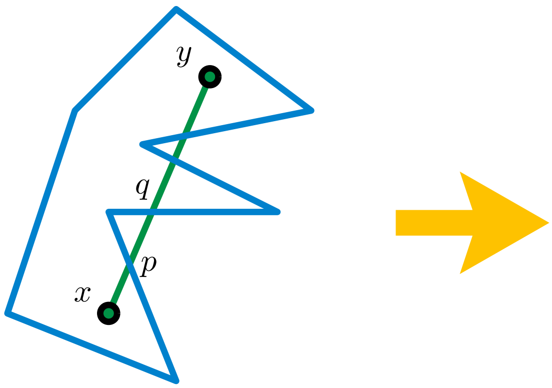
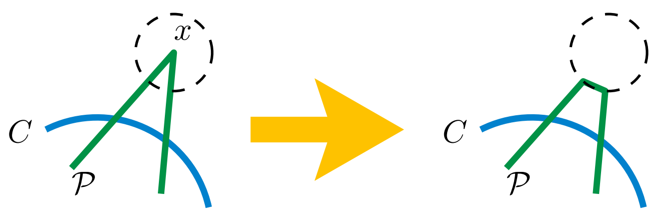
1.1 (a) Let A and B be convex sets, and $C = A \cap B$. If $x, y \in C$ and z is a point on the segment with endpoints x and y, then $z \in A$ and $z \in B$ as $x, y \in A$ and $x, y \in B$, which implies $z \in C$. Thus, C is a convex set.

(b) Let $\mathcal{P}$ be a non convex simple polygon. Since $\mathcal{P}$ is not convex there exists points $x, y \in \mathcal{P}$ such that the segment ℓ from x to y is not contained in P. Let p and q be the first and second intersection of ℓ with $\mathcal{P}$, order from x to y. Now shortcut the path from p to q on $\mathcal{P}$ with straight line segment between them. This operation shortens the perimeter of the polygon without removing any points the original polygon contained. Thus, the polynomial of smallest perimeter containing a set points must be convex.



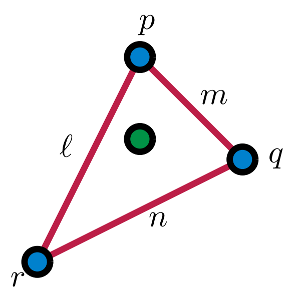
(c) Let $\mathcal{P}$ be the smallest perimeter polygon containing P, and C a convex set containing P. Suppose that P is not fully contained in C. Then there must be a vertex of $\mathcal{P}$ not in C, say x. For a small enough r we can draw a circle around x that does not intersect C. Within this circle we can shorten the perimeter of $\mathcal{P}$ be removing x and connecting the two intersection points of the circle, a contradiction.

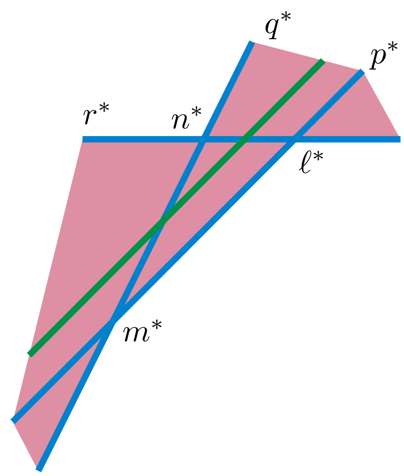


1.9 From S construct the point set $P = \{(x, x^2) \mid x \in S\}$. The convex hull of P contains every point in P and they are sorted by their x value. Thus we have sorted S.

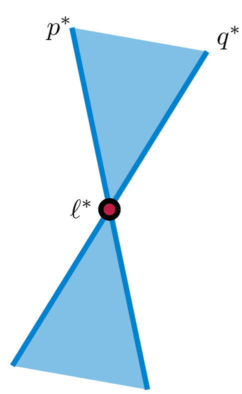
Comment: There are many other solutions to this problem.

8.2 From the book we know that the dual of a face of the triangle is a double wedge. So the boundary of the triangles is the union of three double wedges. Interior points of the triangle dualize to lines contained in this region.





The intersection point ℓ^* of the vertical double wedge corresponds to a line ℓ in the primal plane containing both of the duals of the boundary lines p and q of the double wedge. The object that dualizes to the vertical double wedge is the infinite line ℓ minus the segment from p to q.



8.6 The dual problem is to determine if a line in S^* contains a point from L^*, where S^* contains the duals of points in S and L^* contains the duals of lines in L.

1.6 (a) Let C be the convex hull of the n line segments and let D be the convex hull of $2n$ endpoints. Each of the segments must be contained in D, as the endpoints are in D and D is convex, thus $C \subseteq D$. Conversely, since each of the $2n$ endpoints must be contained in C, as C contains the entire segment, we have $D \subseteq C$. Therefore, $C = D$, which was to be shown.

(b) This problem is a little tricky, and has an interesting history of incorrect and correct solutions, see `http://cgm.cs.mcgill.ca/~athens/cs601/` for more details.

1.10 (a) We will use the following fact

A point p is on the boundary of the convex hull of A if and only if there exists a line ℓ through p such that all of A is contained in one of the closed half-planes defined by ℓ.

If such a line ℓ intersects only one circle, then an arc around that point is on the boundary. If the line intersects more than one point, then the line segment between the furthest two endpoint is on the boundary. Since these are the only cases possible the boundary is made up of arcs and line segments.

(b) Given an arbitrary circle in S we show how to determine which points if any are on the boundary. If C_1 and C_2 are two circles in S, then we can form their convex hull by connecting their two common tangent lines. The closed half circles between the tangent lines will not be on the boundary. So for a fixed circle C every other circle will remove a closed half. The union of all these closed halves is a closed arc. The complement of this arc, is an arc on the boundary. since the complement of an arc is another arc, at most one arc from a given circle is on the boundary.

(c) Suppose a a point p on the circle C in on the boundary. Then there is a line ℓ through p with all the circles in S contained on one side of ℓ. Consider the line ℓ' through the center of C parallel to ℓ. The gap between ℓ and ℓ' has width 1. So all the centers must lie on one side of ℓ' for otherwise they would be in the gap between ℓ and ℓ' and violate the defining property of ℓ. Hence the center of C is on the boundary of the convex hull of centers. The other direction follows by a trivial reversal of the argument.

(d) By part (c) we can compute the convex hull of the centers, then convert the centers into circles. To get the line segments we traverse the circles in order taking the appropriate common tangent.

(e) See the paper "Incremental algorithms for finding the convex hulls of circles and the lower envelopes of parabolas" by Olivier Devillersa and Mordecai J. Golin.