

Homework 1
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CS 274B: Spring & 2016
Due: April 15, 2016

Problem 1:

I will assume that each random variable can take on d values.

Part A

To satisfy $p(W|X, Y, Z)$, we need $d - 1$ parameters for values of W and each of those values are conditioned on d^3 possible configurations of X, Y, Z , thus we need $d^3(d - 1)$ parameters.

To satisfy $p(Z|X, Y)$, we need $d^2(d - 1)$ parameters because we have $d - 1$ parameter values each conditioned on d^2 configurations.

To satisfy $p(Y|X)$, we need $d(d - 1)$ parameters because we have $d - 1$ parameter values each conditioned on d configurations.

To satisfy $p(X)$, we need $d - 1$ parameters because we have $d - 1$ parameter values.

Our total is thus

$$\frac{d^4 - 1}{d - 1}(d - 1)$$

Simplifying, our final total is

$$d^4 - 1$$

Thus this Bayesian network does not simplify the joint distribution

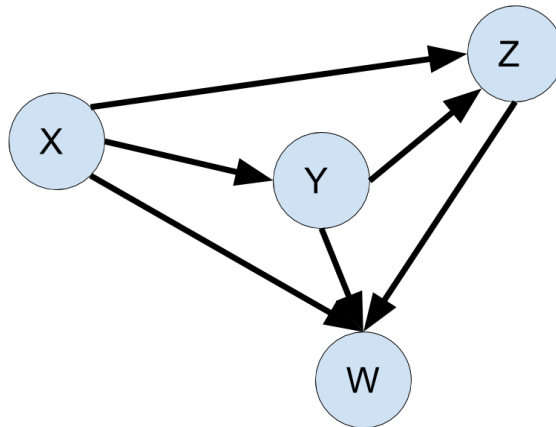


Figure 1: Minimal Directed Graphical Model for Part A

Part B

For each random variable, we need $d - 1$ parameters.

They are all independent

Thus our total is just $4(d - 1)$

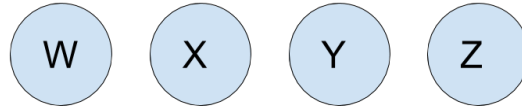


Figure 2: Minimal Directed Graphical Model for Part B

Part C

The variables $p(Z|Y)$, $p(W|Y)$, $p(X|Y)$ each need $d(d - 1)$ parameters since we have $d - 1$ parameter values conditioned on d configurations.

The factor $p(Y)$ needs $d - 1$ parameters

Thus our total is $(3d + 1)(d - 1)$

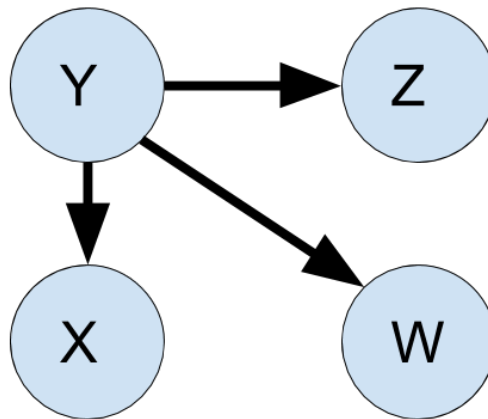


Figure 3: Minimal Directed Graphical Model for Part C

Part D

To satisfy $p(X)$ and $p(Y)$ we need $d - 1$ parameters for each of them

To satisfy $p(W|X)$ we need $d(d - 1)$ parameters since there are $d - 1$ parameter values conditioned on d configurations.

To satisfy $p(Z|X, Y)$ we need $d^2(d - 1)$ parameters since there are $d - 1$ parameter values conditioned on d^2 configurations.

Our total is thus

$$(d^2 + d + 2)(d - 1) = (d^2 + d + 1)(d - 1) + (d - 1) = (d^3 - 1) + (d - 1) = d^3 + d - 2$$

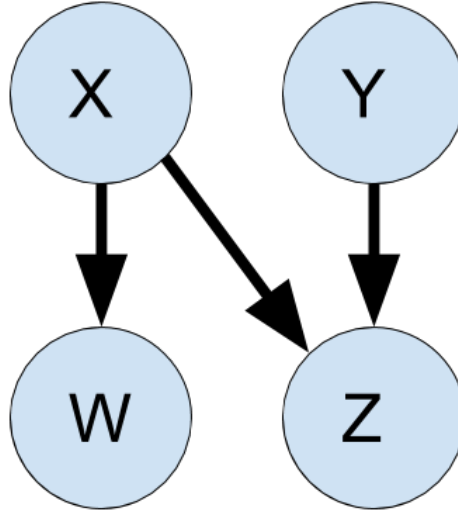


Figure 4: Minimal Directed Graphical Model for Part D

Part E

To satisfy $p(Z)$ we need $d - 1$ parameters

To satisfy $p(Y|Z)$ we need $d(d - 1)$ parameters as there are $d - 1$ values conditioned on d configurations.

To satisfy $p(X|Y)$ we need $d(d - 1)$ parameters for same reason as $p(Y|Z)$

To satisfy $p(W|X)$ we need $d(d - 1)$ parameters for same reason as $p(Y|Z)$

Our total is thus $(3d + 1)(d - 1)$ parameters

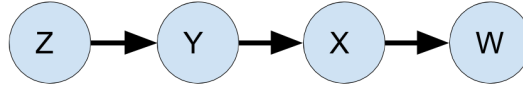


Figure 5: Minimal Directed Graphical Model for Part E

Part F

To satisfy $p(X)$ we need $d - 1$ parameters

To satisfy the other three factors, we need $d(d - 1)$ parameters for each of them for the same reason as $p(Y|Z)$ in part E

Our total is thus $(3d + 1)(d - 1)$ parameters

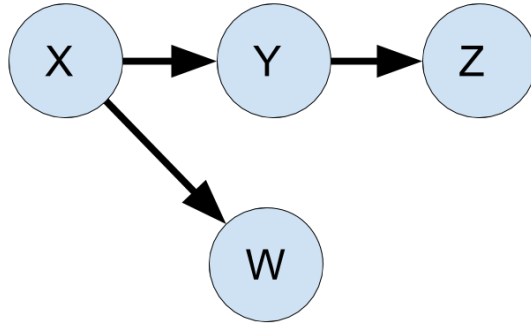


Figure 6: Minimal Directed Graphical Model for Part F

Problem 2:

Part A

No

There is a "vee" structure between them and the variables are unobserved

Thus all paths are inactive and they are conditionally independent

Part B

Yes

This allows you to infer new values for *power_in_building*

This will in turn mean new probabilities for *Sam_reading_book*

Part C

Yes

Observing a value for *screen_lit_up* lets you infer information about the probability of *power_in_wire*. Knowledge of *projector_plugged_in* combined with the information about *power_in_wire* will affect *power_in_building*. This will in turn affect *Sam_reading_book* because it is connected through a chain to *power_in_building*.

Part D

If *lamp_works* was observed, then we would update the probabilities for *projector_lamp_on*

We would then have to update the probabilities for *screen_lit_up*

This would cause us to update probabilities for *ray_says_screen_is_dark*

Part E

If we observe just *power_in_projector* then the same variables from Part D will have their probabilities changed.

We would also update the probability for *projector_switch_on*

We would also have to update *power_in_building* and *power_in_wire*

It would propagate and affect *Sam_reading_book* and *room_light_on*

Problem 3:

Part A

We need to solve the following

$$p(0, 0; \theta) + p(0, 1; \theta) + p(1, 0; \theta) + p(1, 1; \theta) = 1$$

This ends up being the following:

$$\exp(-A(\theta)) + \exp(-A(\theta)) + \exp(\theta_x - A(\theta)) + \exp(\theta_x + \theta_{xy} - A(\theta)) = 1$$

After doing some factoring

$$\frac{\exp(\theta_x) + \exp(\theta_{xy} + \theta_x) + 2}{\exp(A(\theta))} = 1$$

After cross multiplying and solving for $A(\theta)$

$$A(\theta) = \log(\exp(\theta_x) + \exp(\theta_{xy} + \theta_x) + 2)$$

Part B

After letting $\theta_{xy} = 1$ we have the following

$$A(\theta) = \log(\exp(\theta_x) + \exp(1 + \theta_x) + 2)$$

After some factoring

$$A(\theta) = \log(\exp(\theta_x)(1 + \exp(1)) + 2)$$

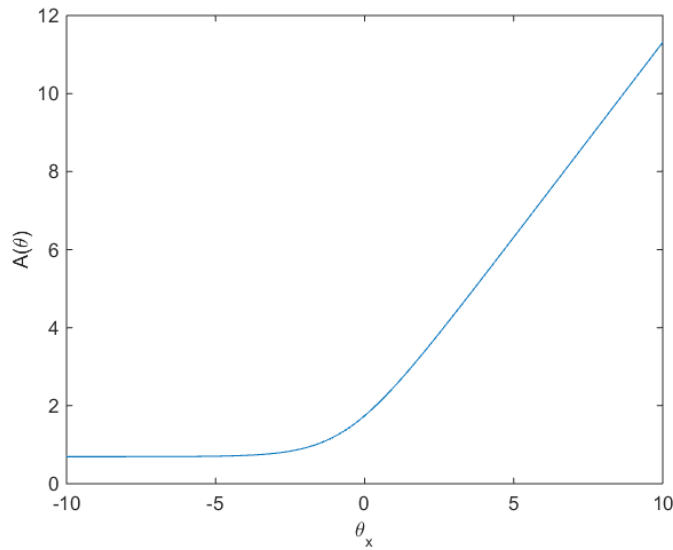


Figure 7: Plot for $A(\theta)$. It appears convex as expected

Part C

This is the partial with respect to θ_x

$$\frac{\partial A}{\partial \theta_x} = \frac{\exp(\theta_x) + \exp(\theta_x + \theta_{xy})}{\exp(\theta_x) + \exp(\theta_x + \theta_{xy}) + 2}$$

This is the partial with respect to θ_{xy}

$$\frac{\partial A}{\partial \theta_{xy}} = \frac{\exp(\theta_x + \theta_{xy})}{\exp(\theta_x) + \exp(\theta_x + \theta_{xy}) + 2}$$

Thus we have

$$\nabla A(\theta) = \left[\frac{\exp(1) + \exp(3)}{\exp(1) + \exp(3) + 2}, \frac{\exp(3)}{\exp(1) + \exp(3) + 2} \right]$$

Approximately

$$\nabla A(\theta) = [0.91937, 0.80978]$$

This is also the expected value of x and xy respectively if $\theta = [1, 2]$

Problem 4

Part A

Here is the code to compute Part A (it runs after the code from markov_chain.py)

```
p0vals = np.zeros(len(xvals))
for i in range(mSeq):
    curSeq = x[i]
    x0val = curSeq[0]
    p0vals[x0val] += 1
p0vals = np.divide(p0vals,mSeq)
print np.transpose(np.matrix(p0vals))
```

This is the result of the print statement

```
[[ 0.87656353]
 [ 0.         ]
 [ 0.         ]
 [ 0.         ]
 [ 0.03587887]
 [ 0.         ]
 [ 0.0875576 ]
 [ 0.         ]]
```

Part B

Here is the code for Part B (follows the part A code)

```
Tmatrix = np.zeros((len(xvals),len(xvals)))
for i in range(mSeq):
    curSeq = x[i]
    for j in range(1,len(curSeq)):
        xPrev = curSeq[j-1]
        xCurrent = curSeq[j]
        Tmatrix[xPrev,xCurrent] += 1
Tsum = np.matrix(np.sum(Tmatrix,axis=1))
Tsum = np.transpose(Tsum)
TsumTiled = np.matlib.repmat(Tsum,1,8)
Tmatrix = np.divide(Tmatrix,TsumTiled)
print 'Transition Matrix (first 5 states) is as follows:'
print Tmatrix[0:5,0:5]

epsilon = 1e-8
Tmatrix2 = np.matrix(Tmatrix)
curX = np.matrix(p0vals)
for i in range(500):
    prevX = np.copy(curX)
    curX = curX*Tmatrix2
    diffX = np.abs(np.subtract(prevX,curX))
    if np.sum(diffX)<epsilon:
        break
print
print 'Stationary Distribution:'
print np.transpose(np.matrix(curX))
```


This is the output of those print statements

Transition Matrix (first 5 states) is as follows:

```
[[ 6.02215434e-01  3.40558643e-02  1.20877341e-01  3.68453980e-02
   1.02646009e-01]
 [ 4.60651517e-01  2.19411172e-02  4.17552894e-02  2.52994515e-02
   4.52255681e-02]
 [ 8.53601579e-02  3.56129514e-03  8.14666698e-01  4.89824037e-03
   5.41783917e-03]
 [ 1.61422755e-01  7.77332840e-03  2.13345896e-02  7.02897331e-01
   3.12952182e-02]
 [ 5.62093936e-02  5.89229505e-04  7.55919431e-04  3.38806965e-03
   9.18310307e-01]]
```

Stationary Distribution:

```
[[ 1.78394852e-01]
 [ 1.16882378e-02]
 [ 2.24024192e-01]
 [ 3.89020939e-02]
 [ 3.36189664e-01]
 [ 9.78638026e-05]
 [ 2.10385915e-01]
 [ 3.17181525e-04]]
```

Part C

Here is the code for it (follows part B code)

```
Omatrix = np.zeros((len(xvals),len(ovals)))
for i in range(mSeq):
    curSeq = x[i]
    curObs = o[i]
    for j in range(len(curSeq)):
        xt = curSeq[j]
        ot = curObs[j]
        Omatrix[xt,ot] += 1
Osum = np.matrix(np.sum(Omatrix,axis=1))
Osum = np.transpose(Osum);
OsumTiled = np.matlib.repmat(Osum,1,20)
Omatrix = np.divide(Omatrix,OsumTiled)
print
print 'Emission Probability Matrix (first 5 states) is as follows:'
print Omatrix[0:5,0:5]
```

This is the output of the print statements

Emission Probability Matrix (first 5 states) is as follows:

```
[[ 0.05946997  0.01452542  0.06708915  0.04987037  0.03195305]
 [ 0.05294974  0.02418001  0.04701668  0.03918057  0.05093474]
 [ 0.06182642  0.01925435  0.03359294  0.0473769  0.05593569]
 [ 0.09200121  0.01224888  0.07588249  0.09230407  0.03970791]
 [ 0.11633794  0.01160627  0.04784776  0.08957451  0.0400521 ]]
```

Part D

For each sequence, we will append a value at the end signifying the end state. In the transition matrix, we will add a row and column that signify the "end" value.