Data III & Integers I

CSE 351 Spring 2019

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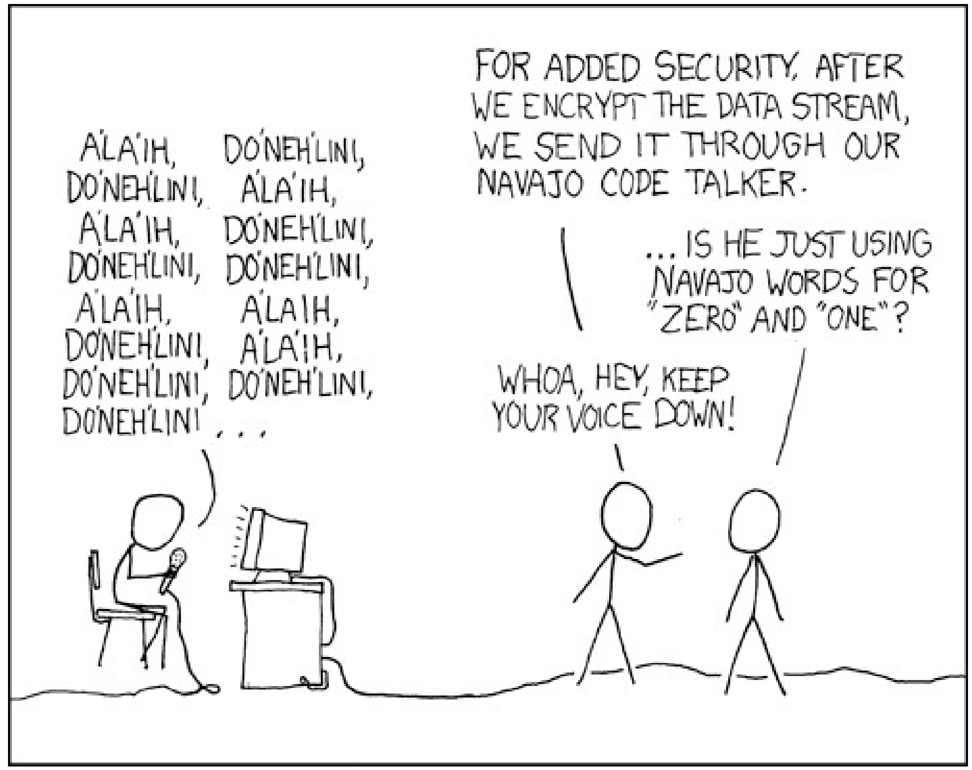


http://xkcd.com/257/

Administrivia

- Lab 0 due TODAY @ 11:59 pm
  - *You will be revisiting this program throughout this class!*
- Homework 1 due Wednesday
  - Reminder: autograded, 20 tries, no late submissions
- Lab 1a released
  - Workflow:
    1) Edit `pointer.c`
    2) Run the Makefile (`make`) and check for compiler errors & warnings
    3) Run ptest (`./ptest`) and check for correct behavior
    4) Run rule/syntax checker (`python dlc.py`) and check output
  - Due Monday 4/15, will overlap a bit with Lab 1b
    - We grade just your *last* submission

Lab Reflections

- All subsequent labs (after Lab 0) have a "reflection" portion
  - The Reflection questions can be found on the lab specs and are intended to be done *after* you finish the lab
  - You will type up your responses in a `.txt` file for submission on Canvas
  - These will be graded "by hand" (read by TAs)

- Intended to check your understand of what you should have learned from the lab
  - Also great practice for short answer questions on the exams

Memory, Data, and Addressing

- Hardware - High Level Overview
- Representing information as bits and bytes
  - Memory is a byte-addressable array
  - Machine “word” size = address size = register size
- Organizing and addressing data in memory
  - Endianness – ordering bytes in memory
- Manipulating data in memory using C
- **Boolean algebra and bit-level manipulations**

Boolean Algebra

- Developed by George Boole in 19th Century
  - Algebraic representation of logic (True → 1, False → 0)
  - AND: `A&B=1` when both A is 1 and B is 1
  - OR: `A|B=1` when either A is 1 or B is 1
  - XOR: `A^B=1` when either A is 1 or B is 1, but not both
  - NOT: `~A=1` when A is 0 and vice-versa
  - DeMorgan's Law: `~(A|B) = ~A & ~B`
    `~(A&B) = ~A | ~B`

AND

&	0	1
0	**0**	**0**
1	**0**	**1**

OR

|	0	1
0	**0**	**1**
1	**1**	**1**

XOR

^	0	1
0	**0**	**1**
1	**1**	**0**

NOT

~	
0	**1**
1	**0**

General Boolean Algebras

- Operate on bit vectors
  - Operations applied bitwise
  - All of the properties of Boolean algebra apply

```
  01101001      01101001      01101001
& 01010101    | 01010101    ^ 01010101    ~ 01010101
  01000001      01111101      00111100      10101010
```

- Examples of useful operations:

$$x \,\hat{}\, x = 0$$

"sets to 1"

$x \mid 1 = 1,$

$0 \mid 1 = 1$

$1 \mid 1 = 1$

"leaves as is"

$x \mid 0 = x$

$0 \mid 0 = 0$

$1 \mid 0 = 1$

```
  01010101
^ 01010101
  00000000   ← creates 0
```

```
  01010101   ← data of interest
| 11110000   ← bit mask (specifically chosen)
  11110101
```

set   left as is

Bit-Level Operations in C

bitwise & is the same as an
bitwise "product"

- & (AND), | (OR), ^ (XOR), ~ (NOT)
  - View arguments as bit vectors, apply operations bitwise
  - Apply to any "integral" data type
    - (8 bytes) (4 bytes) (2 bytes) (1 byte)
    - long, int, short, char, unsigned

bit vector will be width of datatype

- Examples with `char a, b, c;`

C code | Internally | Result

```
a = (char) 0x41;    // 0x41->0b 0100 0001
b = ~a;             //        0b 1011 1110->0x BE
```

```
a = (char) 0x69;    // 0x69->0b 0110 1001
b = (char) 0x55;    // 0x55->0b 0101 0101
c = a & b;          //        0b 0100 0001->0x 41
```

```
a = (char) 0x41;    // 0x41->0b 0100 0001
b = a;              //        0b 0100 0001
c = a ^ b;          //        0b 0000 0000->0x 00
```

Contrast: Logic Operations

- Logical operators in C: `&&` (AND), `||` (OR), `!` (NOT)
  - **0** is False, **anything nonzero** is True
  - **Always** return 0 or 1
  - Early termination (a.k.a. short-circuit evaluation) of `&&`, `||`

0xCC = 0b 1100 1100
0x33 = 0b 0011 0011

- Examples (`char` data type)

0xCC & 0x33 -> 0x00

  - $!\overset{T}{\texttt{0x41}} \;\texttt{->}\; \overset{F}{\texttt{0x00}}$
  - $!\overset{F}{\texttt{0x00}} \;\texttt{->}\; \overset{T}{\texttt{0x01}}$
  - $!(!\overset{T}{\texttt{0x41}}) \;\texttt{->}\; \overset{T}{\texttt{0x01}}$
  - $\overset{T}{\texttt{0xCC}} \;\texttt{\&\&}\; \overset{T}{\texttt{0x33}} \;\texttt{->}\; \texttt{0x01}$
  - $\overset{F}{\texttt{0x00}} \;\texttt{||}\; \overset{T}{\texttt{0x33}} \;\texttt{->}\; \texttt{0x01}$
  - $\overset{①}{\texttt{p}} \;\texttt{\&\&}\; \texttt{*}\overset{②}{\texttt{p}}$
    - If `p` is the null pointer (0x0), then `p` is never dereferenced! normally trying to dereference a null pointer would cause an error

If ① determines output of logical operator, then ② is never evaluated

Roadmap

C:

```
car *c = malloc(sizeof(car));
c->miles = 100;
c->gals = 17;
float mpg = get_mpg(c);
free(c);
```

Java:

```
Car c = new Car();
c.setMiles(100);
c.setGals(17);
float mpg =
    c.getMPG();
```

Assembly language:

```
get_mpg:
    pushq   %rbp
    movq    %rsp, %rbp
    ...
    popq    %rbp
    ret
```

Machine code:

```
0111010000011000
100011010000010000000010
1000100111000010
110000011111101000011111
```

OS:

Windows 10

OS X Yosemite

Computer system:

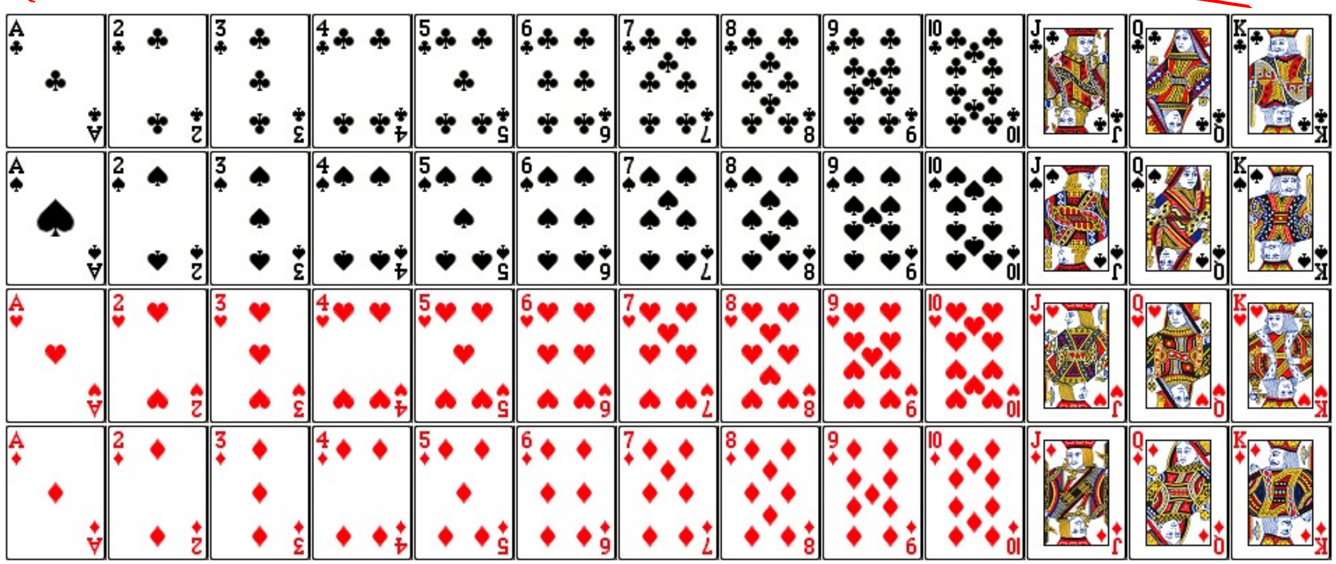
Memory & data
Integers & floats
x86 assembly
Procedures & stacks
Executables
Arrays & structs
Memory & caches
Processes
Virtual memory
Memory allocation
Java vs. C

But before we get to integers….

- Encode a standard deck of playing cards
- 52 cards in 4 suits
  - How do we encode suits, face cards?
- What operations do we want to make easy to implement?
  - Which is the higher value card?
  - Are they the same suit?

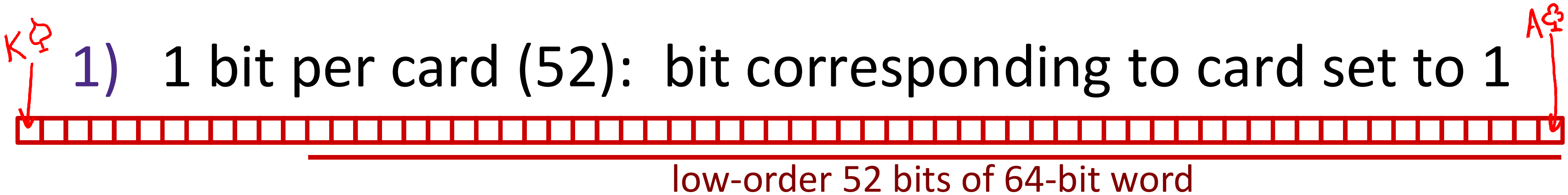
13 values

4 suits



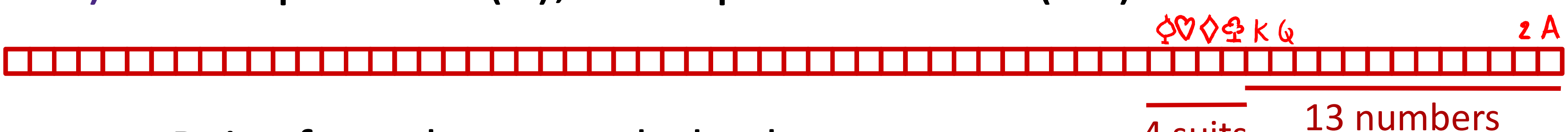
Two possible representations

1) 1 bit per card (52): bit corresponding to card set to 1



- "One-hot" encoding (similar to set notation)
- Drawbacks:
  - Hard to compare values and suits
  - Large number of bits required 52 bits $\xrightarrow{\text{fits in}}$ 7 bytes (56 bits)

2) 1 bit per suit (4), 1 bit per number (13): 2 bits set

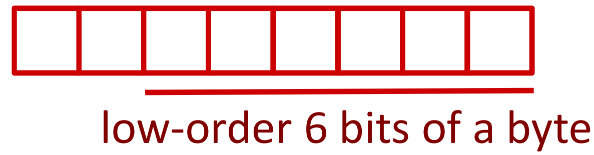


17 bits → 3 bytes

- Pair of one-hot encoded values
- Easier to compare suits and values, but still lots of bits used

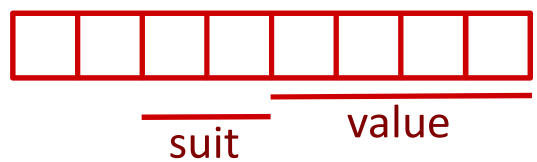
Two better representations

3) Binary encoding of all 52 cards – only 6 bits needed
- $2^6 = 64 \geq 52$
  $2^5 = 32 < 52$



- Fits in one byte (smaller than one-hot encodings)
- How can we make value and suit comparisons easier?

4) Separate binary encodings of suit (2 bits) and value (4 bits)



- Also fits in one byte, and easy to do comparisons

K	Q	J	. . .	3	2	A
1101	1100	1011	. . .	0011	0010	0001
13			. . .			1

C	♣	00
D	♦	01
H	♥	10
S	♠	11

Compare Card Suits

e.g. a filter

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.
Here we turn all *but* the bits of interest in *v* to 0.

```
char hand[5];          // represents a 5-card hand
char card1, card2;     // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( sameSuitP(card1, card2) ) { ... }
```

text substitution

```
#define SUIT_MASK  0x30

int sameSuitP(char card1, char card2) {
  return (!((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK)));
  //return (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}
```

returns int

equivalent

SUIT_MASK = 0x30 =

0	0	1	1	0	0	0	0

suit (keep) value (discard)

x & 0 = 0

x & 1 = x

Compare Card Suits

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.
Here we turn all *but* the bits of interest in *v* to 0.

```
#define SUIT_MASK  0x30

int sameSuitP(char card1, char card2) {
  return (!((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK)));
  //return (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}
```

card1

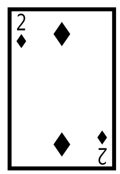
0	0	0	1	0	0	1	0

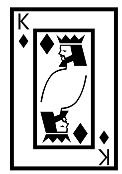
1 &

0	0	1	1	0	0	0	0

=

0	0	0	1	0	0	0	0





SUIT_MASK

x & 0 = 0

x & 1 = x

card2

0	0	0	1	1	1	0	1

& 1

0	0	1	1	0	0	0	0

=

0	0	0	1	0	0	0	0

2 ^

0	0	0	0	0	0	0	0

3 ! ← logical

0	0	0	0	0	0	0	1

If there is any bit where the two sequences don't agree, the XOR will produce a one in that bit. Hence the XOR is identically zero IFF the two bit sequences are the same.

`!(x^y)` equivalent to `x==y`

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.

Compare Card Values

```
char hand[5];         // represents a 5-card hand
char card1, card2;    // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( greaterValue(card1, card2) ) { ... }
```

```
#define VALUE_MASK  0x0F

int greaterValue(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE_MASK) >
          (unsigned int)(card2 & VALUE_MASK));
}
```

casting to an unsigned int will use the two-encoding formula sum x_i 2^i to convert to a nonnegative integer

VALUE_MASK = 0x0F =

0	0	0	0	1	1	1	1

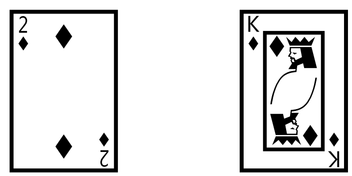
suit (discard) value (keep)

Compare Card **Values**

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.

```
#define VALUE_MASK  0x0F

int greaterValue(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE_MASK) >
          (unsigned int)(card2 & VALUE_MASK));
}
```



0	0	1	0	0	0	1	0

1 &

0	0	0	0	1	1	1	1

=

0	0	0	0	0	0	1	0

VALUE_MASK

0	0	1	0	1	1	0	1

& 1

0	0	0	0	1	1	1	1

=

0	0	0	0	1	1	0	1

2

$2_{10} > 13_{10}$

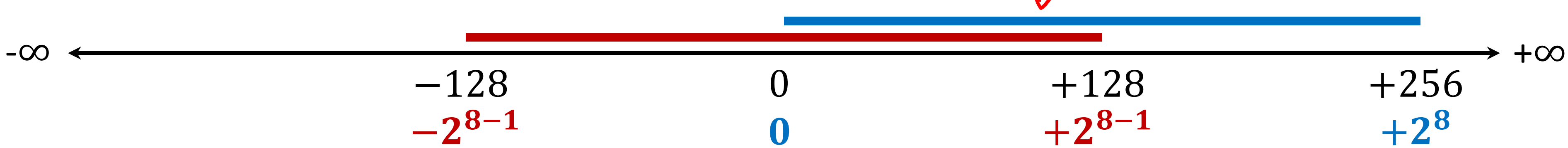
0 (false)

Integers

- **Binary representation of integers**
  - **Unsigned and signed**
  - Casting in C
- Consequences of finite width representation
  - Overflow, sign extension
- Shifting and arithmetic operations

Encoding Integers

- The hardware (and C) supports two flavors of integers
  - *unsigned* – only the non-negatives
  - *signed* – both negatives and non-negatives
- Cannot represent all integers with w bits
  - Only 2^w distinct bit patterns   w → 8 bits
  - Unsigned values: $0 \ldots 2^w - 1$ 0 . . . 255
  - Signed values: $-2^{w-1} \ldots 2^{w-1} - 1$ -128 . . . 127
    same widths, just shifted
- **Example:** 8-bit integers (*e.g.* `char`)



Unsigned Integers

- Unsigned values follow the standard base 2 system
  - $b_7b_6b_5b_4b_3b_2b_1b_0 = b_7 2^7 + b_6 2^6 + \cdots + b_1 2^1 + b_0 2^0$
- Add and subtract using the normal "carry" and "borrow" rules, just in binary

$$\begin{array}{r} 63 \\ +\ \underline{\ \ 8} \\ 71 \end{array} \qquad \begin{array}{r} \scriptstyle 32\ 16\ 8\ 4\ 2\ 1 \\ 00111111 \\ +\underline{00001000} \\ 01000111 \end{array}$$

← 6 1's in a row

This "adding binary" only makes sense in the interpretation of the binary numbers as unsigned integers. Here, the addition is compatible with the above encoding. So instead of converting both the binary numbers to unsigned integers, finding the sum, and converting it back this gives a convenient way to just add the binary numbers

- Useful formula: $2^{N-1} + 2^{N-2} + \ldots + 2 + 1 = 2^N - 1$
  - *i.e.* N ones in a row = $2^N - 1$
- How would you make *signed* integers?

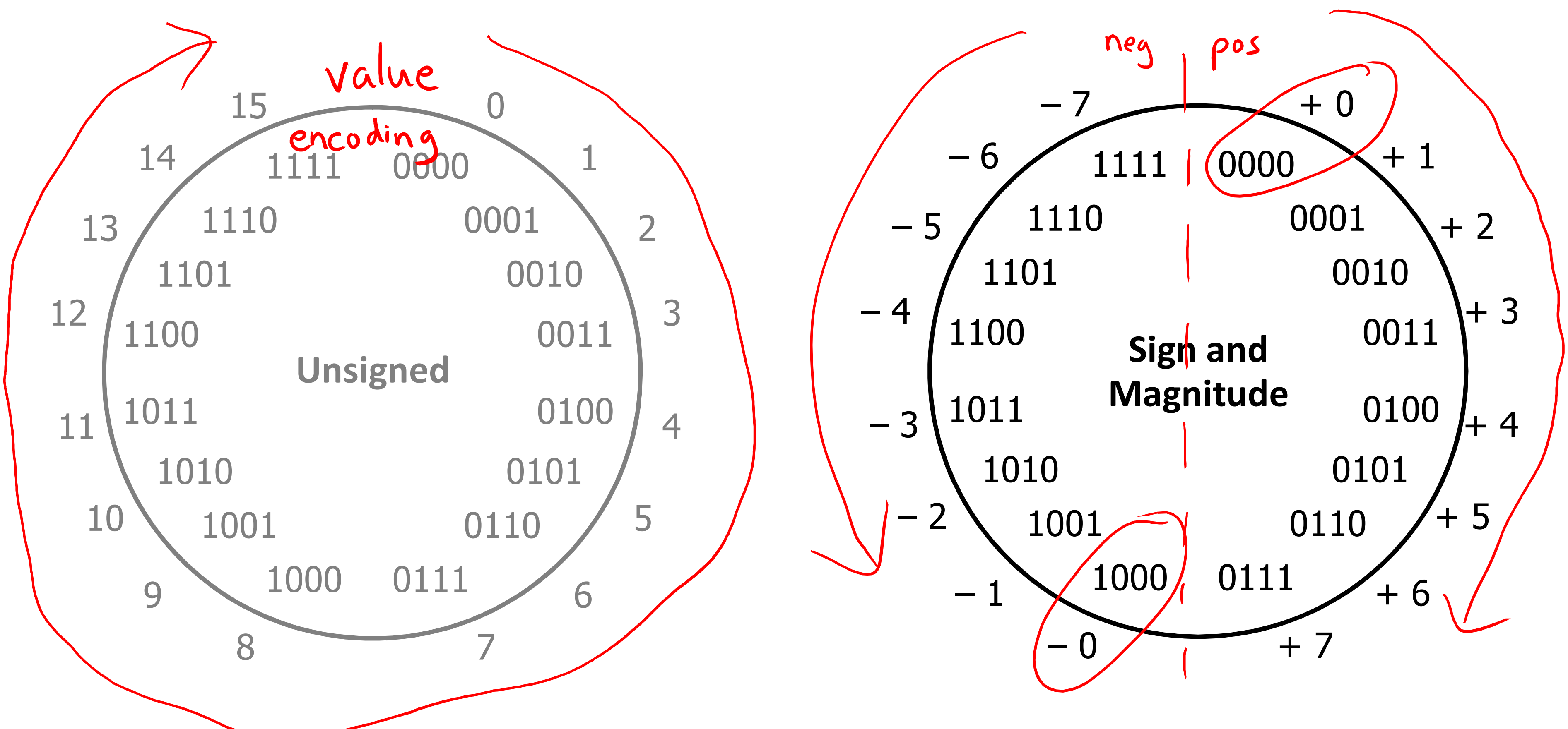
Sign and Magnitude

Most Significant Bit

- Designate the high-order bit (MSB) as the "sign bit"
  - `sign=0`: positive numbers; `sign=1`: negative numbers
- Benefits:
  - Using MSB as sign bit matches positive numbers with unsigned unsigned: $0b\,0010 = 2^1 = 2$; sign + mag: $0b\,0010 = +2^1 = 2$ ☑
  - All zeros encoding is still = 0
- Examples (8 bits):
  - 0x00 = 00000000_2 is non-negative, because the sign bit is 0
  - 0x7F = 01111111_2 is non-negative ($+127_{10}$)
  - 0x85 = 10000101_2 is negative (-5_{10})
  - 0x80 = 10000000_2 is negative... zero???

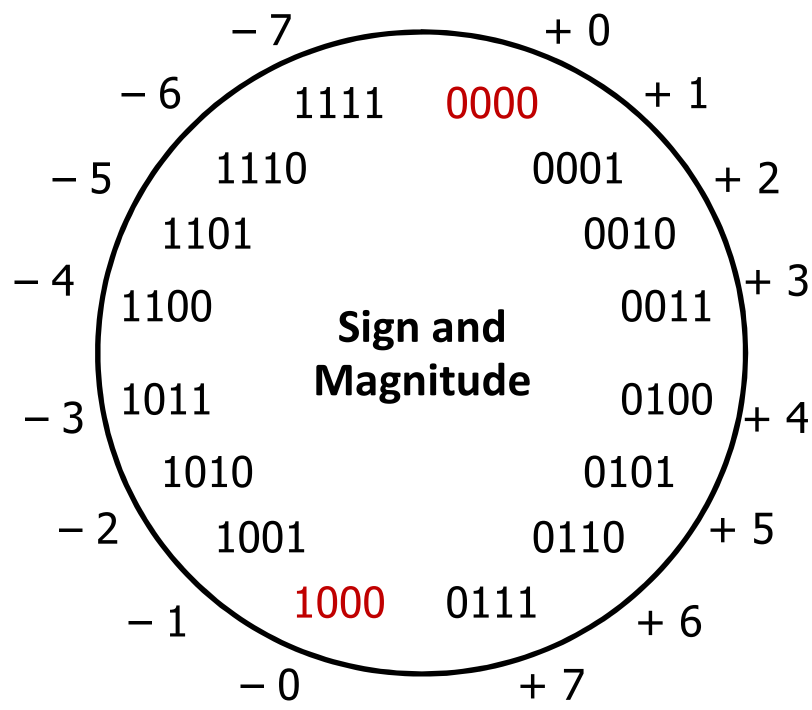
Sign and Magnitude

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks?



Sign and Magnitude

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)



Sign and Magnitude

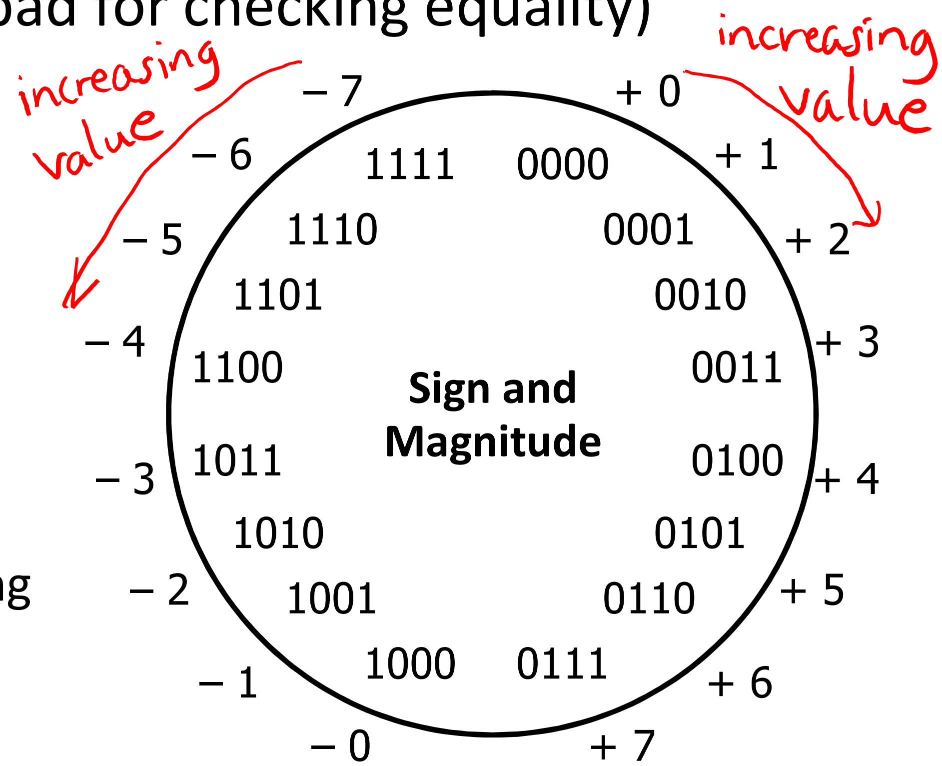
- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)
  - Arithmetic is cumbersome
    - Example: `4-3 != 4+(-3)`

4	0100
− 3	− 0011
1	0001

✓

4	0100
+ −3	+ 1011
−7	1111

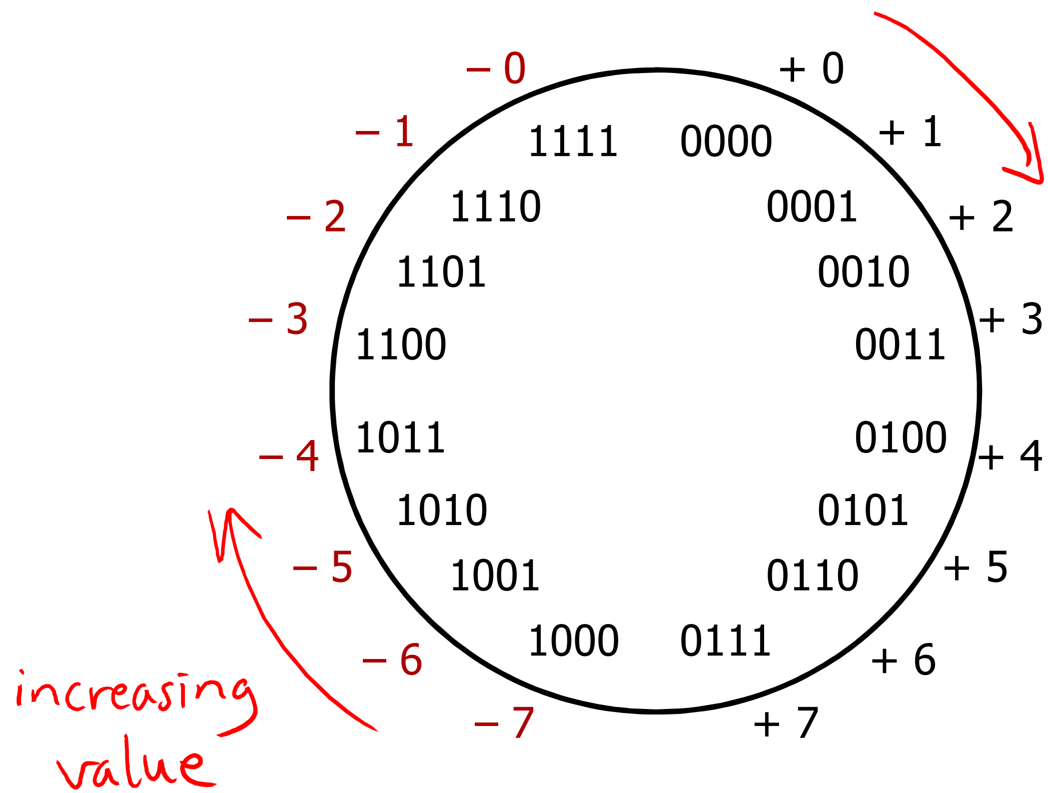
✗



- Negatives "increment" in wrong direction!

Two’s Complement

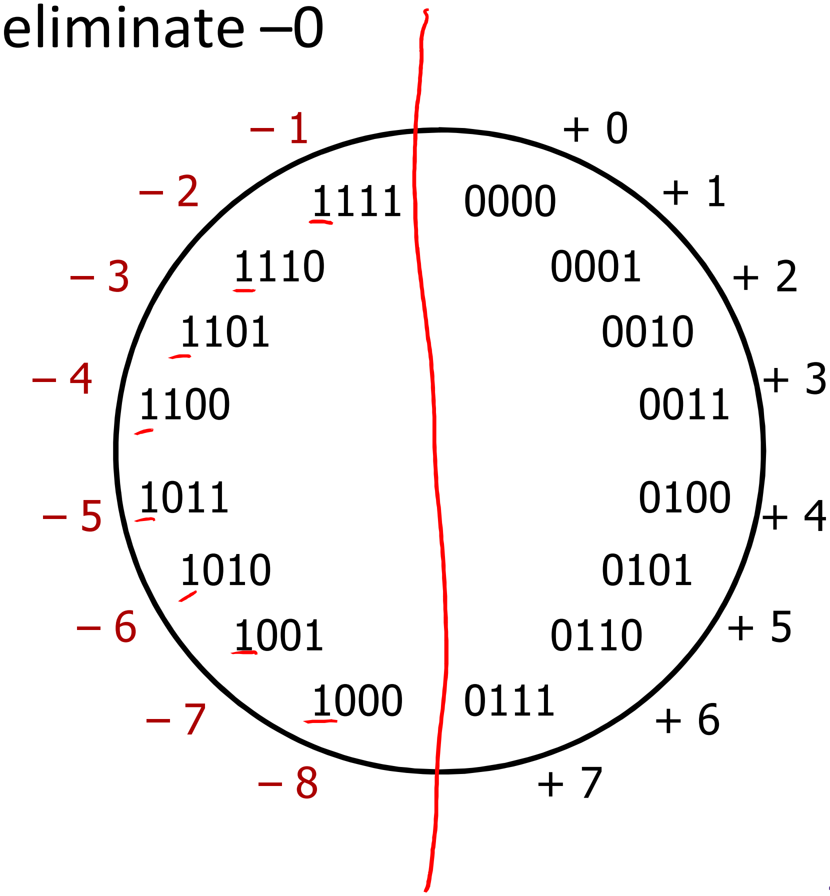
- Let’s fix these problems:
  1) “Flip” negative encodings so incrementing works



Two's Complement

- Let's fix these problems:
  1) "Flip" negative encodings so incrementing works
  2) "Shift" negative numbers to eliminate –0

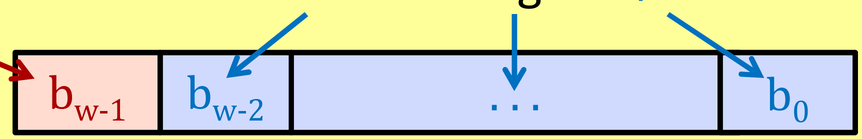
- MSB *still* indicates sign!
  - This is why we represent one more negative than positive number (-2^{N-1} to 2^{N-1} –1)



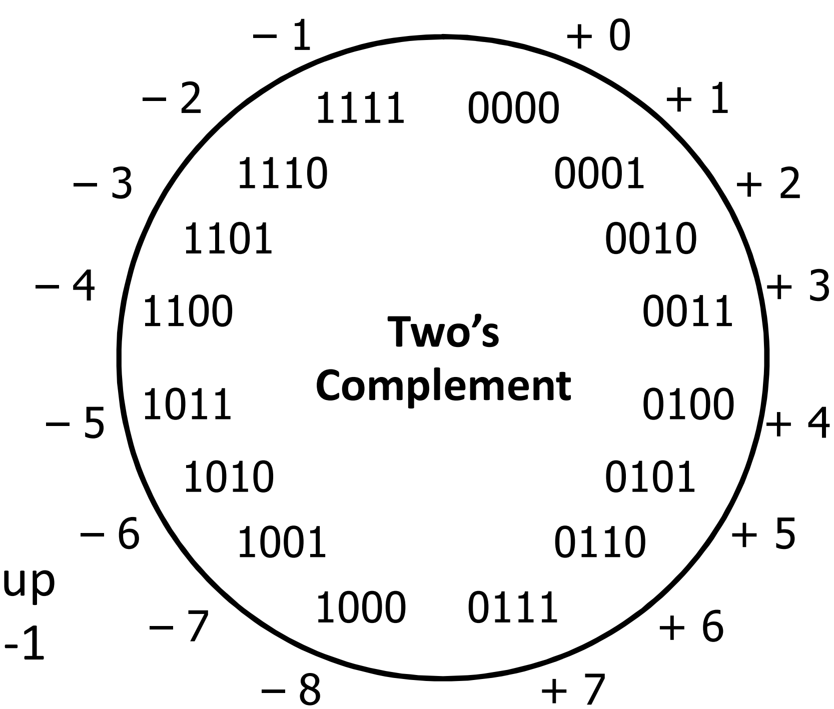
Two's Complement Negatives

- Accomplished with one neat mathematical trick!

b_{w-1} has weight -2^{w-1}, other bits have usual weights $+2^{i}$

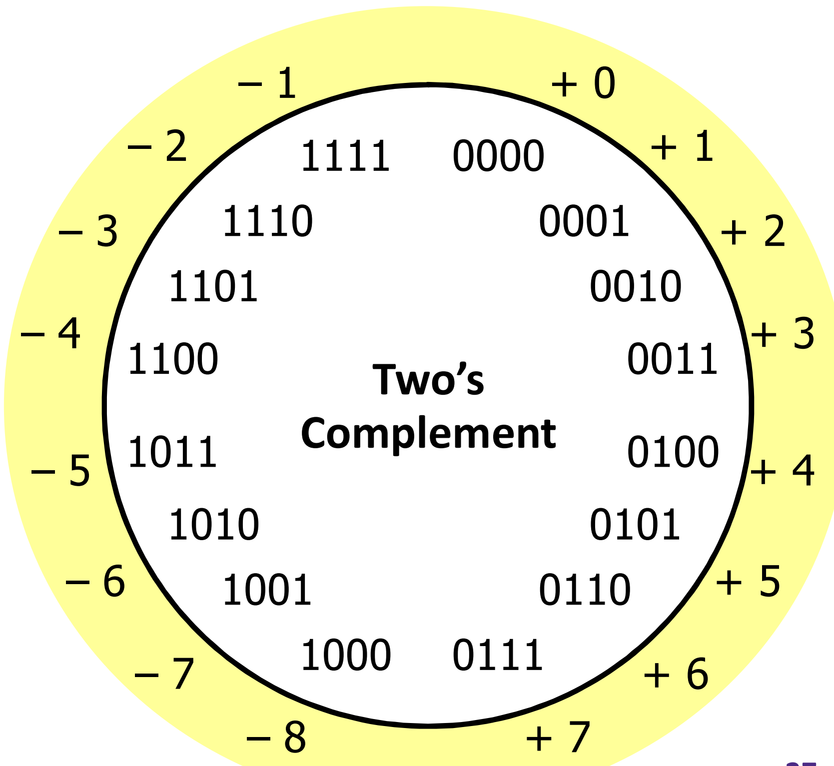


- 4-bit Examples:
  - 8 4 2 1
  - 1010_2 unsigned:
    $1*2^3+0*2^2+1*2^1+0*2^0 =$ **10**
  - 8 4 2 1
  - 1010_2 two's complement:
    $-1*2^3+0*2^2+1*2^1+0*2^0 =$ **–6**
- -1 represented as:
  $1111_2 = -2^3+(2^3-1)$
  3 one's in a row
  - MSB makes it super negative, add up all the other bits to get back up to -1



Why Two's Complement is So Great

- Roughly same number of (+) and (–) numbers
- Positive number encodings match unsigned
- Single zero
- All zeros encoding = 0



- Simple negation procedure:
  - Get negative representation of any integer by taking bitwise complement and then adding one!

```
( ~x + 1 == -x )
```

complement the bit encoding of the number x, convert it to an integer via the two's complement encoding, and then add one. This gives the negative of the original integer value

Peer Instruction Question

- Take the 4-bit number encoding `x = 0b1011` MSB
- Which of the following numbers is NOT a valid interpretation of `x` using any of the number representation schemes discussed today?
  - Unsigned, Sign and Magnitude, Two's Complement
  - Vote at http://PollEv.com/rea

$\boxed{\text{A. -4}}$

~~B. -5~~

~~C. 11~~

~~D. -3~~

E. We're lost…

unsigned: $8+2+1=11$

sign + mag: $1011 \rightarrow -(2+1)=-3$

two's: $-8+2+1=-5$

$-x = 0b\ 0100+1 = 5 \rightarrow x=-5$

Summary

- Bit-level operators allow for fine-grained manipulations of data
  - Bitwise AND (`&`), OR (`|`), and NOT (`~`) different than logical AND (`&&`), OR (`||`), and NOT (`!`)
  - Especially useful with bit masks
- Choice of *encoding scheme* is important
  - Tradeoffs based on size requirements and desired operations
- Integers represented using unsigned and two's complement representations
  - Limited by fixed bit width
  - We'll examine arithmetic operations next lecture