Caches IV

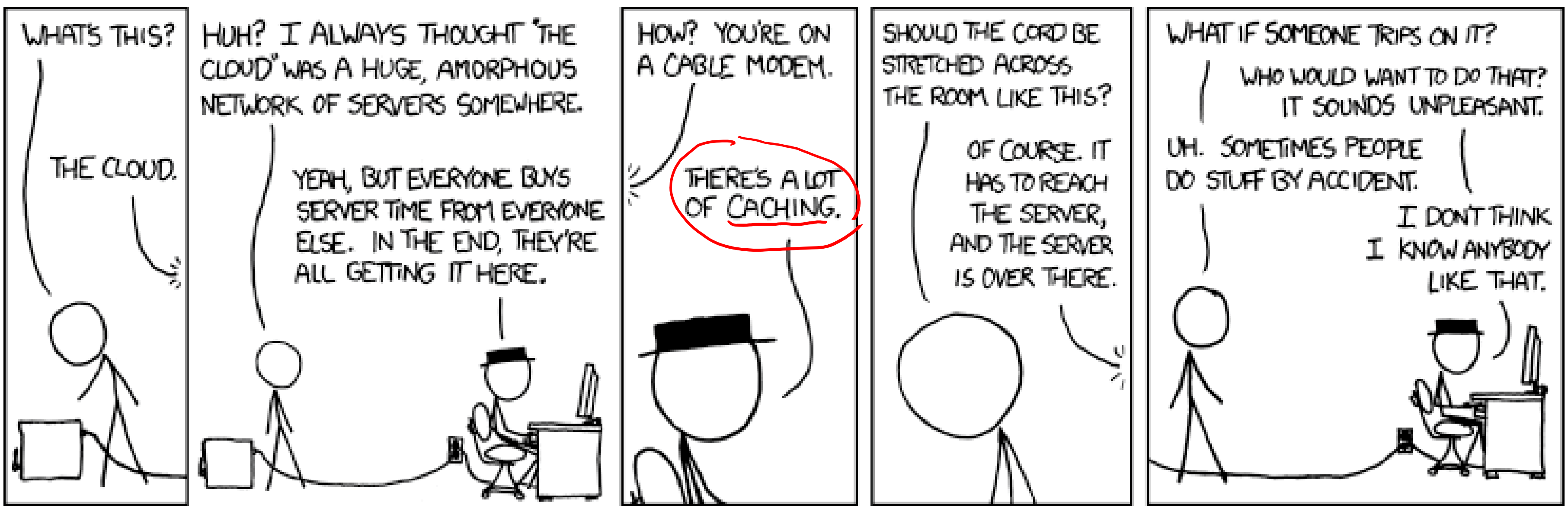
CSE 351 Spring 2019

Instructor:

Ruth Anderson

Teaching Assistants:

Gavin Cai, Jack Eggleston, John Feltrup, Britt Henderson, Richard Jiang, Jack Skalitzky, Sophie Tian, Connie Wang, Sam Wolfson, Casey Xing, Chin Yeoh



http://xkcd.com/908/

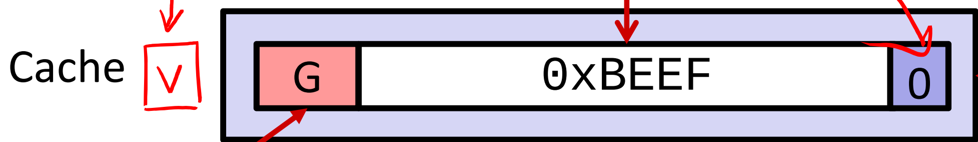
Administrivia

- Lab 3, due TONIGHT, Wednesday (5/15)
- Homework 4 , due Wed (5/22) (Structs, Caches)
- Lab 4, Coming soon!
  - Cache parameter puzzles and code optimizations

Write-back, write-allocate example

valid bit not shown,
but assume 1 for all of example

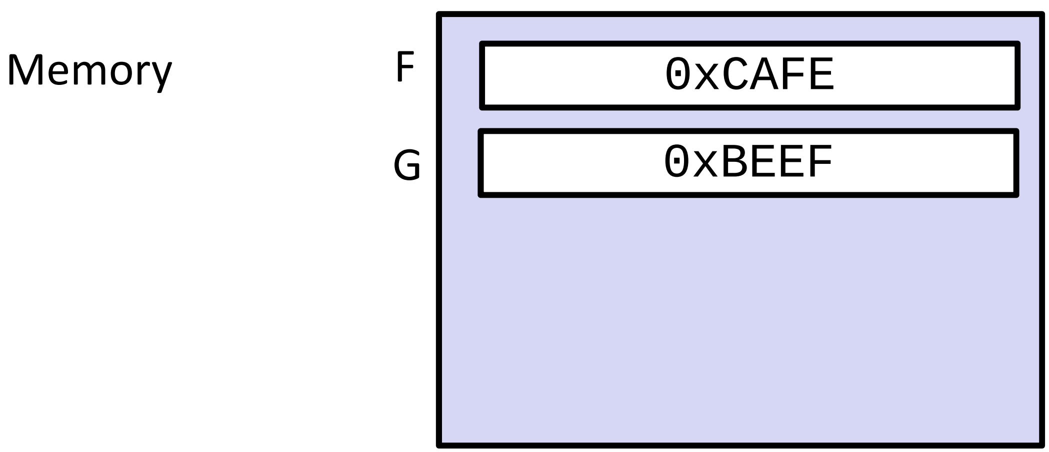
Contents of memory stored at address G



dirty bit

dirty bit keeps track of whether the data for address G in the cache matches that at address G in memory (0 if same, 1 if not). This is in case we write over the data in the cache so we know we have to go back and adjust the data in memory (a write back)

tag (there is only one set in this tiny cache, so the tag is the entire block address!)



In this example we are sort of ignoring block offsets. Here a block holds 2 bytes (16 bits, 4 hex digits).

Normally a block would be much bigger and thus there would be multiple items per block. While only one item in that block would be written at a time, the entire line would be brought into cache.

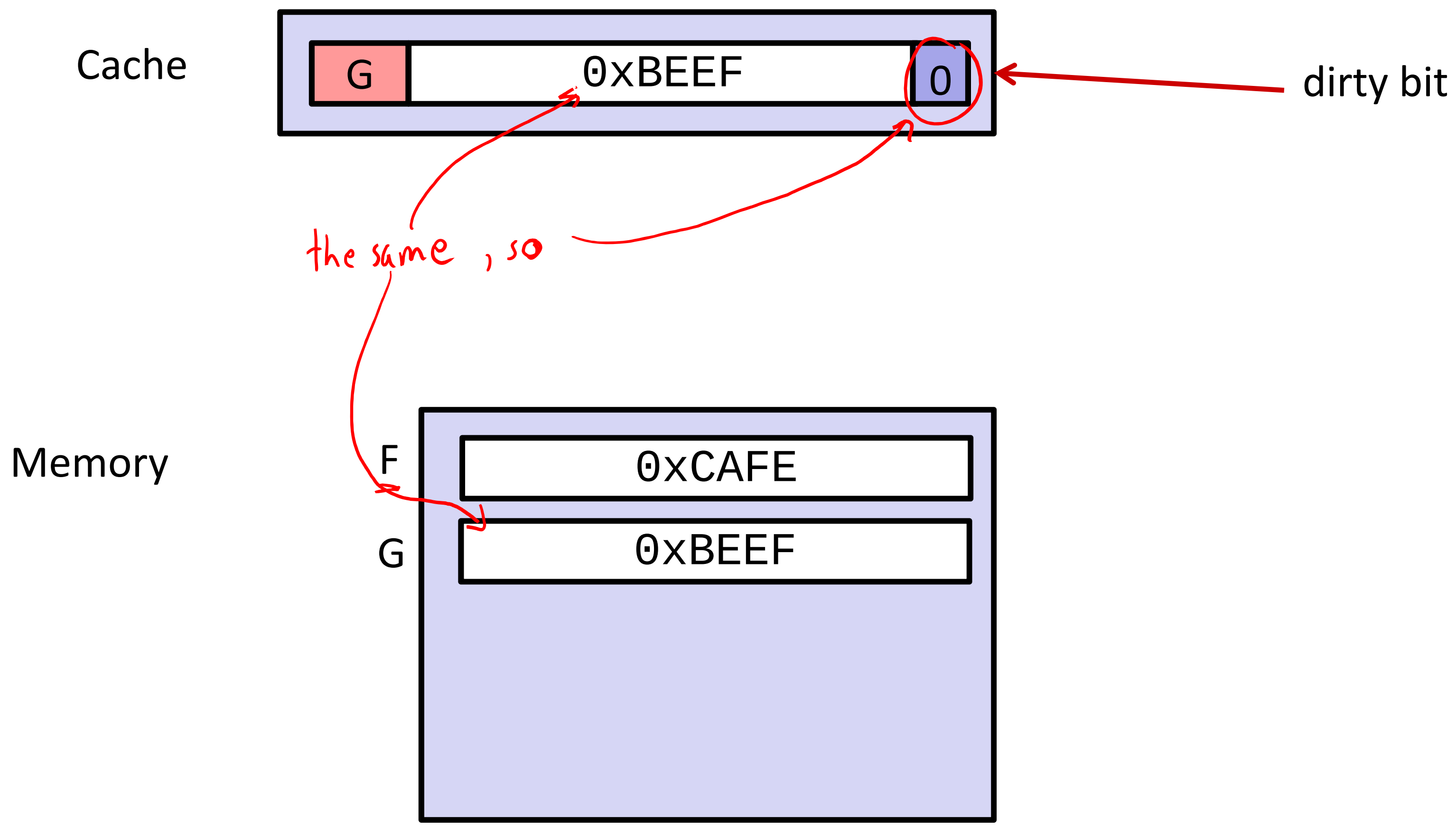
Write-back, write-allocate example

write miss

```
mov 0xFACE, F
```

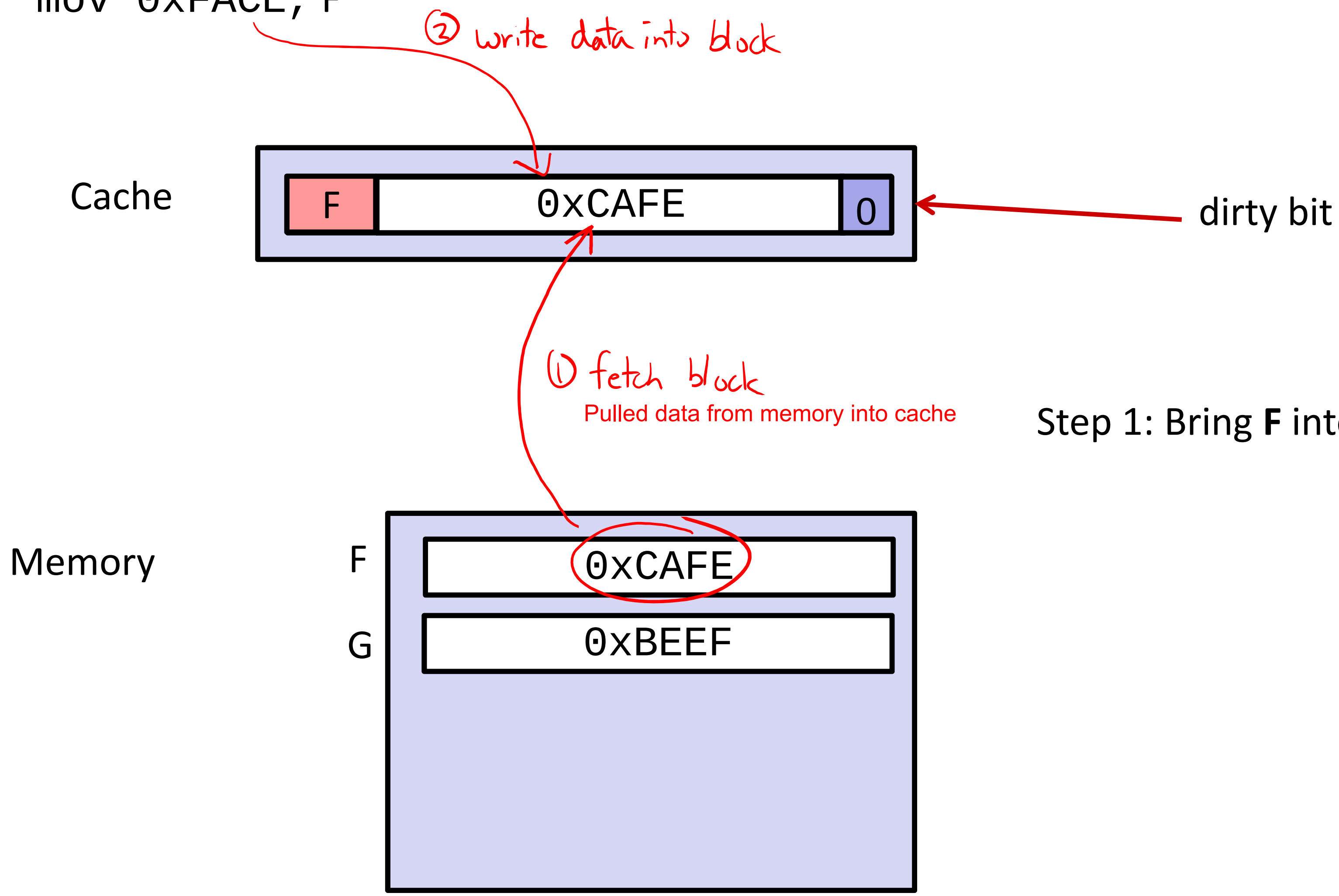
① check cache for F → miss

② pull block into $, then write



Write-back, write-allocate example

`mov 0xFACE, F`

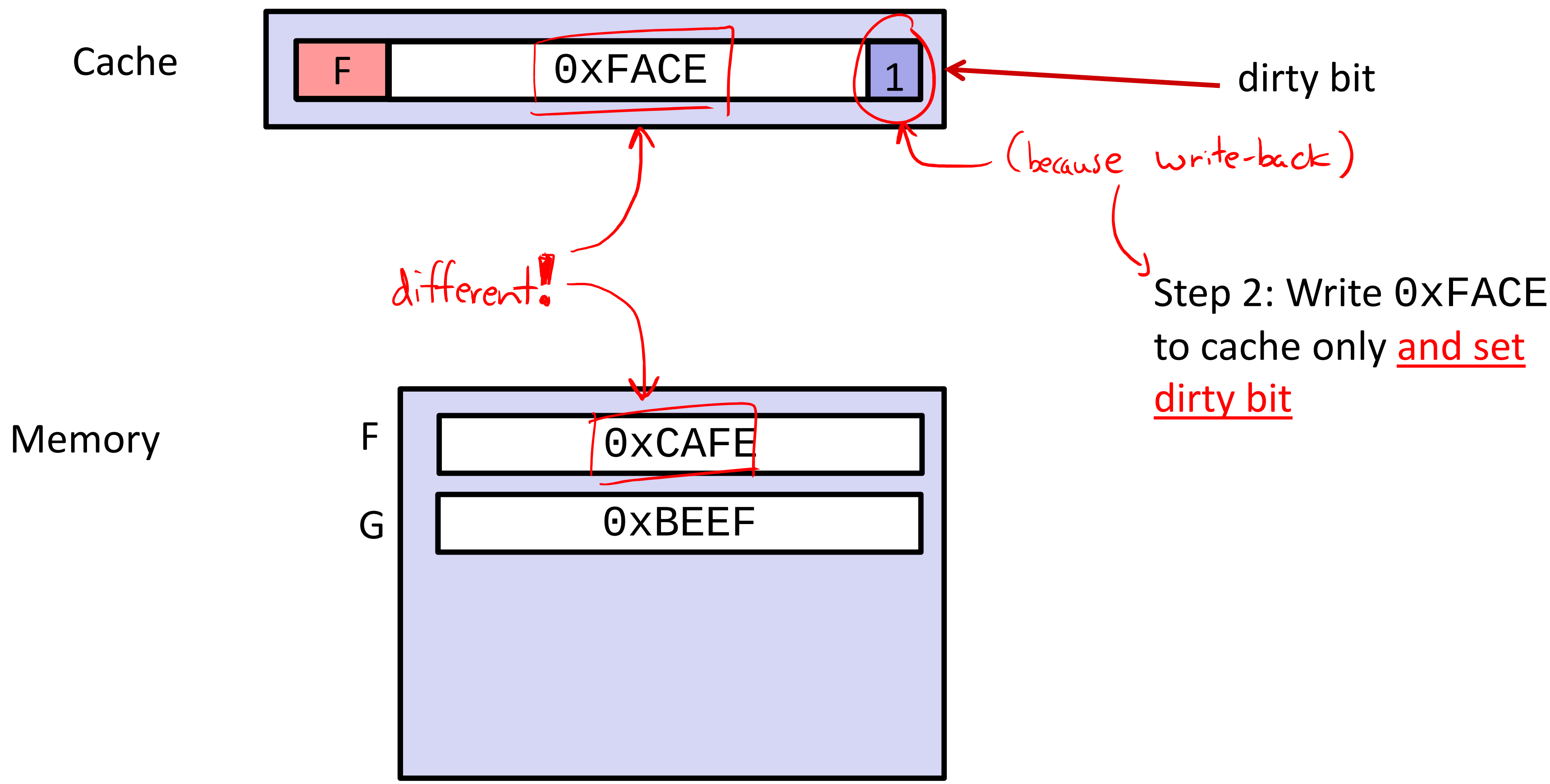


Step 1: Bring **F** into cache

Write-back, write-allocate example

`mov 0xFACE, F`

Write the new data to the cache (0xFACE); change the dirty bit to a 1 because now this data in cache no longer matches data in memory, so eventually we will have to go back and update the value in memory (once this data gets evicted from cache; it may change again before that occurs so no need to write to memory now)



Write-back, write-allocate example

```
mov 0xFACE, F        mov 0xFEED, F
```

write mis   write hit

Cache: F | ~~0xFACE~~ 0xFEED | 1 ← dirty bit

Again, we haven't adjusted the value at address F in memory yet, only in the cache, because this data has not been evicted from the cache yet (may be overwritten again so why waste time adjusting value in memory first?)

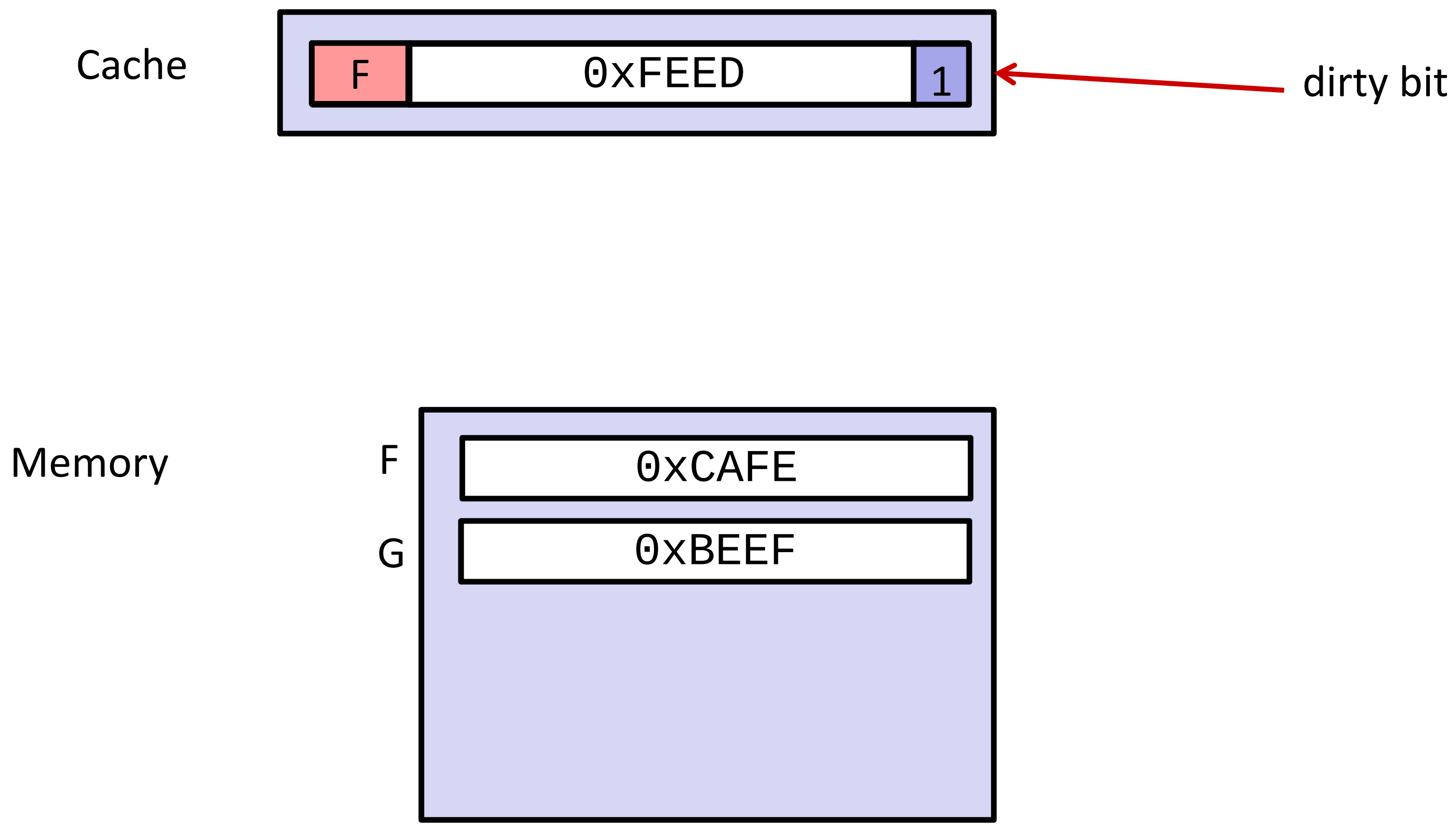
Memory:
F: 0xCAFE
G: 0xBEEF

Write hit!
Write `0xFEED` to cache only

Write-back, write-allocate example

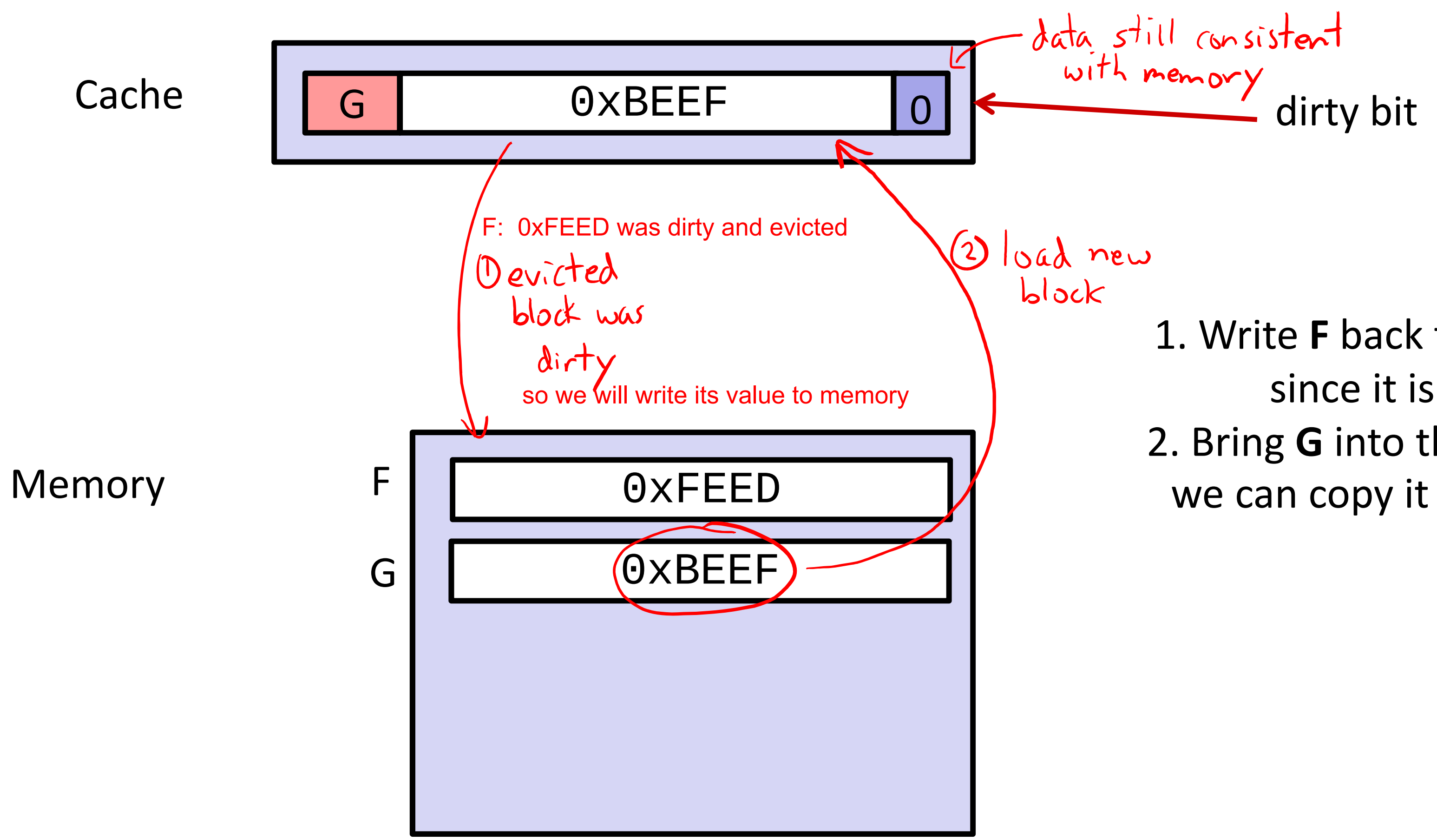
`mov 0xFACE, F` `mov 0xFEED, F` `mov G, %rax`

read miss



Write-back, write-allocate example

`mov 0xFACE, F`   `mov 0xFEED, F`   `mov G, %rax`



1. Write **F** back to memory since it is dirty
2. Bring **G** into the cache so we can copy it into `%rax`

Peer Instruction Question

see slide 21 of lecture 18

- Which of the following cache statements is FALSE?
  - Vote at http://pollev.com/rea

False A. We can reduce compulsory misses by decreasing our block size
smaller block size pulls fewer bytes into $ on a miss

True B. We can reduce conflict misses by increasing associativity
more options to place blocks before evictions occur

True C. A write-back cache will save time for code with good temporal locality on writes
frequently-used blocks rarely get evicted, so fewer write-backs

True D. A write-through cache will always match data with the memory hierarchy level below it
yes, its main goal is data consistency

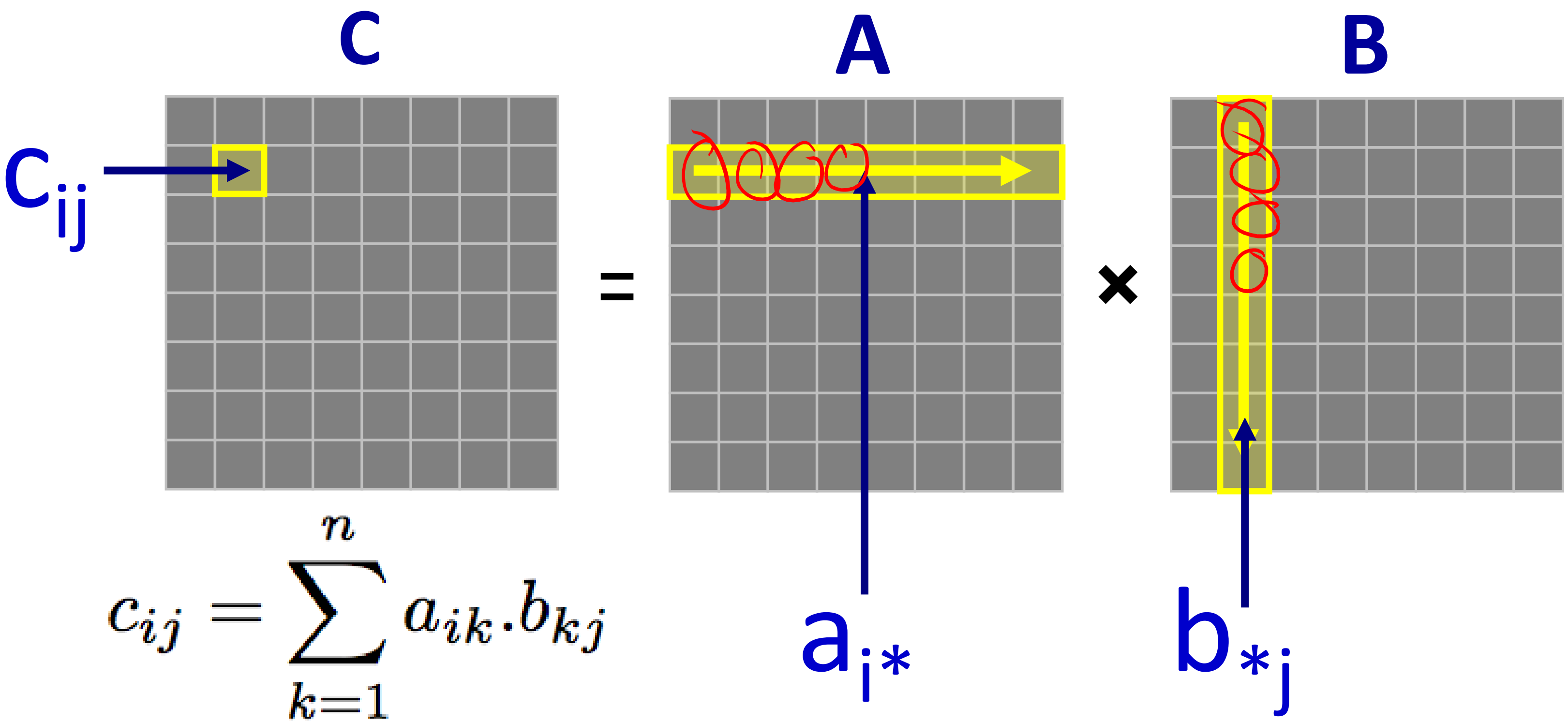
E. We're lost…

Optimizations for the Memory Hierarchy

- Write code that has locality!
  - Spatial: access data contiguously
  - Temporal: make sure access to the same data is not too far apart in time

- How can you achieve locality?
  - Adjust memory accesses in *code* (software) to improve miss rate (MR)
    - Requires knowledge of *both* how caches work as well as your system's parameters
  - Proper choice of algorithm
  - Loop transformations

Example: Matrix Multiplication



$$c_{ij} = \sum_{k=1}^{n} a_{ik}.b_{kj}$$

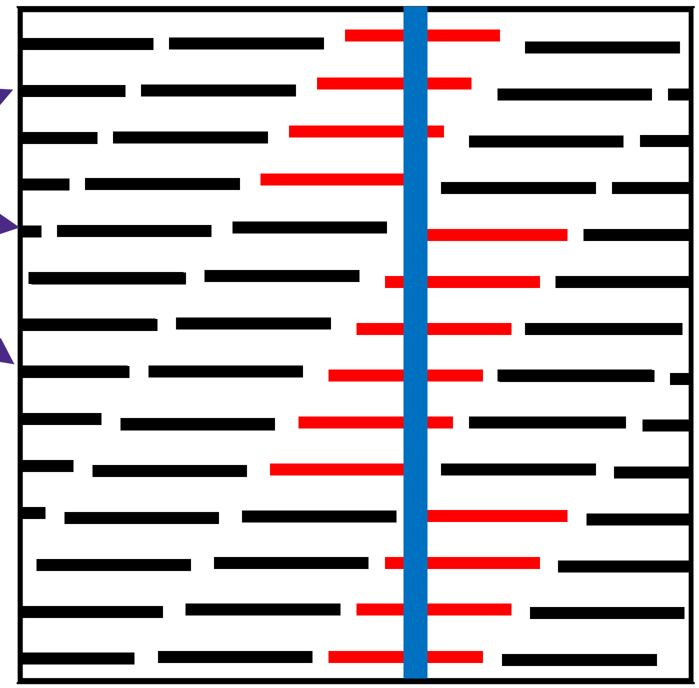
Matrices in Memory

- How do cache blocks fit into this scheme?
  - Row major matrix in memory:

all thjis means is that matrices are stored row-wise contiguously in memory. Namely, memory looks like

(row 0)(row1)(row2)….

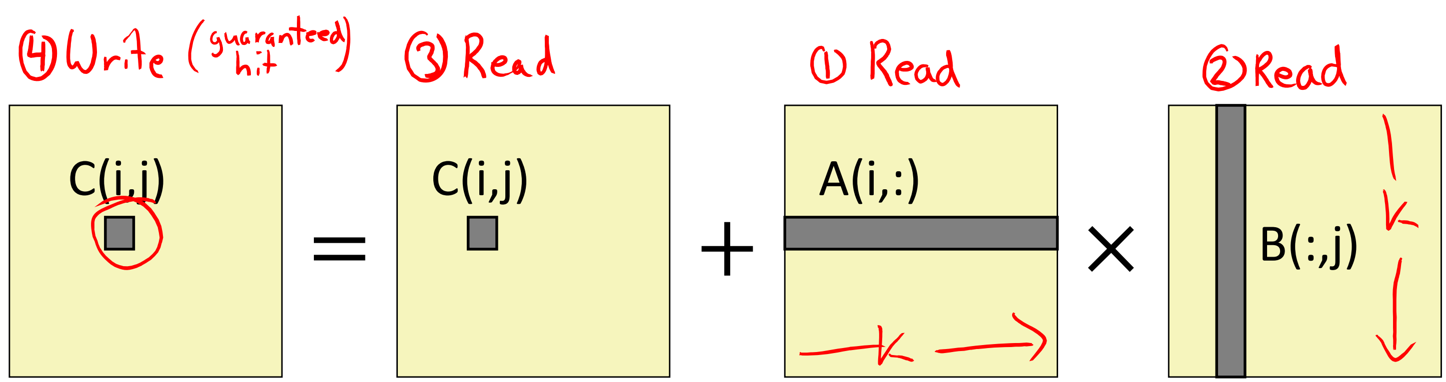
Cache blocks



COLUMN of matrix (blue) is spread among cache blocks shown in red

Naïve Matrix Multiply

```
# move along rows of A
for (i = 0; i < n; i++)
  # move along columns of B
  for (j = 0; j < n; j++)
    # EACH k loop reads row of A, col of B
    # Also read & write c(i,j) n times
    for (k = 0; k < n; k++)
      c[i*n+j] += a[i*n+k] * b[k*n+j];
```



c[i][j] += a[i][k] * b[k][j]

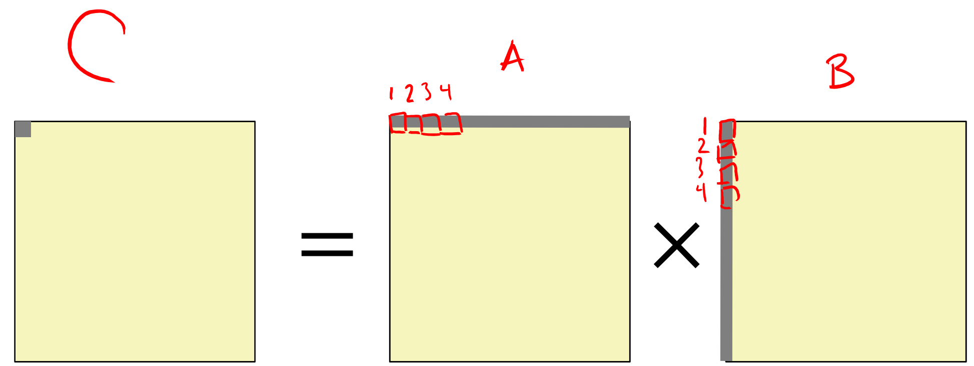
Cache Miss Analysis (Naïve)

Ignoring matrix c

- Scenario Parameters:
  - Square matrix ($n \times n$), elements are `doubles`
  - Cache block size K = 64 B = 8 doubles

    8 matrix elements per cache block
  - Cache size $C \ll n$ (much smaller than n)

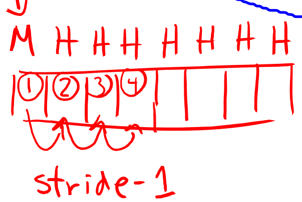
    key assumption!



- Each iteration:
  - $\overset{A}{\frac{n}{8}} + \overset{B}{n} = \frac{9n}{8}$ misses

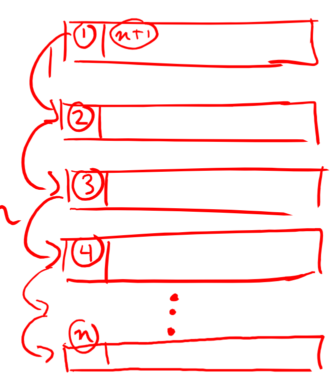
compulsory

A has good spatial locality



B has bad spatial locality

stride-n



by the time we get to n+1, block has been kicked out of $

e.g. each time we run through the inner loop (over k) on the previous page, we are traversing matrix A rowwise (and 2D arrays are stored rowwise in memory) so we have good spatial locality: when we load in a block of 8 elements, those are the next 8 elements used in the loop.

On the other hand, we traverse B column wise. As matrices are stored rowwise in memory, if a full row does not fit in cache then we will miss every time. Thjs is because the next element to be extracted is more than the block size away from the current one (so no two consecutive entries of a column are in same block)

Cache Miss Analysis (Naïve)

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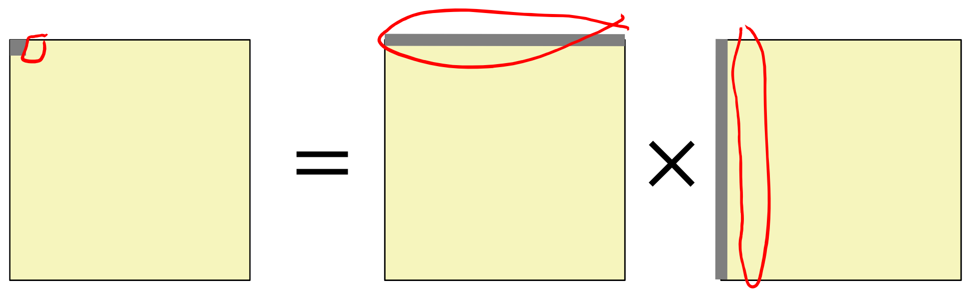
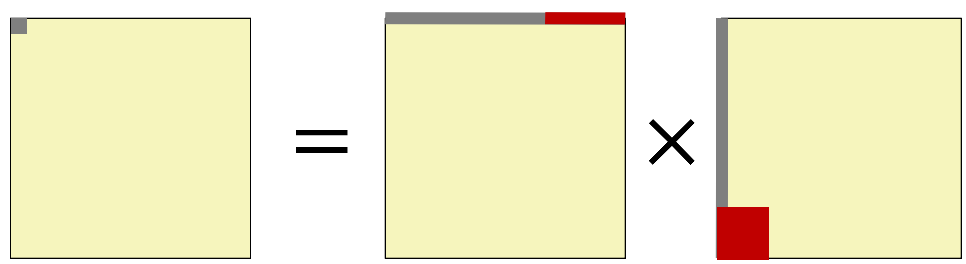
  - Cache size $C \ll n$ (much smaller than n)

- Each iteration:
  - $\frac{n}{8} + n = \frac{9n}{8}$ misses



  - Afterwards in cache: (schematic)

red showing blocks remaining in the $



8 doubles wide

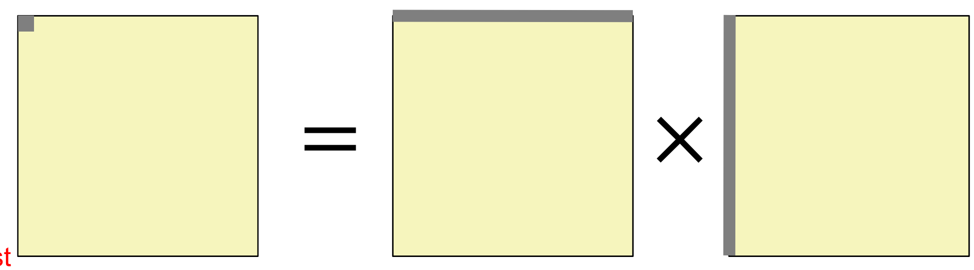
Cache Miss Analysis (Naïve)

Ignoring matrix c

- Scenario Parameters:
  - Square matrix ($n \times n$), elements are `doubles`
  - Cache block size K = 64 B = 8 doubles
  - Cache size $C \ll n$ (much smaller than n)

- Each iteration:
  - $\frac{n}{8} + n = \frac{9n}{8}$ misses

    e.g. misses in each runthrough of the innermost loop over k



- Total misses: $\frac{9n}{8} \times n^2 = \frac{9}{8}n^3$

once per element

e.g. for the loops over i,j on slide 14

Linear Algebra to the Rescue (1)

This is extra (non-testable) material

- Can get the same result of a matrix multiplication by splitting the matrices into smaller submatrices (matrix "blocks")

The idea here is that if we can fit these smaller blocks entirely into memory, we won't have as big an issue with constantly evicting data from the cache and poor spatial locality on B

- For example, multiply two 4×4 matrices:

$$\begin{array}{c} \quad A_{11} \qquad\qquad A_{12} \\ A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \\ \quad A_{21} \qquad\qquad A_{22} \end{array} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \text{ with } B \text{ defined similarly.}$$

$$\boxed{AB = \begin{bmatrix} (A_{11}B_{11} + A_{12}B_{21}) & (A_{11}B_{12} + A_{12}B_{22}) \\ (A_{21}B_{11} + A_{22}B_{21}) & (A_{21}B_{12} + A_{22}B_{22}) \end{bmatrix}}$$

Linear Algebra to the Rescue (2)

This is extra (non-testable) material

C_{11}	C_{12}	C_{13}	C_{14}
C_{21}	C_{22}	C_{23}	C_{24}
C_{31}	C_{32}	C_{43}	C_{34}
C_{41}	C_{42}	C_{43}	C_{44}

A_{11}	A_{12}	A_{13}	A_{14}
A_{21}	A_{22}	A_{23}	A_{24}
A_{31}	A_{32}	A_{33}	A_{34}
A_{41}	A_{42}	A_{43}	A_{144}

B_{11}	B_{12}	B_{13}	B_{14}
B_{21}	B_{22}	B_{23}	B_{24}
B_{32}	B_{32}	B_{33}	B_{34}
B_{41}	B_{42}	B_{43}	B_{44}

Matrices of size $n \times n$, split into 4 blocks of size r (n=$4r$)

$$C_{22} = A_{21}B_{12} + A_{22}B_{22} + A_{23}B_{32} + A_{24}B_{42} = \sum_k A_{2k}*B_{k2}$$

- Multiplication operates on small "block" matrices
  - Choose size so that they fit in the cache!
  - This technique called "*cache blocking*"

Blocked Matrix Multiply

6 nested loops may seem less efficient, but leads to much faster code!

- Blocked version of the naïve algorithm:

```
# move by rxr BLOCKS now
for (i = 0; i < n; i += r)
  for (j = 0; j < n; j += r)
    for (k = 0; k < n; k += r)
      # block matrix multiplication
      for (ib = i; ib < i+r; ib++)
        for (jb = j; jb < j+r; jb++)
          for (kb = k; kb < k+r; kb++)
            c[ib*n+jb] += a[ib*n+kb]*b[kb*n+jb];
```

walk thru entire matrix a block at a time

walk thru one block

  - r = block matrix size (assume r divides n evenly)

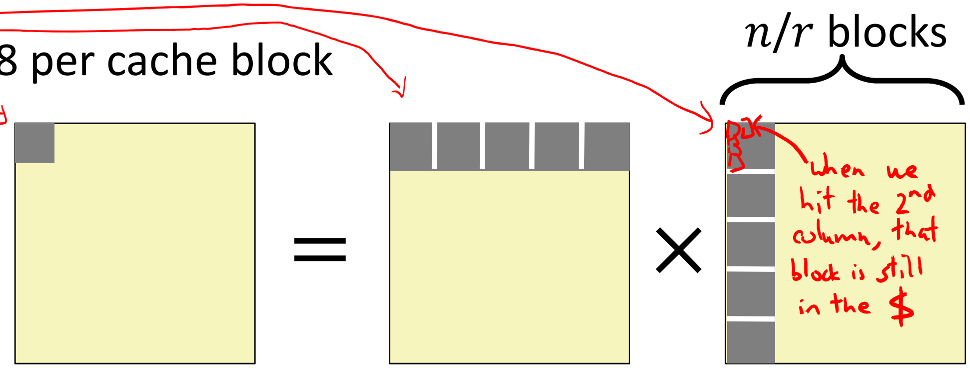
Cache Miss Analysis (Blocked)

Ignoring matrix c

- Scenario Parameters:
  - Cache block size K = 64 B = 8 doubles
  - Cache size $C \ll n$ (much smaller than n)
  - Three blocks ■ $(r \times r)$ fit into cache: $3r^2 < C$

r^2 elements per block, 8 per cache block

- Each block iteration:
  - $r^2/8$ misses per block
  - $2n/r \times r^2/8 = nr/4$



n/r blocks in row and column

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n/r blocks

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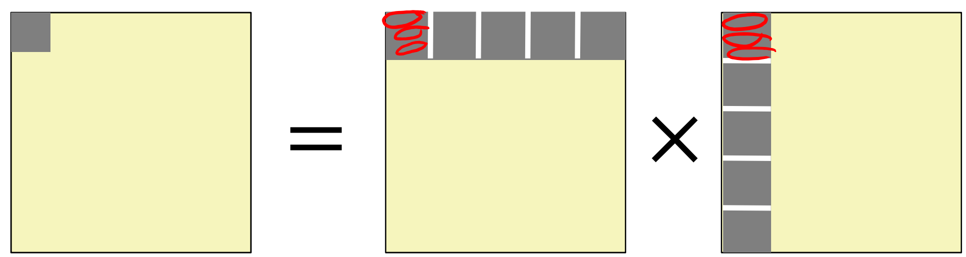
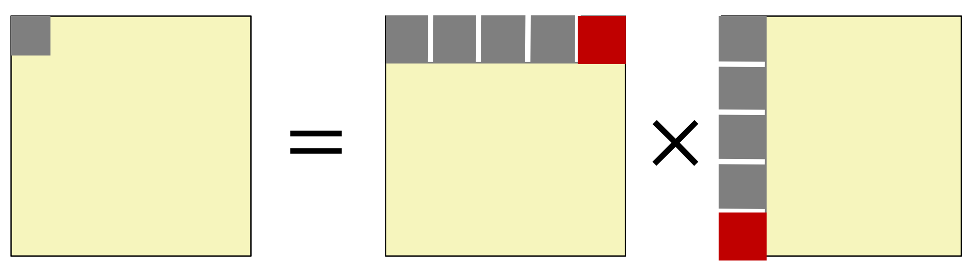


- Afterwards in cache (schematic)



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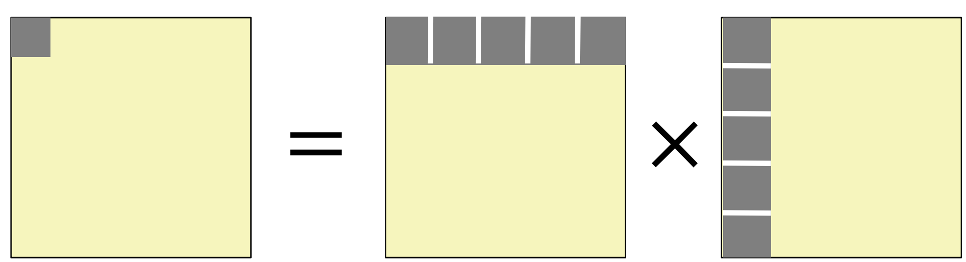


n/r blocks in row and column

- Total misses:
  - $nr/4 \times (n/r)2 = n^3/(4r)$ vs. $9n^3/8$

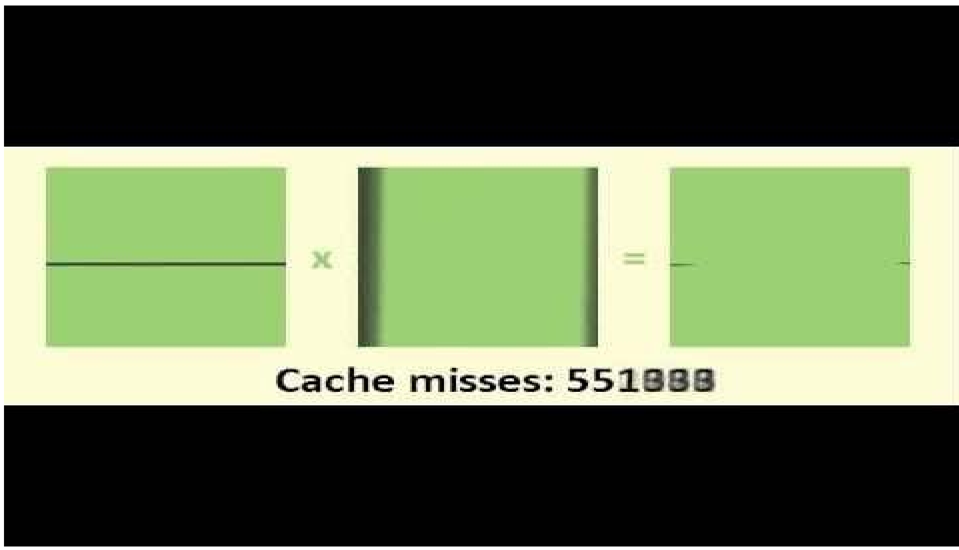
again we cant just choose r to be arbitrarily large as the block matrices have to fit in the cache for this approach to be worthwhile

Matrix Multiply Visualization

shaded areas show blocks stored in the $

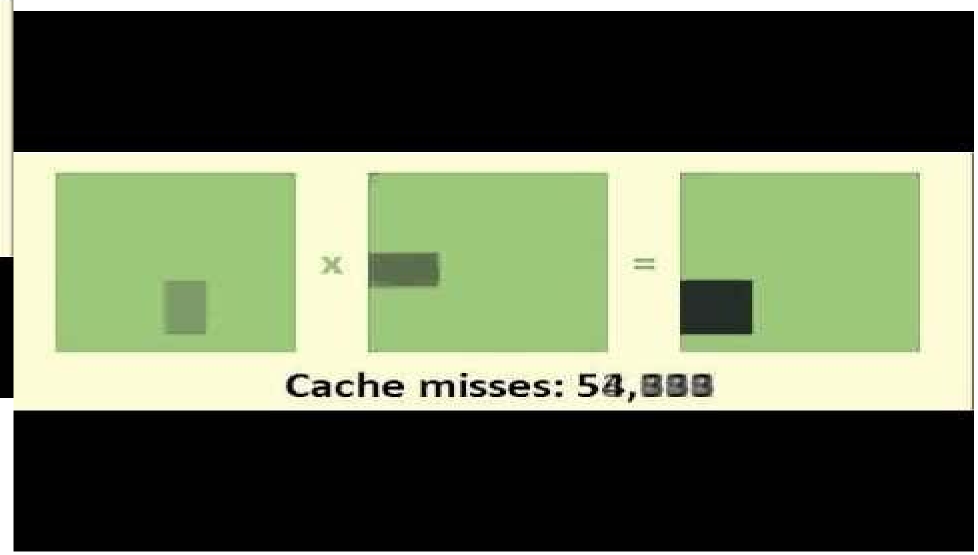
- Here n = 100, C = 32 KiB, r = 30

Naïve:



≈ 1,020,000
cache misses

Blocked:



≈ 90,000
cache misses

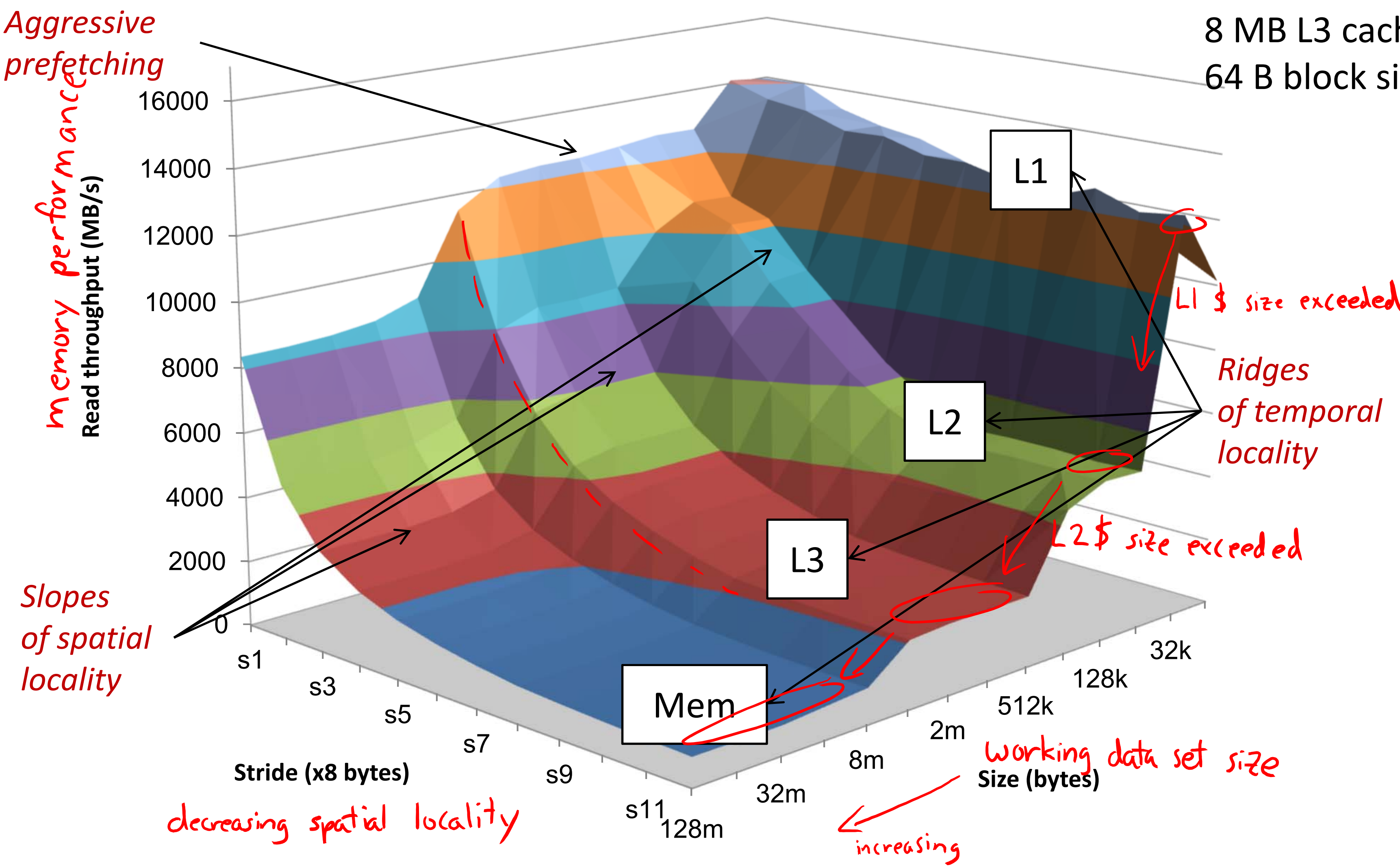
Cache-Friendly Code

- Programmer can optimize for cache performance
  - How data structures are organized
  - How data are accessed
    - Nested loop structure
    - Blocking is a general technique
- All systems favor "cache-friendly code"
  - Getting absolute optimum performance is very platform specific
    - Cache size, cache block size, associativity, etc.
  - Can get most of the advantage with generic code
    - Keep working set reasonably small (temporal locality)
    - Use small strides (spatial locality)
    - Focus on inner loop code

great general rules of thumb!

The Memory Mountain

Core i7 Haswell
2.1 GHz
32 KB L1 d-cache
256 KB L2 cache
8 MB L3 cache
64 B block size



Learning About Your Machine

- **Linux:**
  - `lscpu`
  - ls /sys/devices/system/cpu/cpu0/cache/index0/
    - Ex: cat /sys/devices/system/cpu/cpu0/cache/index*/size
- **Windows:**
  - `wmic memcache get <query>` (all values in KB)
  - Ex: `wmic memcache get MaxCacheSize`
- Modern processor specs: http://www.7-cpu.com/