Contents

Chapter 1

Path Loss Models

Path loss (PL) models are formulas which allow for modeling the received power P_r given the transmit power P_t, distance d, and signal carrier wavelength λ. The resulting PL is due to signal propagation through the environment.

1.1 Friss Formula

For a transmitter and receiver spaced d distance apart, the received power Pr follows:

$$P_r = P_t \frac{1}{4\pi d} A_e \tag{1.1}$$

where $A_e = \frac{\lambda}{4\pi}$ for an omnidirectional antenna and a signal with a wavelength λ. Substituting A_e in (1.1) provides the Friis Formula, which applies for a wireless channel in a vacuum. Further adding terms G_t and G_r for the transmit and receive antenna gains, we reach the generalized Friis formula.

$$P_r = \frac{G_t G_r \lambda^2 P_t}{(4\pi d)^2} \tag{1.2}$$

1.2 Path Loss Exponent Model

Since wireless channels exist in environments without a vacuum, a generalized model is needed to cover a variety of cases. A generalized such model is an exponential model using the received power formula below.

$$P_r = P_t G_t G_r K \left[\frac{d_0}{d}\right]^\alpha \tag{1.3}$$

The parameter d_0 is a unit normalization parameter, usually set to 1 m. The parameter α is the PL exponent. The parameter K is the nominal PL at 1 m. Notice when $K = [\frac{\lambda}{4\pi}]^2$ defined as K_0 and $\alpha = 2$, the formula reveals the Friis Formula from (1.2).

Chapter 2

Multipath Effect Modeling

Other than the natural landscape of the environment, there are other effects that may further increase PL.

2.1 Blocking

Blocking allows for a multi-slope PL model. Essentially, the PL model has multiple definitions, and follows each one with a certain probability. This usually translates to following a certain PL for a Line of sight (LOS) path, and following another model for the Non-line of sight (NLOS) path. A common model for this is the Exponential Blocking model

$$P_r = \begin{cases} PL_{LOS} & \text{w.p.} \quad e^{-d/\beta} \\ PL_{NLOS} & \text{w.p.} \quad 1 - e^{-d/\beta} \end{cases} \tag{2.1}$$

where β acts as the mean distance before blocking occurs (i.e. by a building), and the PL_{LOS} and PL_{NLOS} terns are functions for PL models.

Blocking example

Imagine a scenario in which a LOS path is in free space and the NLOS is modeled as an exponential PL with an exponent $\alpha = 2.5$ and a distance of $d = 100$ between the transmitter and receiver. The blocking exponent $\beta = 25$. What does the $SNR = P_r/P_t$ model finalize to if $G = G_t G_r = 1$, $K_{\text{LOS}} = K_{\text{NLOS}} = K_0 = -40$ dB and $d_0 = d_1 = 1$.

Answer:

$$SNR = \frac{P_r}{P_t} = \begin{cases} K_0 \left[\frac{1}{100}\right]^2 & \text{w.p.} \quad e^{-100/25} \\ K_0 \left[\frac{1}{100}\right]^{2.5} & \text{w.p.} \quad 1 - e^{-100/25} \end{cases} = \begin{cases} -80 \text{ dB} & \text{w.p. } 0.0183 \\ -90 \text{ dB} & \text{w.p. } 0.9817 \end{cases}$$

2.2 Shadowing

Shadowing is attenuation caused by objects between the transmitter and receiver that reduce the received signal's power. It is modeled as random and does not depend on the distance of the wireless communication link. By modeling shadowing we account for the constructive and destructive interference cause by the transmitted signal reflecting, diffracting and scattering, the components of which are called multipath components.

The shadowing effect is modeled as lognormal. The received power equation from a PL model is simply multiplied with a new variable $\mathcal{X}$, where $x_{\text{dB}} = 10log_{10}\mathcal{X} \sim \mathcal{N}(0, \sigma^2_{\text{dB}})$.

Alternatively, some use a model where $P_r = P_t\Psi$ where $\psi_{\text{dB}} = 10log_{10}\Psi \sim \mathcal{N}(PL_{\text{dB}}(d), \sigma^2_{\text{dB}})$ for some PL model.

Shadowing example

Imagine a scenario in which we wish to evaluate Wi-Fi coverage for a desired range of 100 m, with a minimum $SNR = 5$ dB and outage constraint at 1% for the 5 GHz frequencies. The PL is modeled as exponential with $\alpha = 3$, $K = K_0 = -50$ dB (for $F_c \approx 5$ GHz), no TX and RX gain, a noise power of $P_n = -100$ dBm and a shadowing standard deviation of $\sigma_{\text{dB}} = 6$. What is the mimum transmition power to meet the outage constraint?

Answer:

$$
\begin{aligned}
SNR &= \frac{P_r}{P_t} \geq 5 \text{ dB} \\
P_r &\geq -95 \text{ dB} \\
P_r &= P_{t,\text{dB}} + K_{0,\text{dB}} + x_{\text{dB}} - \alpha 10 log_{10} d \\
P_{t,\text{dB}} &\geq -95 + 50 - x_{\text{dB}} + 60 \\
P_{t,\text{dB}} &\geq 15 - x_{\text{dB}}
\end{aligned}
$$

Since x_{dB} is normally distributed, we simply use the CDF of a Gaussian distribution to evaluate transmit power that would allow for the required outage constraint of 0.01:

$$
\begin{aligned}
P(P_{t,\text{dB}} \geq 15 - x_{\text{dB}}) &= 1 - 0.01 \\
15 - P_{t,\text{dB}} &= \sigma_{\text{dB}} Q^{-1}(0.99) \\
P_{t,\text{dB}} &\geq 29 \text{ dBm}
\end{aligned}
$$

Chapter 3

Multipath Channel Models