Here I show the cooling and heating effects of a rods on a plate. Here is the system:

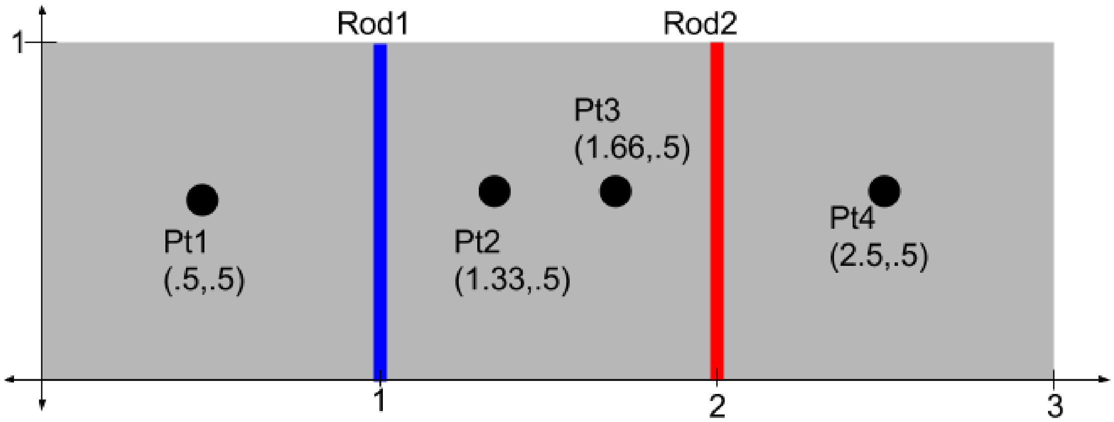

Figure 1: A line and square joined together

0.1 Studying the square

The square will have a heat equation $v(x, y, t)$.
Here is the square's diffusion equation:

$$\alpha_1\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) = \frac{\partial v}{\partial t}$$

This is the initial condition:

$$v(x, y, 0) = -20x(x - 1.5)(x - 3)y(y - 1)$$

The boundary conditions are as follows:

$$v(0, y, t) = 0$$

$$v(3, y, t) = 0$$

$$v(x, 0, t) = 0$$

$$v(x, 1, t) = 0$$

The boundary conditions were chosen so the system will equilibrate to zero over time. The initial condition was chosen so the left half of the square has a high temperature and the right half of the square has a low temperature. Therefore, the left rod will have a cooling effect and the right rod will have a heating effect as the system equilibrates to zero.

0.2 Heat Equation Over Rod

The heat equation for the rods $u(y,t)$ is as follows:

$$\alpha_2 \frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial t}$$

The boundary conditions will be as follows:

$$u(0,t) = 0$$

$$u(3,t) = 0$$

The initial condition for the left rod is as follows:

$$u_1(y,0) = v(1,y,0) = y(y-1)$$

The initial condition for the right rod is as follows:

$$u_2(y,0) = v(2,y,0) = -y(y-1)$$

0.3 Heat Equation Over Square with rods

For this I split the domain into three parts. The initial and boundary conditions remain the same unless one of the boundaries contains a rod in which case the boundary condition is the value for $u(y,t)$. The rest of the details are in the mathematica notebook.