Contents

Chapter 1

Path Loss Models

Path loss (PL) models are formulas which allow for modeling the received power P_r given the transmit power P_t, distance d, and signal carrier wavelength λ. The resulting PL is due to signal propagation through the environment.

1.1 Friss Formula

For a transmitter and receiver spaced d distance apart, the received power Pr follows:

$$P_r = P_t \frac{1}{4\pi d} A_e \tag{1.1}$$

where $A_e = \frac{\lambda}{4\pi}$ for an omnidirectional antenna and a signal with a wavelength λ. Substituting A_e in (1.1) provides the Friis Formula, which applies for a wireless channel in a vacuum. Further adding terms G_t and G_r for the transmit and receive antenna gains, we reach the generalized Friis formula.

$$P_r = \frac{G_t G_r \lambda^2 P_t}{(4\pi d)^2} \tag{1.2}$$

1.2 Path Loss Exponent Model

Since wireless channels exist in environments without a vacuum, a generalized model is needed to cover a variety of cases. A generalized such model is an exponential model using the received power formula below.

$$P_r = P_t G_t G_r K \left[\frac{d_0}{d}\right]^\alpha \tag{1.3}$$

The parameter d_0 is a unit normalization parameter, usually set to 1 m. The parameter α is the PL exponent. The parameter K is the nominal PL at 1 m. Notice when $K = [\frac{\lambda}{4\pi}]^2$ defined as K_0 and $\alpha = 2$, the formula reveals the Friis Formula from (1.2).

Chapter 2

Multipath Effect Modeling

Other than the natural landscape of the environment, there are other effects that may further increase PL.

2.1 Blocking

Blocking allows for a multi-slope PL model. Essentially, the PL model has multiple definitions, and follows each one with a certain probability. This usually translates to following a certain PL for a Line of sight (LOS) path, and following another model for the Non-line of sight (NLOS) path. A common model for this is the Exponential Blocking model

$$P_r = \begin{cases} PL_{LOS} & \text{w.p.} \quad e^{-d/\beta} \\ PL_{NLOS} & \text{w.p.} \quad 1 - e^{-d/\beta} \end{cases} \tag{2.1}$$

where β acts as the mean distance before blocking occurs (i.e. by a building), and the PL_{LOS} and PL_{NLOS} terns are functions for PL models.

Blocking example

Imagine a scenario in which a LOS path is in free space and the NLOS is modeled as an exponential PL with an exponent $\alpha = 2.5$ and a distance of $d = 100$ between the transmitter and receiver. The blocking exponent $\beta = 25$. What does the $SNR = P_r/P_t$ model finalize to if $G = G_tG_r = 1$, $K_{\text{LOS}} = K_{\text{NLOS}} = K_0 = -40$ dB and $d_0 = d_1 = 1$.

Answer:

$$SNR = \frac{P_r}{P_t} = \begin{cases} K_0 \left[\frac{1}{100}\right]^2 & \text{w.p.} \quad e^{-100/25} \\ K_0 \left[\frac{1}{100}\right]^{2.5} & \text{w.p.} \quad 1 - e^{-100/25} \end{cases} = \begin{cases} -80 \text{ dB} & \text{w.p. } 0.0183 \\ -90 \text{ dB} & \text{w.p. } 0.9817 \end{cases}$$

2.2 Shadowing

Shadowing is attenuation caused by objects between the transmitter and receiver that reduce the received signal's power. It is modeled as random and does not depend on the distance of the wireless communication link. By modeling shadowing we account for the constructive and destructive interference cause by the transmitted signal reflecting, diffracting and scattering, the components of which are called multipath components.

The shadowing effect is modeled as lognormal. The received power equation from a PL model is simply multiplied with a new variable $\mathcal{X}$, where $x_{\text{dB}} = 10log_{10}\mathcal{X} \sim \mathcal{N}(0, \sigma_{\text{dB}}^2)$.

Alternatively, some use a model where $P_r = P_t\Psi$ where $\psi_{\text{dB}} = 10log_{10}\Psi \sim \mathcal{N}(PL_{\text{dB}}(d), \sigma_{\text{dB}}^2)$ for some PL model.

Shadowing example

Imagine a scenario in which we wish to evaluate Wi-Fi coverage for a desired range of 100 m, with a minimum $SNR = 5$ dB and outage constraint at 1% for the 5 GHz frequencies. The PL is modeled as exponential with $\alpha = 3$, $K = K_0 = -50$ dB (for $F_c \approx 5$ GHz), no TX and RX gain, a noise power of $P_n = -100$ dBm and a shadowing standard deviation of $\sigma_{\text{dB}} = 6$. What is the mimum transmition power to meet the outage constraint?

Answer:

$$
\begin{aligned}
SNR = \frac{P_r}{P_t} &\geq 5 \text{ dB} \\
P_r &\geq -95 \text{ dB} \\
P_r &= P_{t,\text{dB}} + K_{0,\text{dB}} + x_{\text{dB}} - \alpha 10log_{10}d \\
P_{t,\text{dB}} &\geq -95 + 50 - x_{\text{dB}} + 60 \\
P_{t,\text{dB}} &\geq 15 - x_{\text{dB}}
\end{aligned}
$$

Since x_{dB} is normally distributed, we simply use the CDF of a Gaussian distribution to evaluate transmit power that would allow for the required outage constraint of 0.01:

$$
\begin{aligned}
P(P_{t,\text{dB}} \geq 15 - x_{\text{dB}}) &= 1 - 0.01 \\
15 - P_{t,\text{dB}} &= \sigma_{\text{dB}} Q^{-1}(0.99) \\
P_{t,\text{dB}} &\geq 29 \text{ dBm}
\end{aligned}
$$

Chapter 3

Multipath Channel Models

A transmitted signal $s(t)$ propagates through an environment in multiple paths. Neglecting noise, the received signal is simply a sum of attenuated, delayed, and phase shifted versions of the original transmitted signal. The different paths a signal takes to get to a receiver results in a sum with patterns of high or low destructive interference, an effect called multipath fading.

3.1 Multipath Model

We model the channel $c(\tau, t)$ as a time-varying (t) impulse response function evaluated at $t - \tau$. As with any system, the received signal is modeled as a convolution of $c(\tau, t)$ with the transmitted signal $s(t)$.

A way to interpret the parameters is as follows:

- t is the time an impulse response is observed
- τ is the offset from the observation time in which the impulse was transmitted

The traditional impulse response of a time-invariant system can be expressed as thinking of the channel's impulse response as periodic $c(\tau, t + T)$ where $T = -t$ such that $c(\tau, 0) = c(\tau)$, which is the impulse response at time 0.

Whether NLOS received signals are received in a large spread of time compared to the LOS component or not, depends on the inverse of the transmitted signal's Bandwidth (BW). This spread is called the delay spread T_m. Depending on whether it is large or small changes the model significantly, so it is explored as Narrowband and Wideband fading.

3.2 Narrowband Fading Model

Under this regime, the delay spread T_m is significantly smaller than the inverse of the baseband BW of the signal B_{bb}, $T_m << B_{bb}^{-1}$. Essentially, this model assumes that all paths have no delay as mentioned in Section 3, but still have random phases. For a received signal $r(t)$, the following model is assumed,

$$r(t) = \mathrm{Re}\left\{ s(t)e^{j2\pi f_c t}\left(\sum_i \alpha_i(t)e^{-j\phi_i(t)}\right)\right\} \tag{3.1}$$

where f_c is the carrier frequency, α_i are the attenuation on each path i with their respective phase offset ϕ_i.

For large $N(t)$ (time dependent number of multipath components) if α_i and ϕ_i are independent and identically-distributed (i.i.d.), then by the Central Limit Theorem (CLT), the real and imaginary components of the baseband equivalent received signal are approximately joint Gaussian random variables (GRVs). If some path is larger in amplitude, like the LOS path, then the paths are no longer i.i.d. so the CLT no longer holds and the baseband components are not joint GRV.

TODO Autocorrelation

Chapter 4

Channel Reliability

4.1 Signal Modulation

TODO

Table 4.1

	MPAM	MPSK	MQAM	FSK
E_s	$\frac{d_{min}^2}{12}(M^2-1)$	d^2	$\frac{d_{min}^2}{6}(M^2-1)$	$d^2/2$
d_{min}	$\sqrt{\frac{12E_s}{M^2-1}}$	$\sqrt{E_s}$	$\sqrt{\frac{6}{M^2-1}}$	$\sqrt{2E_s}$

4.2 Signal-to-noise ratio

If the complex noise $n(t) \sim \mathcal{CN}(0, 2\sigma^2)$, meaning each of the two channels is distributed as a zero-mean GRV with variance σ, this relates to the noise power spectral density (PSD) N_0 (in watts per hertz W/Hz) as $\sigma^2 = N_0/2$. If the energy per dimension is $\bar{E_s} = E_s/2$, then the signal-to-noise ratio (SNR) is defined as:

$$\gamma_s = SNR = \frac{\bar{E_s}}{\sigma} = \frac{E_s}{2\sigma} = \frac{E_s}{N_0} \tag{4.1}$$

SNR can also be defined in terms of the power of the received signal P_s over the power of the noise $P_n = N_0 B$, where $B = 1/T$ is the BW of the receiver. Note, the received signal power P_s relates to the energy per symbol as $P_s = E_s/T$ for a symbol time T.

$$\gamma_s = SNR = \frac{P_s}{P_n} = \frac{E_s/T}{N_0 B} = \frac{E_s}{N_0} \tag{4.2}$$

The SNR also has an equivalent metric, the per-bit SNR $Ebb - N_0$ expressed in terms of the energy per bit E_b over the noise PSD N_0. The energy per bit is $E_b = E_s/\log_2 M$. Also, define the bit-rate as $R = \frac{\log_2 M}{T}$.

$$\gamma_b = Ebb - N_0 = \frac{E_b}{N_0} = \frac{E_s/\log_2 M}{N_0} = \frac{\gamma_s}{\log_2 M} = \gamma_s \frac{B}{R} \tag{4.3}$$

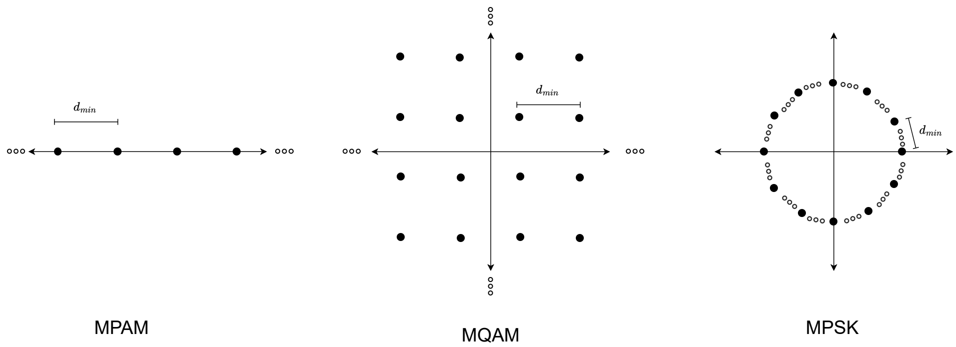

Figure 4.1: Constellation schemes where Quadrature and In-Phase possible discrete options are displayed in terms of their energy.

4.3 Probability of Error for fixed SNR

TODO

The probability of error P_s is defined in a per-symbol basis. Each constellation has a different P_s, seen in the table below based on the Nearest Neighbor bound in an additive white Gaussian noise (AWGN) channel.

Table 4.2

	$P_s(\gamma_s)$	$P_b(\gamma_b)$
MPAM	$2(1-1/M)Q(\sqrt{\frac{6\gamma_s}{M^2-1}})$	$\frac{2(1-1/M)}{\log_2 M}Q(\sqrt{\frac{6\gamma_b \log_2 M}{M^2-1}})$
MPSQ	$2Q(\sqrt{2\gamma_s}\sin(\pi/M))$	$\frac{2}{\log_2 M}Q(\sqrt{2\gamma_b \log_2 M}\sin(\pi/M))$
MQAM	$4(1-1/\sqrt{M})Q(\sqrt{\frac{3\gamma_s}{M-1}})$	$\frac{4(1-1/\sqrt{M})}{\log_2 M}Q(\sqrt{\frac{3\gamma_b \log_2 M}{M-1}})$
FSK	$(M-1)Q(\sqrt{\gamma_s/2})$	$\frac{(M-1)}{\log_2 M}Q(\sqrt{\gamma_b \log_2 M/2})$

Where generally $P_b = P_s/\log 2(M)$ for Gray coded constellations.
Useful approximations for $0 \leq \gamma_s \leq 30$ dB:

- MQAM $M \geq 4$ has $P_b(\gamma_s) \approxeq 0.2e^{\frac{-1.5\gamma_s}{M-1}}$
- BPSK has $P_b(\gamma_s) = P_s(\gamma_s) \approxeq 2e^{-1.5\gamma_s}$

Another way to more easily calculate

4.4 Probability of Error for stochastic SNR

Since the SNR depends on the received power, which in turn depends on the affects of blocking, shadowing, and fading, the SNR is a random variable. It becomes useful then to define the average bit error rate (BER).

$$\overline{P_b} \triangleq \mathbb{E}\left[P_b\right] = \int_0^\infty P_b(\gamma) f_\gamma(\gamma) d\gamma \tag{4.4}$$

Average BPSK in Rayleigh Fading

Express the average BER of a BPSK signal in a Rayleigh fading channel in terms of the averge SNR $\overline{\gamma}$.

Answer:
For Rayleigh fading:

$$f_{\gamma_s}(\gamma) = \frac{1}{\overline{\gamma}} e^{-\gamma/\overline{\gamma}}$$

And for BPSK:

$$P_b(\gamma_s) = 2Q(\sqrt{2\gamma_s})$$

So the BER is:

$$\overline{P_b} = \int_0^{\infty} 2Q(\sqrt{2\gamma}) \frac{1}{\overline{\gamma}} e^{-\gamma/\overline{\gamma}} d\gamma$$

with change of variables $x = \gamma/\overline{\gamma}$ such that $d\gamma/\overline{\gamma} = dx$

$$\begin{aligned}\overline{P_b} &= \int_0^{\infty} 2Q(\sqrt{2x\overline{\gamma}}) e^{-x} dx \\ &= \frac{2}{2\pi} \int_0^{\infty} e^{-x} \int_{\sqrt{2x\overline{\gamma}}}^{\infty} e^{-t^2/2} dt dx\end{aligned}$$

and switching the order of the integrals

$$\begin{aligned}\overline{P_b} &= \frac{1}{\pi} \int_0^{\infty} \int_0^{t^2/(2\overline{\gamma})} e^{-t^2/2} e^{-x} dx dt \\ &= \frac{1}{\pi} \int_0^{\infty} e^{-t^2/2} (1 - e^{t^2/(2\overline{\gamma})}) dt \\ &= \frac{1}{2} - \frac{1}{1\pi} \int_0^{\infty} e^{-t^2(1+1/\overline{\gamma})/2} dt \\ &= \frac{1}{2} \left(1 - \sqrt{\frac{\overline{\gamma}}{1+\overline{\gamma}}}\right) \\ &\approx \frac{1}{4\overline{\gamma}}\end{aligned}$$

As seen in the example above it can become cumbersome to calculate the average bit error rate. To expedite the process we can use the moment generating functions of the channels and simplify the integral. Most constellation bit error probabilities can be approximated or precisely expressed as

$$P_b(\gamma) = aQ(\sqrt{\beta\gamma})$$

Using [1], we can express the Q-function as

$$Q(x) = \frac{1}{\pi} \int_0^{\pi/2} e^{-\frac{x^2}{2\sin^2\phi}} d\phi \tag{4.5}$$

TODO

Chapter 5

Diversity

Chapter 6

Channel Capacity

6.1 AWGN Channel Capacity, no fading

The capacity of a channel is a measure of the amount of information that can be transmitted over the channel, and is denoted with C. Another representation is with channel efficiency where the capacity is divided by the signal BW denoted with B.

$$\begin{aligned} C &= Blog_2(1+\gamma)\ \ bps/\text{complex channel} \\ C/B &= log_2(1+\gamma)\ \ bps/Hz/\text{complex channel} \end{aligned} \tag{6.1}$$

Shannon Capacity/ AWGN Capacity derivation

Derive the Shannon capacity using mutual information.

Answer: If an AWGN channel $y = x + w$ where $w \sim \mathcal{N}(0, N_0)$ then the capacity is equal to the maximum amount of mutual information $I(x, y)$ of the channel input and output.

$$\begin{aligned} C/B &= \max_x I(x;y) = \max_x \left[h(y) - h(y|x)\right] \\ &= \max_x \left[h(x+w) - h(x+w|x)\right] \\ &= \max_x \left[h(x+w) - h(w)\right] \end{aligned}$$

For a GRV w, the entropy $h(w) = \frac{1}{2}\log_2(2\pi e N_0)$. Also, the distribution which maximizes differential entropy h for random variables of the same variance is Gaussian. This means maximizing C can be boiled down to maximizing $h(x+w)$, which happens when $x + w \sim \mathcal{N}(\mu, \sigma^2)$. This is true only when $x \sim \mathcal{N}(0, P_x)$.

$$\begin{aligned} \therefore C/B &= \max_x I(x;y) = \frac{1}{2}\log_2(2\pi e(E_s + N_0)) - \frac{1}{2}\log_2(2\pi e N_0) \\ &= \frac{1}{2}\log_2(1 + \frac{E_s}{N_0}) = \frac{1}{2}\log_2(1+\gamma) \end{aligned} \tag{6.2}$$

For x a complex GRV the capacity is doubled.

6.2 Flat Fading Channel Capacity

In this regime, the SNR of the channel is a random variable (r.v.) due to fading. Since the noise is always modeled as Gaussian, what we model making the SNR of a channel a r.v. is the time-varying gain of a channel. The instantaneous received SNR at time i is a function of the average transmit power $\overline{P_x}$, the noise PSD, signal BW and instantaneous gain of the channel at time i $g[i]$.

The capacity of the channel, then, depends on the knowledge of the channel gain, called the channel side information (CSI). There are three choices regarding the CSI:

- channel distribution information (CDI): Both transmitter and receiver known the distribution of $g[i]$
- Receiver CSI: Only the receiver knows the value of $g[i]$, but both transmitter and receiver known the distribution of $g[i]$
- Receiver & transmitter CSI: Both transmitter and receiver known the distribution and the value of $g[i]$

6.2.1 Channel Distribution Information

CDI is an open problem. It can become difficult to find the capacity-achieving distribution for the input. Literature exists on specific cases, such as the Rayleigh fading channel, in which the power gain is exponentially distrubuted and changes with each channel use at time i. An optimal solution exists, but the optimal distribution and capacity must be found numerically and lacks a closed-form solution. TODO

6.2.2 Receiver Channel Side Information

In this case the capacity is defined by the Shannon capacity. In practical systems the rate cannot be maintained constant as is expected in Shannon capacity, so a reduction from the Shannon capacity is expected.

Shannon (ergodic) capacity can be computed by integrating over the distribution of the SNR, which is known by both the transmitter and receiver.

$$C = \mathbb{E}\left[B\log_2(1+\gamma)\right] = \int_0^\infty B\log_2(1+\gamma)p(\gamma)d\gamma \tag{6.3}$$

Since the realization of γ is known only to the receiver, the ergotic capacity is constant, regardless of γ. A bound on the capacity can be roughly formed by using Jensen's inequality and switching the expectation and logarithm to achieve

$$C \leq B\log_2(1+\mathbb{E}\left[\gamma\right]) = B\log_2(1+\overline{\gamma}) \tag{6.4}$$

where $\overline{\gamma}$ is the average channel SNR. This shows that for receiver CSI, the achievable capacity is always less than the Shannon capacity.

6.2.3 Receiver & Transmitter Channel Side Information

In this regime, the receiver can feed back information about the channel to the transmitter, and in turn the transmitter can take actions and adapt it's transmission strategy.

There are two main knobs which the transmitter has access to in order to change it's strategy: power and rate. If we allow the transmit power to vary with some function of γ subject to some average power constraint $\overline{P}$, we arrive at the power adaptation strategy.

Water filling: Optimal Strategy

TODO Solving the resulting lagrangian from

$$C = \max_{P(\gamma)} \left[\int_0^\infty B \log_2(1 + \frac{P(\gamma)}{\overline{P}}\gamma) p(\gamma) d\gamma \right]$$
$$\text{s.t.} \int P(\gamma)p(\gamma)d\gamma = \overline{P} \tag{6.5}$$

results in a waterfilling (WF) solution where

$$\frac{P(\gamma)}{\overline{P}} = \begin{cases} 1/\gamma_0 - 1/\gamma & \gamma \geq \gamma_0 \\ 0 & \gamma < \gamma_0 \end{cases} \tag{6.6}$$

for the value of γ_0 that satisfies the power constraint from the lagrangian. The overall capacity in the receiver and transmitter CSI is then:

$$C = \int_{\gamma_0}^\infty B \log_2(\frac{\gamma}{\gamma_0}) p(\gamma) d\gamma \tag{6.7}$$

Since the power adaptation policy in (6.6) has a dependence on the threshold, it can only be achieved with a time-varying data rate achieved for the instantaneous SNR. For Rayleigh fading, this capacity can exceed the AWGN channel capacity for the same average SNR, different to the CSI case where fading always indicated decreased capacity.

For an arbitrary power adaptation policy, the channel capacity is

$$C = \int_{\gamma_0}^\infty B \log_2(1 + \frac{P(\gamma)}{\overline{P}}\gamma) p(\gamma) d\gamma \tag{6.8}$$

Discrete Rate adaptation

Since the WF is not practically achievable, a discrete Rate adaptation method is described where the transmitter can choose the rate between discrete scemes (i.e. MQAM with varying M). TODO

Power Adaptation at fixed rate

The power adaptation scheme employed here is:

$$\frac{P(\gamma)}{\overline{P}} = \begin{cases} \gamma_t/\gamma & \gamma \geq \gamma_0 \\ 0 & \gamma < \gamma_0 \end{cases} \tag{6.9}$$

for some target SNR γ_t.

Bibliography

[1] J. Craig, "A new, simple and exact result for calculating the probability of error for two-dimensional signal constellations," in *MILCOM 91 - Conference record*, 1991, pp. 571–575 vol.2.