Example in Washington/Trapp, page 189-191.

Factorization of $n = 3837523$ using the remainders of some squares $\pmod n$. Note, in the book there is a 7th column for the prime 17, which is all zeros and so we can safely delete it and we only consider the seven primes $2, 3, 5, 7, 11, 13, 19$ to factor the remainders. The following matrix gives the exponent vectors for the remainders of the selected squares $\pmod n$. The integers, whose squares we compute, are in the first column. They are all of the form $[\sqrt{in} + j]$ as discussed in the book and i, j are listed in column $2, 3$.

$$F := \left(\begin{array}{rl|rr|rrrrrrr|r}
\multicolumn{2}{r|}{n = 3837523} & i & j & 2 & 3 & 5 & 7 & 11 & 13 & 19 & \text{remainder} \\
\hline
9398^2 & \pmod n & 23 & 4 & 0 & 0 & 5 & 0 & 0 & 0 & 1 & 59375 \\
19095^2 & \pmod n & 95 & 2 & 2 & 0 & 1 & 0 & 1 & 1 & 1 & 54340 \\
1964^2 & \pmod n & 1 & 6 & 0 & 2 & 0 & 0 & 0 & 3 & 0 & 19773 \\
17078^2 & \pmod n & 76 & 1 & 6 & 2 & 0 & 0 & 1 & 0 & 0 & 6336 \\
8077^2 & \pmod n & 17 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 38 \\
3397^2 & \pmod n & 3 & 4 & 5 & 0 & 1 & 0 & 0 & 2 & 0 & 27040 \\
14262^2 & \pmod n & 53 & 1 & 0 & 0 & 2 & 2 & 0 & 1 & 0 & 15925
\end{array}\right)$$

So for example $9398^2 \pmod n = 59375 = 2^0 \cdot 3^0 \cdot 5^5 \cdot 7^0 \cdot 11^0 \cdot 13^0 \cdot 19^1$. To find a product of the remainders, which is a square, we look compute the following matrix $A := F \pmod 2$ by replacing an even number in an exponent vector by 0 and an odd number in an exponent vector by 1:

$$A := \begin{pmatrix}
0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0
\end{pmatrix}$$

A basis for left nullspace of A is given by the following three vectors of the matrix N using standard row reduction:(note the last column of A is the sum of the first and the forth column of A).

$$N := \begin{matrix}
1 & 0 & 0 & 0 & 1 & 1 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1
\end{matrix}$$

Taking the sum of the corresponding exponent vectors in F .

2	3	5	7	11	13	19
6	0	6	0	0	2	2
8	4	6	0	2	4	2
0	2	2	2	0	4	0

and dividing the entries by 2 to get the exponent vector of a square root we get:

2	3	5	7	11	13	19
3	0	3	0	0	1	1
4	2	3	0	1	2	1
0	1	1	1	0	2	0

This tells us using the first vector of N and the corresponding numbers $9398, 8077$ and 3397:

$$(9398 \cdot 8077 \cdot 3397)^2 \equiv (2^3 \cdot 3^0 \cdot 5^3 \cdot 7^0 \cdot 11^0 \cdot 13^1 \cdot 19^1)^2 \pmod{n}$$

Note that on the left hand side $X := 9398 \cdot 8077 \cdot 3397 \pmod{n} = 3590523$ and on the right hand side $Y := 2^3 \cdot 3^0 \cdot 5^3 \cdot 7^0 \cdot 11^0 \cdot 13^1 \cdot 19^1 \pmod{n} = 247000$. So we get:

$$3590523^2 \equiv 247000^2 \pmod{n}$$

and we can test $\gcd(X - Y, n)$. It is n.

Applying this recipe to the second vector of N, the left hand side is

$$9398 \cdot 19095 \cdot 1964 \cdot 17078 \pmod{n} = 2230387$$

and the right hand side gives

$$635778000 \pmod{n} = 2586705,$$

so we get

$$2230387^2 \equiv 2586705^2 \pmod{n}$$

and $\gcd(2230387 - 2586705, n) = 1093$.