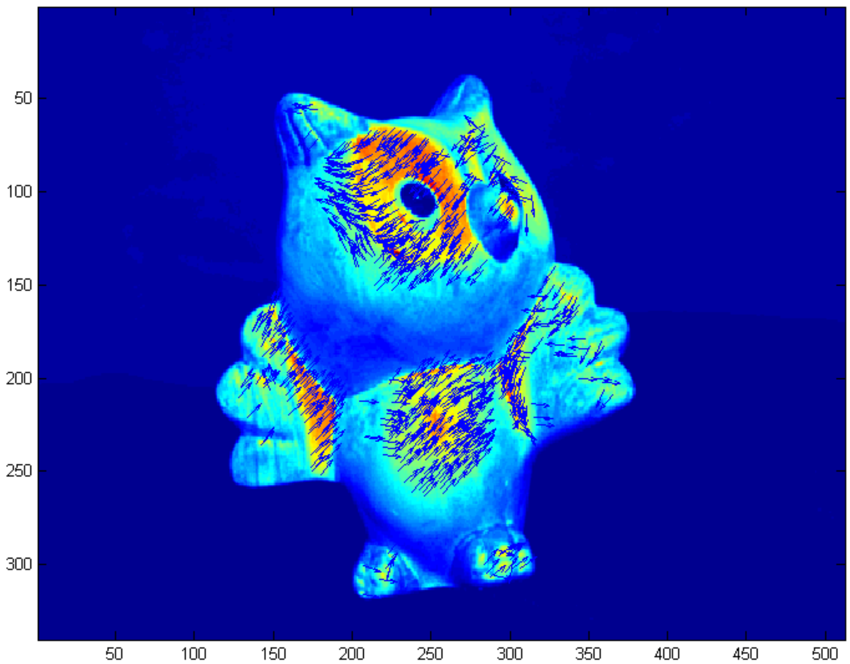
The code that I used to compute this is attached in prob2script.m

## Problem 3

Here are a few of the needle diagrams with the owl images. The value I obtain for $\rho_r$ is 0.5471. The code to compute it is attached in prob3script.m.

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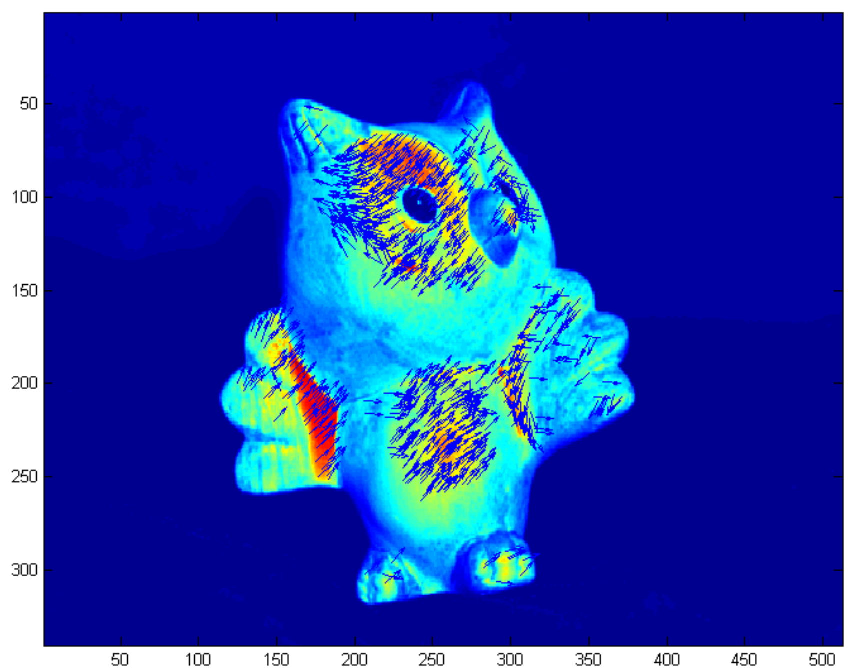
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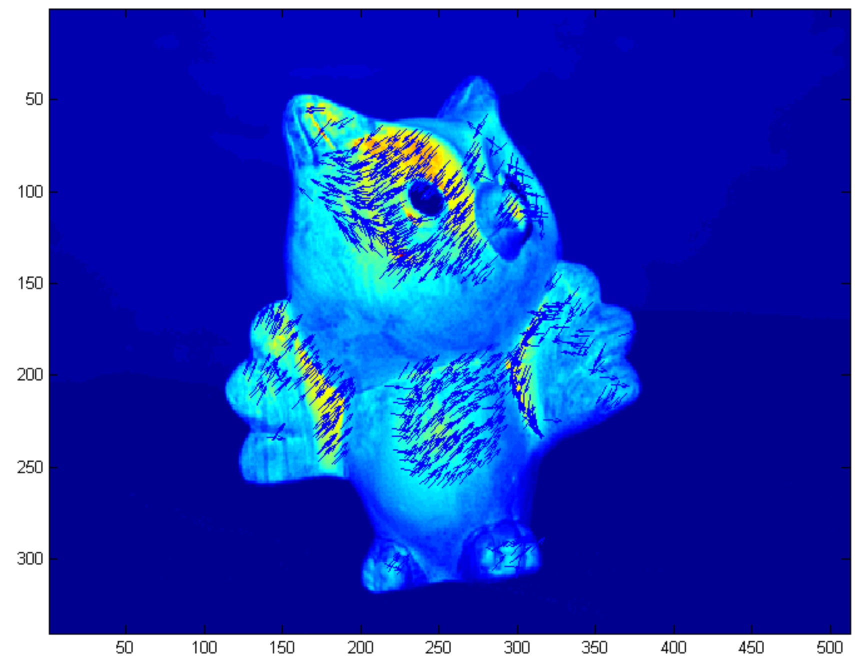
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# Problem 4

I found the following average $\rho$ values for this problem
$\rho_r = 0.5471$
$\rho_g = 0.6722$
$\rho_b = 0.6722$

The code to compute everything is in prob4script.m. It is mostly the same as the code from Problem 3, except that I vary the color channel selection.

Here are the needle diagrams for this problem. As can be observed, the normals point in roughly the same direction when a pixel has a normal in multiple channels.

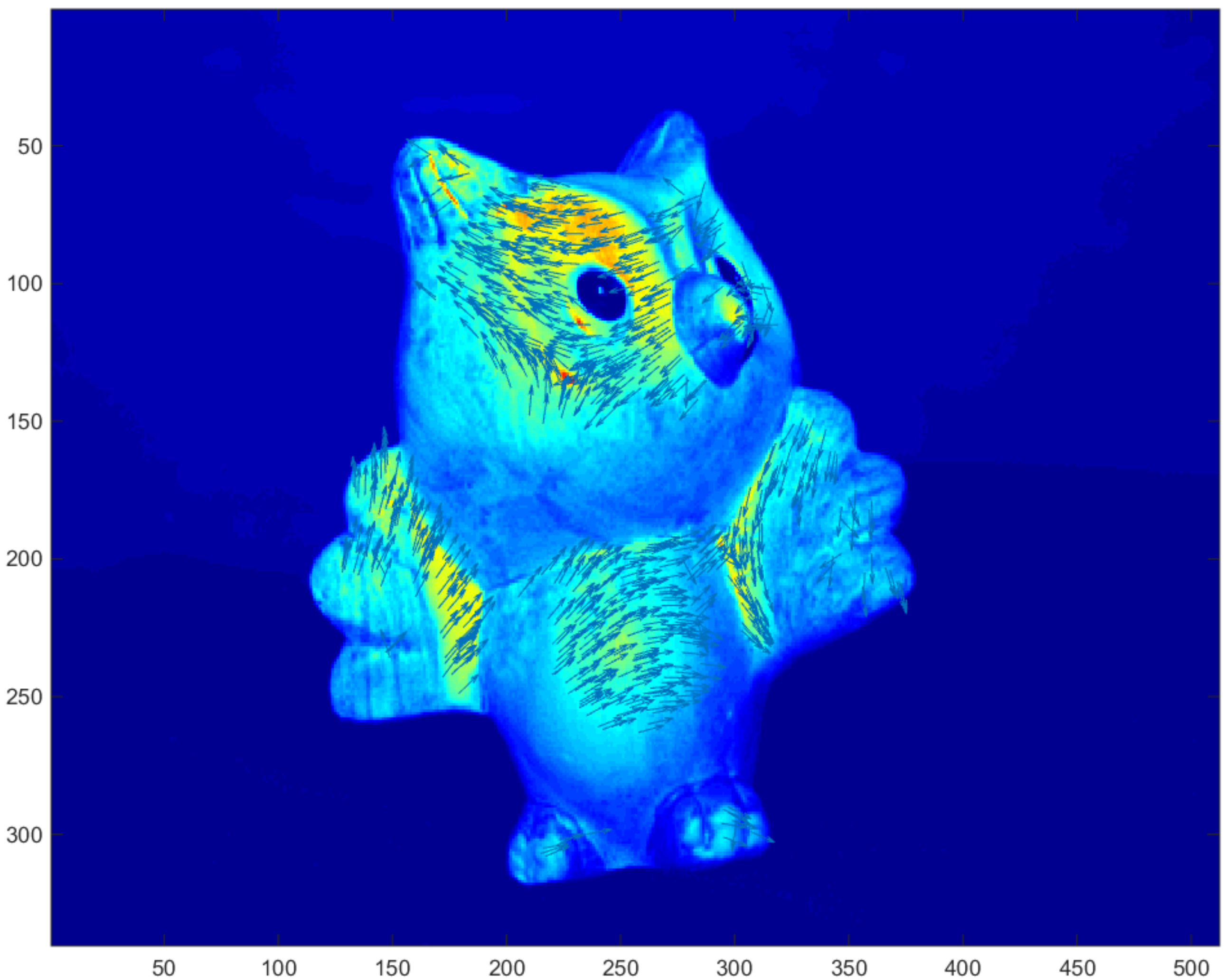


Figure 2: Needle Diagram for Red Channel

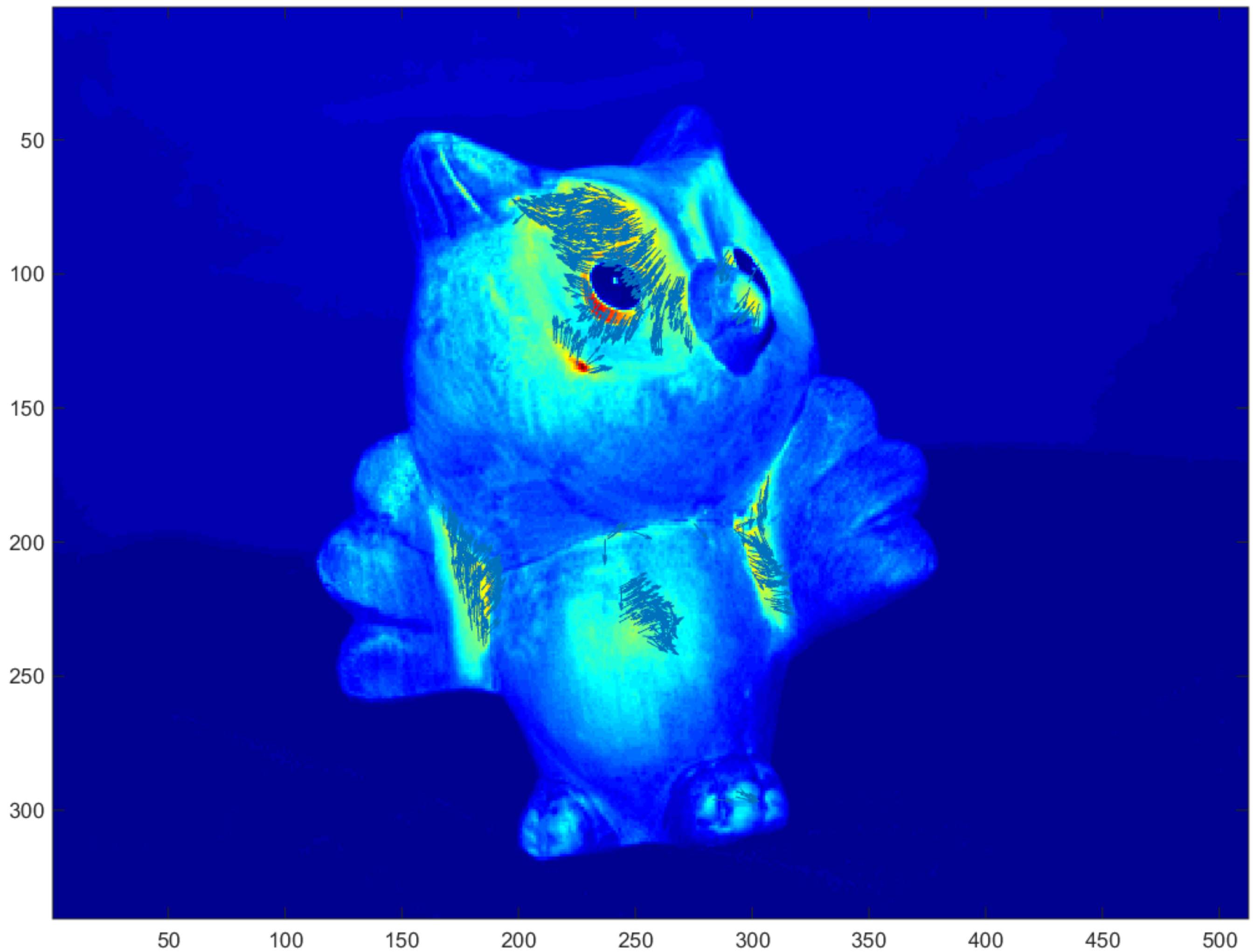


Figure 3: Needle Diagram for Green Channel

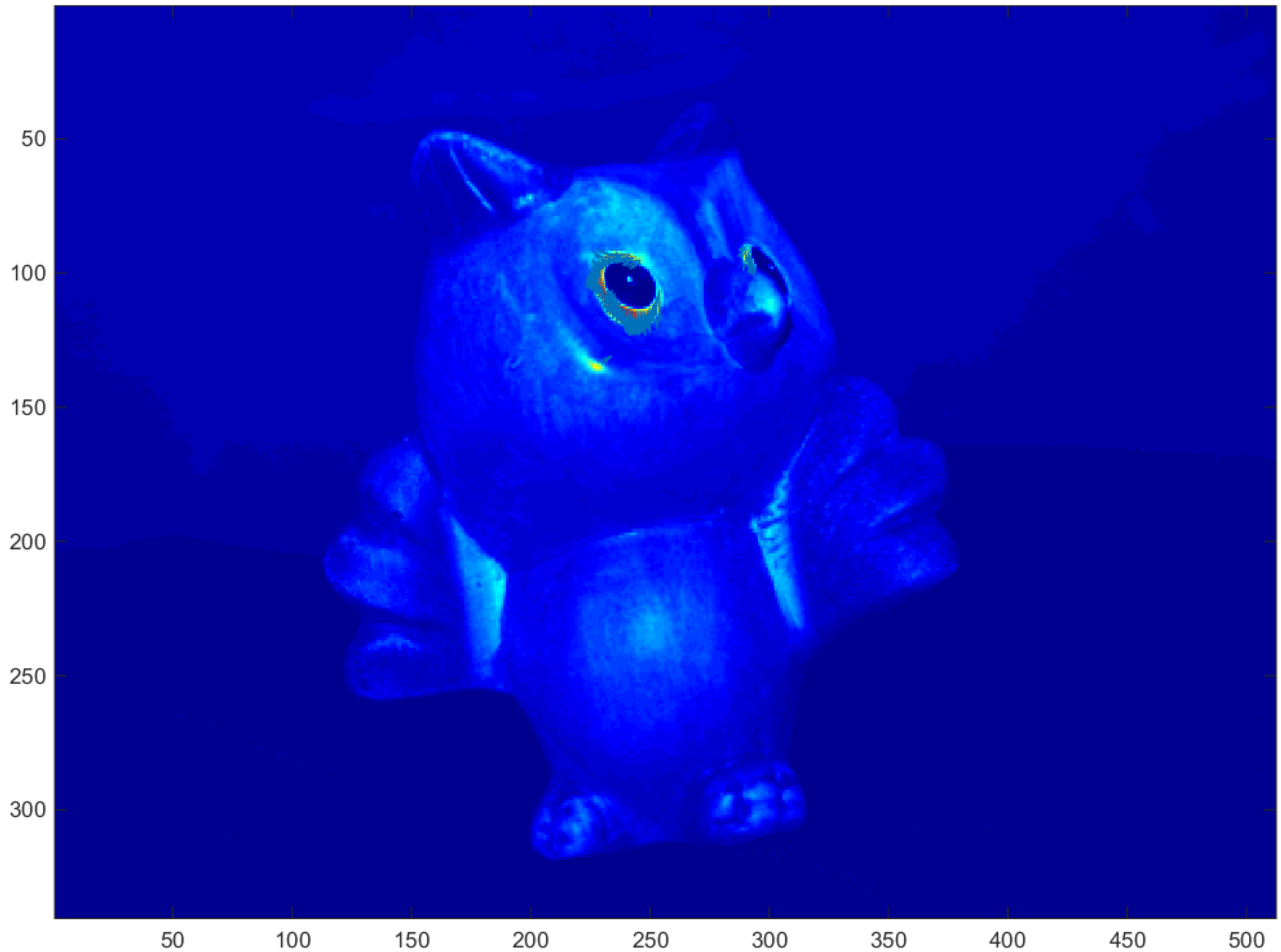


Figure 4: Needle Diagram for Blue Channel

Here are the figures for the $\rho$ values at each pixel with each channel. I set $\rho = 0$ if it could not be computed at that pixel.

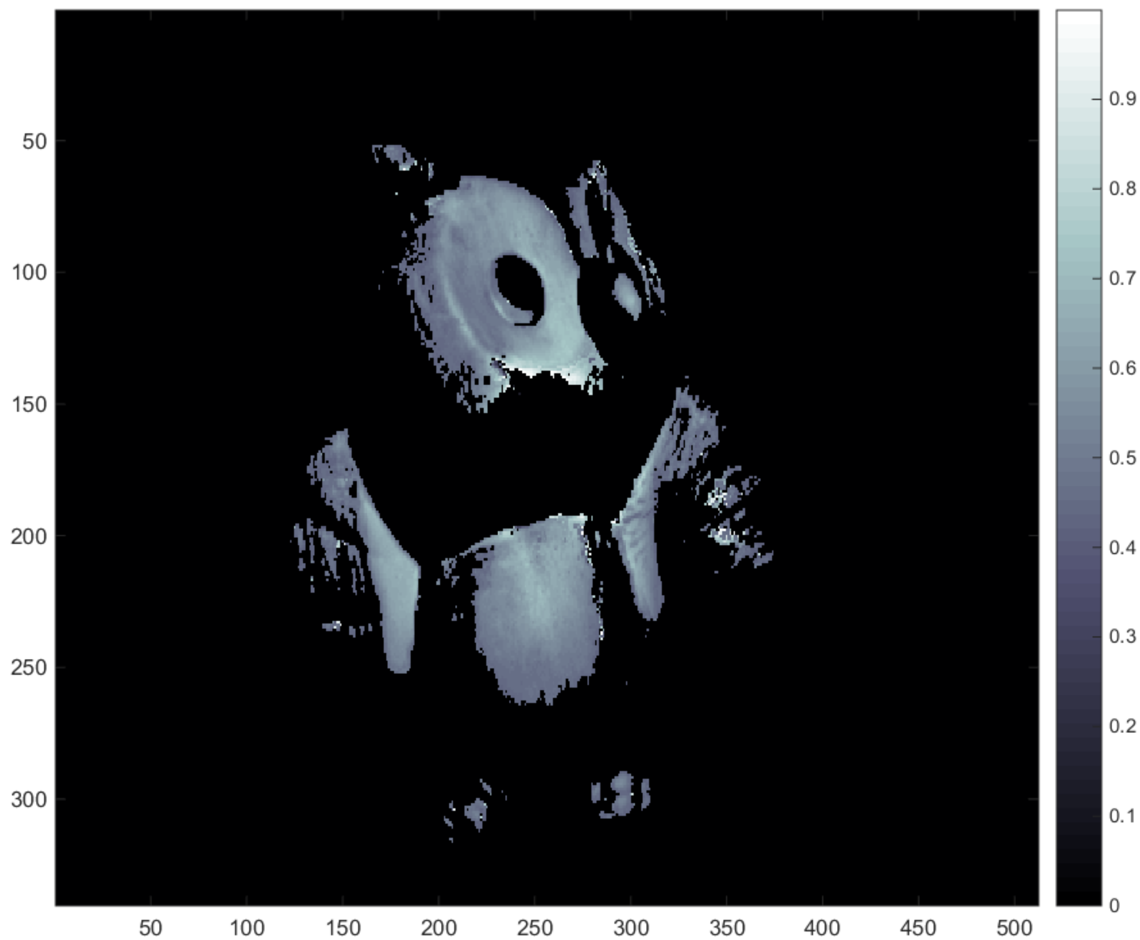


Figure 5: $\rho$ Plot for Red Channel

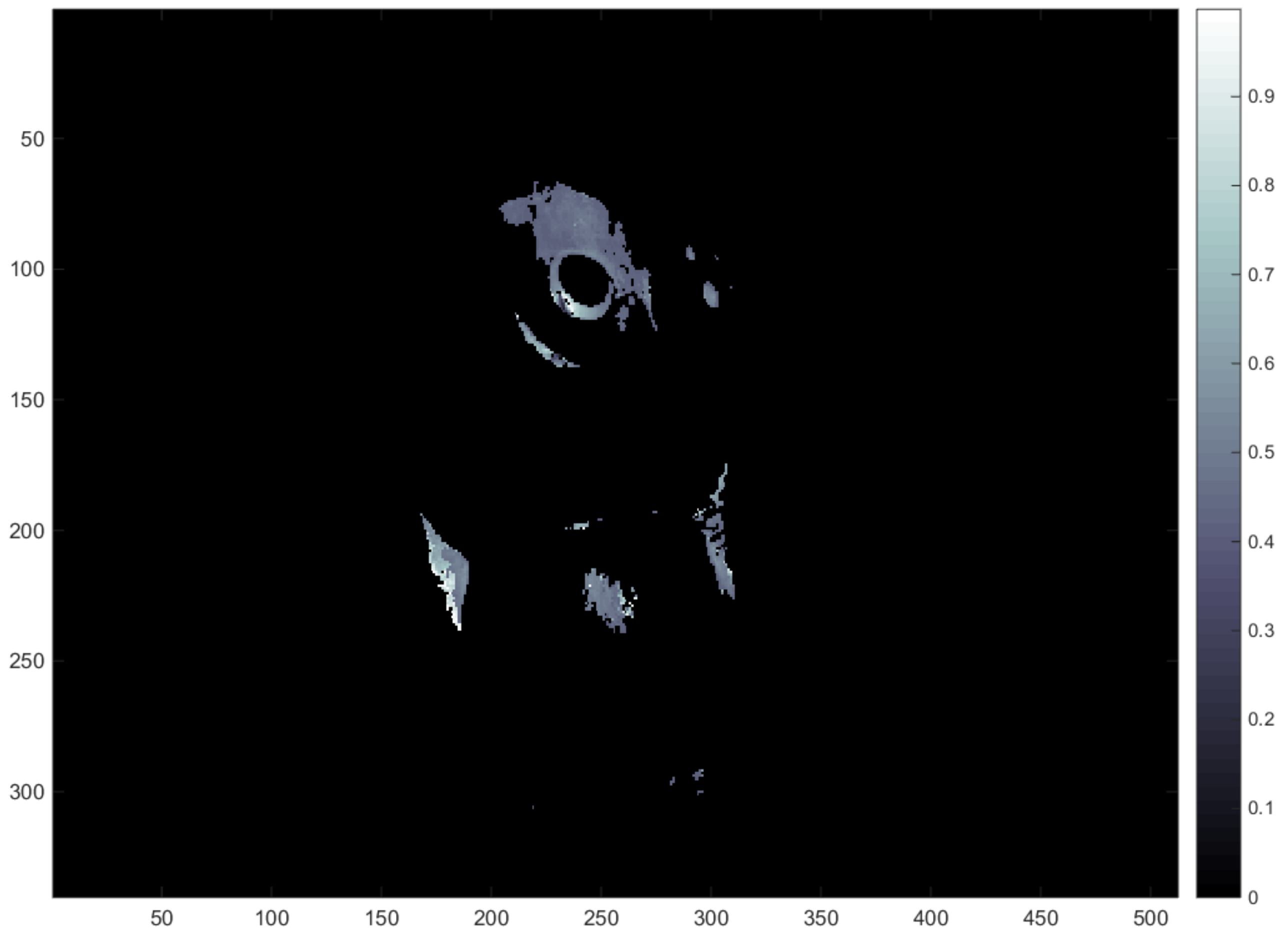


Figure 6: $\rho$ Plot for Green Channel

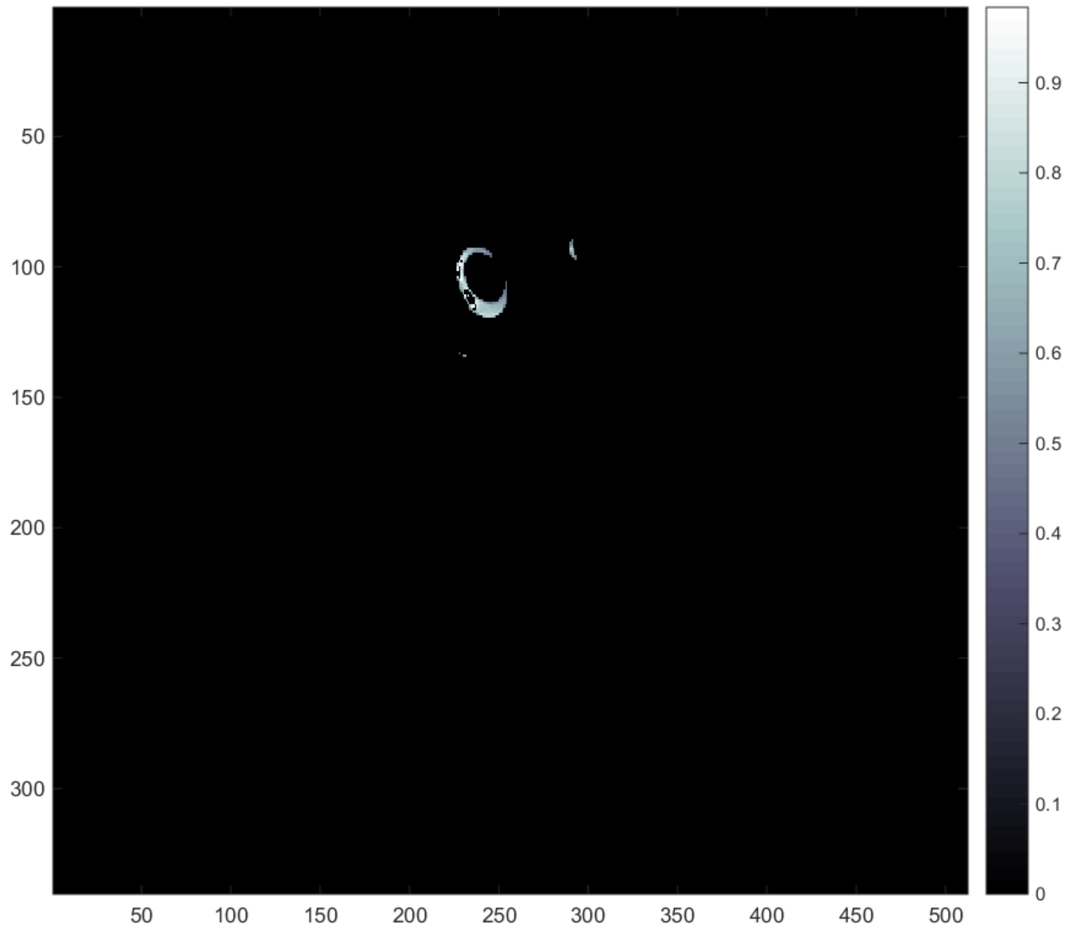


Figure 7: $\rho$ Plot for Blue Channel