CS 266 Homework 2

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Problem 2.3

Change the code of Algorithm FINDINTERSECTIONS (and of the procedures that it calls) such that the working storage is O(n) instead of O(n+k).

Problem 2.11

Let S be a set of n circles in the plane. Describe a plane sweep algorithm to compute all intersection points between the circles. (Because we deal with circles, not discs, two circles do not intersect if one lies entirely inside the other.) Your algorithm should run in O((n+k) logn) time, where k is the number of intersection points.
First, two circles will intersect in at most two points if they are not the exact same circle. Here is the proof: Any two circles can be translated and rotated so that one of the centers is the origin and the other center is on the x-axis. Thus assume the two circles have centers $(0, 0)$ and $(a, 0)$ and radii of r_1 and r_2. We will assume distinct centers so that $a \neq 0$. The two circles are thus described by:

$$x^2 + y^2 = r_1^2$$
$$(x - a)^2 + y^2 = r_2^2$$

Let $R = r_1^2 - r_2^2$. After subtracting the two equations we have

$$2ax - a^2 = R$$

We can turn this into

$$x = \frac{a^2 + R}{2a}$$

If $x > r_1$ or $x < -r_1$ then we know there is no intersection point. If $x = r_1$ or $x = -r_1$ then there is 1 intersection point. If $-r_1 \leq x \leq r_1$, then from the equation there is one matching x which means two matching (x, y) pairs, thus two intersection points. Since $a \neq 0$ we do not have to worry about any more intersections.

Problem 8.4

Let L be a set of n lines in the plane. Give an O(nlogn) time algorithm to compute an axis-parallel rectangle that contains all the vertices of A(L) in its interior.

Problem 8.14

Let S be a set of n points in the plane. Give an O(n2) time algorithm to find the line containing the maximum number of points in S.