Convolution Theorem for our example

Let the following be true

$$T_1(f) = \int f(x)cos(\phi x)\, dx$$

$$T_2(f) = \int f(x)sin(\phi x)\, dx$$

Take the functions f and g and let $h = f * g$, then it is conjectured that

$$T_1(h) = T_1(f)T_1(g) - T_2(f)T_2(g)$$

$$T_2(h) = T_1(f)T_2(g) + T_1(g)T_2(f)$$

This comes from the fact that in the proof of the convolution theorem, when computing the integral for the convolution, they are able to separate the integral due to the fact that $e^{x+y} = e^x e^y$. In our case,

$$cos(\phi(x + y)) = cos(\phi x)cos(\phi y) - sin(\phi x)sin(\phi y)$$

$$sin(\phi(x + y)) = cos(\phi x)sin(\phi y) + sin(\phi x)cos(\phi y)$$

Applying the above identities to the integral for T_1 and T_2 should lead to the conjecture.

Proving Convolution Theorem in Normal Case

With convolution theorem, it says that

$$F(f * g) = F(f) \cdot F(g)$$

The point-wise multiplication means multiplication of the frequencies.

Here is the procedure for doing it in Matlab

- The variables data,filter will be used

- Let filteredData = conv(data,filter)

- Let paddedData and paddedFilter be data and filter but with zeros added to the end

- Do fft(filteredData) and notice that it is equal to fft(paddedData).*fft(paddedFilter)