

## 0.1 Mixed Dimensional Mesh

Here I show the cooling and heating effects of a rod on a plate. Here is the initial mesh:



Figure 1: A line and square joined together

The topological space has 3 strata:

- 1) The left rod
- 2) The right rod
- 3) The square

## 0.2 Studying the square

The square will have a heat equation  $v(x, y, t)$ .

Here is the square's diffusion equation:

$$\alpha_1 \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) = \frac{\partial v}{\partial t}$$

## 0.3 Square without the rod

For the equations below, I will assume it's a line and square.  
One part of the boundary of the square forms part of the line.

## 0.4 Heat Equation Over Rod

The heat equation for the rod  $u(x, t)$  is as follows:

$$\alpha_1 \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$

The boundary conditions will be as follows:

$$u(0, t) = 0$$

$$u(3, t) = 0$$

The initial condition is as follows:

$$u(x, 0) = -5x(x - 3)$$

## 0.5 Heat Equation Over Square

The square will have a separate heat equation  $v(x, y, t)$ .

Here is the square's diffusion equation:

$$\alpha_2 \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) = \frac{\partial v}{\partial t}$$

The boundary condition on the square will be the values at the rod. Formally this means

$$v(x, 0, t) = u(x, t)$$

I assumed the square was uniformly heated initially in the y-direction. More formally,

$$v(x, y, 0) = u(x, 0)$$