Here is the system:

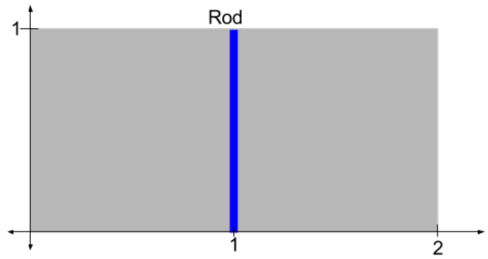

Figure 1: A plate with a rod in the middle

I am using the sequential splitting technique described in section 2.1 of this paper
Let R represent the plate without the rod
Let V represent the plate with the rod temperature as a boundary condition
Let F be the final temperature of the plate with the plate and rod interacting
I am assuming F = R + V

The diffusion equation used will be the following:

$$\frac{\partial F}{\partial t} = \alpha_{plate}(\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2})$$

I will let the initial condition be

$$F(x, y, 0) = 20x(x-2)y(y-1)$$

The boundary conditions will be as follows:

$$F(0, y, t) = F(2, y, t) = F(x, 0, t) = F(x, 1, t) = 0$$

For the rod $u(y, t)$, I will assume its diffusion equation is as follows

$$\frac{\partial u}{\partial t} = \alpha_{rod}\frac{\partial^2 u}{\partial y^2}$$

The initial condition will be $u(y, 0) = F(1, y, 0)$
The boundary conditions will be $u(0, t) = u(1, t) = 0$

When computing R, I will use the same differential as F and the same boundary conditions.
When computing V, I will impose the condition that $V(1, y, t) = u(y, t)$

The pseudo-code and implementation are in the attached mathematica notebook.