Here is the system:

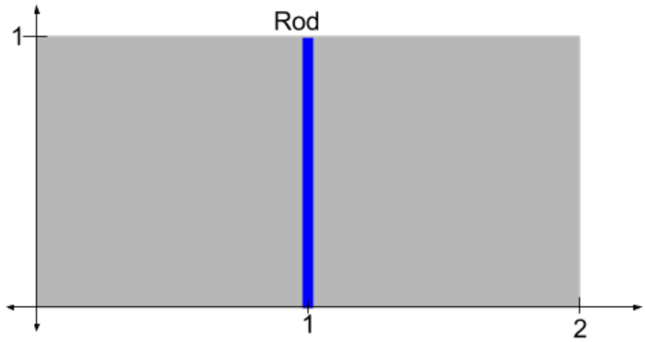

Figure 1: A line and square joined together

I am using the sequential splitting technique described in section 2.1 of this paper
Let R represent the plate without the rod
Let V represent the plate with the rod temperature as a boundary condition
Let F be the final temperature of the plate with the plate and rod interacting
I am assuming F = R + V

The square will have a heat equation $v(x, y, t)$.
Here is the square's diffusion equation:

$$\alpha_1(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}) = \frac{\partial v}{\partial t}$$

This is the initial condition:

$$v(x, y, 0) = -20x(x-2)y(y-1)$$

The boundary conditions are as follows:

$$v(0, y, t) = 0$$

$$v(2, y, t) = 0$$

$$v(x, 0, t) = 0$$

$$v(x, 1, t) = 0$$

0.1 Heat Equation Over Rod

The heat equation for the rod $u(y,t)$ is as follows:

$$\alpha_2 \frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial t}$$

The boundary conditions will be as follows:

$$u(0,t) = 0$$

$$u(1,t) = 0$$

The initial condition for the rod is as follows:

$$u(y,0) = v(1,y,0) = -20y(y-1)$$