Proof of Orthogonality

To prove that our matrix is orthogonal, we need to prove that any two vectors have dot product of zero. In this case the number of data points is n and for simplicity of the proof we will let $N = n - 1$.
Let $\phi = \frac{2\pi}{N}$
We will have to prove that

$$\sum_{k=0}^{N/2} cos(t \cdot k \cdot \phi)cos(t' \cdot k \cdot \phi) + sin(t \cdot k \cdot \phi)sin(t' \cdot k \cdot \phi) = 0$$

By Trig Identities it holds that

$$cos(A - B) = cos(A)cos(B) + sin(A)sin(B)$$

We can then assert that the above equation is equivalent to saying

$$\sum_{k=0}^{N/2} cos((t - t') \cdot k \cdot \phi) = 0$$

Let $C = |t - t'|$. Since $cos(\theta) = cos(-\theta)$ we can assume that $t - t' > 0$. Thus we just have to prove that

$$\sum_{k=0}^{N/2} cos(\frac{2\pi Ck}{N}) = 0$$

For each angle, there is another one on the other side of $\pi/2$. **INSERT DETAILS**