

For our current system we assume the following

$$\frac{\partial u}{\partial t} = \alpha(x, y) \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

where  $\alpha(x, y)$  describes the diffusion constant at point  $(x, y)$

We are also assuming that we are given an initial function  $f(x, y)$  so that

$$u(x, y, 0) = f(x, y)$$

This is being solved over region  $R$

Let  $n_{(x,y)}$  be the normal at point  $(x, y)$ .

We are assuming conservation hence

$$n_{(x,y)} \cdot \nabla_{(x,y)} u = 0$$

for  $(x, y) \in \partial R$

To ensure that conservation happens we need to make sure

$$E(t) = \int_R u(x, y, t) dx dy$$

is a constant for every value of  $t$

We should also know the values of  $\frac{\partial u}{\partial x}$  and  $\frac{\partial u}{\partial y}$

In summary, this is the list of things to know for each system. In each mathematica notebook, these will be computed:

1. Total energy is conserved
2. No energy flowing out at the border
3. Values of energy flow in x and y direction