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Chapter 1

Path Loss Models

Path loss (PL) models are formulas which allow for modeling the received power P_r given the transmit power P_t, distance d, and signal carrier wavelength λ. The resulting PL is due to signal propagation through the environment.

1.1 Friss Formula

For a transmitter and receiver spaced d distance apart, the received power Pr follows:

$$P_r = P_t \frac{1}{4\pi d} A_e \tag{1.1}$$

where $A_e = \frac{\lambda}{4\pi}$ for an omnidirectional antenna and a signal with a wavelength λ. Substituting A_e in (1.1) provides the Friis Formula, which applies for a wireless channel in a vacuum. Further adding terms G_t and G_r for the transmit and receive antenna gains, we reach the generalized Friis formula.

$$P_r = \frac{G_t G_r \lambda^2 P_t}{(4\pi d)^2} \tag{1.2}$$

1.2 Path Loss Exponent Model

Since wireless channels exist in environments without a vacuum, a generalized model is needed to cover a variety of cases. A generalized such model is an exponential model using the received power formula below.

$$P_r = P_t G_t G_r K \left[\frac{d_0}{d}\right]^\alpha \tag{1.3}$$

The parameter d_0 is a unit normalization parameter, usually set to 1 m. The parameter α is the PL exponent. The parameter K is the nominal PL at 1 m. Notice when $K = [\frac{\lambda}{4\pi}]^2$ defined as K_0 and $\alpha = 2$, the formula reveals the Friis Formula from (1.2).

Chapter 2

Multipath Effect Modeling

Other than the natural landscape of the environment, there are other effects that may further increase PL.

2.1 Blocking

Blocking allows for a multi-slope PL model. Essentially, the PL model has multiple definitions, and follows each one with a certain probability. This usually translates to following a certain PL for a Line of sight (LOS) path, and following another model for the Non-line of sight (NLOS) path. A common model for this is the Exponential Blocking model

$$P_r = \begin{cases} PL_{LOS} & \text{w.p.} \quad e^{-d/\beta} \\ PL_{NLOS} & \text{w.p.} \quad 1 - e^{-d/\beta} \end{cases} \tag{2.1}$$

where β acts as the mean distance before blocking occurs (i.e. by a building), and the PL_{LOS} and PL_{NLOS} terns are functions for PL models.

Blocking example

Imagine a scenario in which a LOS path is in free space and the NLOS is modeled as an exponential PL with an exponent $\alpha = 2.5$ and a distance of $d = 100$ between the transmitter and receiver. The blocking exponent $\beta = 25$. What does the $SNR = P_r/P_t$ model finalize to if $G = G_t G_r = 1$, $K_{\text{LOS}} = K_{\text{NLOS}} = K_0 = -40$ dB and $d_0 = d_1 = 1$.

Answer:

$$SNR = \frac{P_r}{P_t} = \begin{cases} K_0 \left[\frac{1}{100}\right]^2 & \text{w.p.} \quad e^{-100/25} \\ K_0 \left[\frac{1}{100}\right]^{2.5} & \text{w.p.} \quad 1 - e^{-100/25} \end{cases} = \begin{cases} -80 \text{ dB} & \text{w.p. } 0.0183 \\ -90 \text{ dB} & \text{w.p. } 0.9817 \end{cases}$$

2.2 Shadowing

Shadowing is attenuation caused by objects between the transmitter and receiver that reduce the received signal's power. It is modeled as random and does not depend on the distance of the wireless communication link. By modeling shadowing we account for the constructive and destructive interference cause by the transmitted signal reflecting, diffracting and scattering, the components of which are called multipath components.

The shadowing effect is modeled as lognormal. The received power equation from a PL model is simply multiplied with a new variable $\mathcal{X}$, where $x_{\text{dB}} = 10log_{10}\mathcal{X} \sim \mathcal{N}(0, \sigma^2_{\text{dB}})$.

Alternatively, some use a model where $P_r = P_t\Psi$ where $\psi_{\text{dB}} = 10log_{10}\Psi \sim \mathcal{N}(PL_{\text{dB}}(d), \sigma^2_{\text{dB}})$ for some PL model.

Shadowing example

Imagine a scenario in which we wish to evaluate Wi-Fi coverage for a desired range of 100 m, with a minimum $SNR = 5$ dB and outage constraint at 1% for the 5 GHz frequencies. The PL is modeled as exponential with $\alpha = 3$, $K = K_0 = -50$ dB (for $F_c \approx 5$ GHz), no TX and RX gain, a noise power of $P_n = -100$ dBm and a shadowing standard deviation of $\sigma_{\text{dB}} = 6$. What is the mimum transmition power to meet the outage constraint?

Answer:

$$
\begin{aligned}
SNR = \frac{P_r}{P_t} &\geq 5 \text{ dB} \\
P_r &\geq -95 \text{ dB} \\
P_r &= P_{t,\text{dB}} + K_{0,\text{dB}} + x_{\text{dB}} - \alpha 10log_{10}d \\
P_{t,\text{dB}} &\geq -95 + 50 - x_{\text{dB}} + 60 \\
P_{t,\text{dB}} &\geq 15 - x_{\text{dB}}
\end{aligned}
$$

Since x_{dB} is normally distributed, we simply use the CDF of a Gaussian distribution to evaluate transmit power that would allow for the required outage constraint of 0.01:

$$
\begin{aligned}
P(P_{t,\text{dB}} \geq 15 - x_{\text{dB}}) &= 1 - 0.01 \\
15 - P_{t,\text{dB}} &= \sigma_{\text{dB}} Q^{-1}(0.99) \\
P_{t,\text{dB}} &\geq 29 \text{ dBm}
\end{aligned}
$$

Chapter 3

Multipath Channel Models

A transmitted signal $s(t)$ propagates through an environment in multiple paths. Neglecting noise, the received signal is simply a sum of attenuated, delayed, and phase shifted versions of the original transmitted signal. The different paths a signal takes to get to a receiver results in a sum with patterns of high or low destructive interference, an effect called multipath fading.

3.1 Multipath Model

We model the channel $c(\tau, t)$ as a time-varying (t) impulse response function evaluated at $t - \tau$. As with any system, the received signal is modeled as a convolution of $c(\tau, t)$ with the transmitted signal $s(t)$.

A way to interpret the parameters is as follows:

- t is the time an impulse response is observed
- τ is the offset from the observation time in which the impulse was transmitted

The traditional impulse response of a time-invariant system can be expressed as thinking of the channel's impulse response as periodic $c(\tau, t + T)$ where $T = -t$ such that $c(\tau, 0) = c(\tau)$, which is the impulse response at time 0.

Whether NLOS received signals are received in a large spread of time compared to the LOS component or not, depends on the inverse of the transmitted signal's Bandwidth (BW). This spread is called the delay spread T_m. Depending on whether it is large or small changes the model significantly, so it is explored as Narrowband and Wideband fading.

3.2 Narrowband Fading Model

Under this regime, the delay spread T_m is significantly smaller than the inverse of the baseband BW of the signal B_{bb}, $T_m << B_{bb}^{-1}$. Essentially, this model assumes that all paths have no delay as mentioned in Section 3, but still have random phases. For a received signal $r(t)$, the following model is assumed,

$$r(t) = \mathrm{Re}\left\{ s(t) e^{j2\pi f_c t} \left(\sum_i \alpha_i(t) e^{-j\phi_i(t)} \right) \right\} \tag{3.1}$$

where f_c is the carrier frequency, α_i are the attenuation on each path i with their respective phase offset ϕ_i.

For large $N(t)$ (time dependent number of multipath components) if α_i and ϕ_i are independent and identically-distributed (i.i.d.), then by the Central Limit Theorem (CLT), the real and imaginary components of the baseband equivalent received signal are approximately joint Gaussian random variables (GRVs). If some path is larger in amplitude, like the LOS path, then the paths are no longer i.i.d. so the CLT no longer holds and the baseband components are not joint GRV.

TODO Autocorrelation

Chapter 4

Channel Reliability

4.1 Signal Modulation

TODO

Table 4.1

	MPAM	MPSK	MQAM	FSK
E_s	$\frac{d_{min}^2}{12}(M^2-1)$	d^2	$\frac{d_{min}^2}{6}(M^2-1)$	$d^2/2$
d_{min}	$\sqrt{\frac{12E_s}{M^2-1}}$	$\sqrt{E_s}$	$\sqrt{\frac{6}{M^2-1}}$	$\sqrt{2E_s}$

4.2 Signal-to-noise ratio

If the complex noise $n(t) \sim \mathcal{CN}(0, 2\sigma^2)$, meaning each of the two channels is distributed as a zero-mean GRV with variance σ, this relates to the noise power spectral density (PSD) N_0 (in watts per hertz W/Hz) as $\sigma^2 = N_0/2$. If the energy per dimension is $\bar{E_s} = E_s/2$, then the signal-to-noise ratio (SNR) is defined as:

$$\gamma_s = SNR = \frac{\bar{E_s}}{\sigma} = \frac{E_s}{2\sigma} = \frac{E_s}{N_0} \tag{4.1}$$

SNR can also be defined in terms of the power of the received signal P_s over the power of the noise $P_n = N_0 B$, where $B = 1/T$ is the BW of the receiver. Note, the received signal power P_s relates to the energy per symbol as $P_s = E_s/T$ for a symbol time T.

$$\gamma_s = SNR = \frac{P_s}{P_n} = \frac{E_s/T}{N_0 B} = \frac{E_s}{N_0} \tag{4.2}$$

The SNR also has an equivalent metric, the per-bit SNR $Ebb - N_0$ expressed in terms of the energy per bit E_b over the noise PSD N_0. The energy per bit is $E_b = E_s/\log_2 M$. Also, define the bit-rate as $R = \frac{\log_2 M}{T}$.

$$\gamma_b = Ebb - N_0 = \frac{E_b}{N_0} = \frac{E_s/\log_2 M}{N_0} = \frac{\gamma_s}{\log_2 M} = \gamma_s \frac{B}{R} \tag{4.3}$$

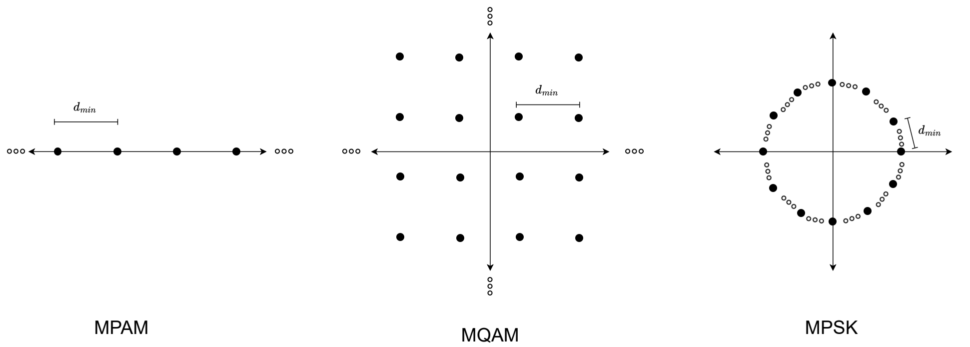

Figure 4.1: Constellation schemes where Quadrature and In-Phase possible discrete options are displayed in terms of their energy.

4.3 Probability of Error for fixed SNR

TODO

The probability of error P_s is defined in a per-symbol basis. Each constellation has a different P_s, seen in the table below based on the Nearest Neighbor bound in an additive white Gaussian noise (AWGN) channel.

Table 4.2

	$P_s(\gamma_s)$	$P_b(\gamma_b)$
MPAM	$2(1-1/M)Q(\sqrt{\frac{6\gamma_s}{M^2-1}})$	$\frac{2(1-1/M)}{\log_2 M}Q(\sqrt{\frac{6\gamma_b \log_2 M}{M^2-1}})$
MPSQ	$2Q(\sqrt{2\gamma_s}\sin(\pi/M))$	$\frac{2}{\log_2 M}Q(\sqrt{2\gamma_b \log_2 M}\sin(\pi/M))$
MQAM	$4(1-1/\sqrt{M})Q(\sqrt{\frac{3\gamma_s}{M-1}})$	$\frac{4(1-1/\sqrt{M})}{\log_2 M}Q(\sqrt{\frac{3\gamma_b \log_2 M}{M-1}})$
FSK	$(M-1)Q(\sqrt{\gamma_s/2})$	$\frac{(M-1)}{\log_2 M}Q(\sqrt{\gamma_b \log_2 M/2})$

Where generally $P_b = P_s/\log 2(M)$ for Gray coded constellations.
Useful approximations for $0 \leq \gamma_s \leq 30$ dB:

- MQAM $M \geq 4$ has $P_b(\gamma_s) \approxeq 0.2e^{\frac{-1.5\gamma_s}{M-1}}$
- BPSK has $P_b(\gamma_s) = P_s(\gamma_s) \approxeq 2e^{-1.5\gamma_s}$

Another way to more easily calculate

4.4 Probability of Error for stochastic SNR

Since the SNR depends on the received power, which in turn depends on the affects of blocking, shadowing, and fading, the SNR is a random variable. It becomes useful then to define the average bit error rate (BER).

$$\overline{P_b} \triangleq \mathbb{E}\left[P_b\right] = \int_0^\infty P_b(\gamma) f_\gamma(\gamma) d\gamma \tag{4.4}$$

Average BPSK in Rayleigh Fading

Express the average BER of a BPSK signal in a Rayleigh fading channel in terms of the averge SNR $\overline{\gamma}$.

Answer:
For Rayleigh fading:

$$f_{\gamma_s}(\gamma) = \frac{1}{\overline{\gamma}} e^{-\gamma/\overline{\gamma}}$$

And for BPSK:

$$P_b(\gamma_s) = 2Q(\sqrt{2\gamma_s})$$

So the BER is:

$$\overline{P_b} = \int_0^\infty 2Q(\sqrt{2\gamma}) \frac{1}{\overline{\gamma}} e^{-\gamma/\overline{\gamma}} d\gamma$$

with change of variables $x = \gamma/\overline{\gamma}$ such that $d\gamma/\overline{\gamma} = dx$

$$\begin{aligned} \overline{P_b} &= \int_0^\infty 2Q(\sqrt{2x\overline{\gamma}}) e^{-x} dx \\ &= \frac{2}{2\pi} \int_0^\infty e^{-x} \int_{\sqrt{2x\overline{\gamma}}}^\infty e^{-t^2/2} dt dx \end{aligned}$$

and switching the order of the integrals

$$\begin{aligned} \overline{P_b} &= \frac{1}{\pi} \int_0^\infty \int_0^{t^2/(2\overline{\gamma})} e^{-t^2/2} e^{-x} dx dt \\ &= \frac{1}{\pi} \int_0^\infty e^{-t^2/2}(1 - e^{t^2/(2\overline{\gamma})}) dt \\ &= \frac{1}{2} - \frac{1}{1\pi} \int_0^\infty e^{-t^2(1+1/\overline{\gamma})/2} dt \\ &= \frac{1}{2}\left(1 - \sqrt{\frac{\overline{\gamma}}{1+\overline{\gamma}}}\right) \\ &\approxeq \frac{1}{4\overline{\gamma}} \end{aligned}$$

As seen in the example above it can become cumbersome to calculate the average bit error rate. To expedite the process we can use the moment generating functions of the channels and simplify the integral. Most constellation bit error probabilities can be approximated or precisely expressed as

$$P_b(\gamma) = aQ(\sqrt{\beta\gamma})$$

Using [1], we can express the Q-function as

$$Q(x) = \frac{1}{\pi} \int_0^{\pi/2} e^{-\frac{x^2}{2\sin^2\phi}} d\phi \tag{4.5}$$

Bibliography

[1] J. Craig, "A new, simple and exact result for calculating the probability of error for two-dimensional signal constellations," in *MILCOM 91 - Conference record*, 1991, pp. 571–575 vol.2.