# Adversarial Search I: Playing: 2-Person, 0-Sum Games

CSE 415: Introduction to Artificial Intelligence
University of Washington
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## Outline

- Two-person, zero-sum games.
- Static evaluation functions.
- Minimax search.



## Two-Person, Zero-Sum, Perfect Information Games

1. A two-person, zero-sum game is a game in which only one player wins and only one player loses. There may be ties ("draws"). There are no "win-win" or "lose-lose" instances.

2. Most 2PZS games involve turn taking. In each turn, a player makes a move. Turns alternate between the players.

3. Perfect information: no randomness as in Poker or bridge.

4. Examples of 2PZS games include Tic-Tac-Toe, Othello, Checkers, and Chess.



## Why Study 2PZS Games in AI?

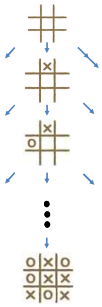
1. Games are idealizations of problems.
2. Some problems involve adversaries, such as companies competing in a marketplace.
3. A game opponent can serve as a proxy for the environment of an agent.
4. We'll transition from non-stochastic games to games with random aspects.



## Tic-Tac-Toe

A tree search in Tic Tac Toe involves alternating levels for players X and O.

state space tree



X

O

X

O



## Static Evaluation Functions

In most of the interesting 2PZS games, state spaces are too large to exhaustively search each alternative evolutionary path to its end.

Thus we assume here that our agent does not have enough computational resources to completely explore the game's search tree starting from the current state, except when the game is almost over. So the search has to be cut off; this is typically based on a decision to look ahead some number *k* of moves to all states reachable in k or fewer moves.

To estimate how good a given game state *s* is, let's compute a real-valued function *h*(*s*) of the state: *h*(*s*) will be high if it is favorable to one player (the player we'll call Max) and unfavorable to the other player (whom we will call Min).

NOT based on how far you are from a goal state but more like how likely you are to reach a goal state

This function *h*(*s*) is called a *static evaluation function*.



the key idea is that cannot know for sure what our opponent will do, so we have to consider all of their possible moves if we were to make a given move, and what kind of states this may bring us to. Much like heuristics, calculating this explicitly by counting all possible states is likely not feasible, so we have to develop some montra for evaluating possible states.

# Tic-Tac-Toe Static Eval. Fn.

Static Evaluation functions

Here is a possible static evaluation function for Tic-Tac-Toe.

h(s) = 100 A + 10 B + C – (100 D + 10 E + F)

A = number of lines of 3 Xs in a row.
B = number of lines of 2 Xs in a row (not blocked by an O)
C = number of lines containing one X and no Os.

XX_
XXO
X_X
XX_
XXO

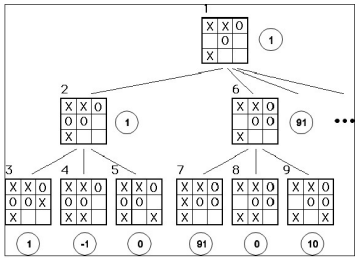
D = number of lines of 3 Os in a row.
E = number of lines of 2 Os in a row (not blocked by an X)
F = number of lines containing one O and no Xs.

X player is trying to maximize h(s) — maximizing players
O player is trying to minimize h(s) — minimizing players



idea: look ahead a few moves, find all states reachable by these moves, evaluate all of these possible states (WITHOUT considering future states) using our "static evaluation function", and consider best-case/worst case scenarios

# Minimax Search (Illustration)






O's turn; wants to minimize
If he chooses 2, he know X will want to maximize so X will choose 3
If he chooses 6 he knows X will want to maximize and X will choose 7
Of these two choices for O, choosing 2 has a much better worst case scenario, so choose 2.

# Minimax Search (Rationale)

If looking ahead one move, generate all successors of the current state, and apply the static evaluation function to each of them, and if we are Max, make the move that goes to the state with the maximum score.

If looking ahead two moves, we will be considering the positions that our opponent can get two in one move, from each of the positions that we can get to in one move.

Assuming that the opponent is playing rationally, the opponent, Min, will be trying to minimize the value of the resulting board.

Therefore, instead of using the static value at each successor of the current state, we examine the successors of each of those, computing their *static* values, and take the minimum of those as the value of our successor.



# Minimax Search (Algorithm)

```
Procedure minimax(board, whoseMove, plyLeft):
        if plyLeft == 0:  return staticValue(board)
        if whoseMove == 'Max': provisional = -100000
        else: provisional = 100000
        for s in successors(board, whoseMove):
                newVal = minimax(s, other(whoseMove), plyLeft-1)
            if (whoseMove == 'Max' and newVal > provisional\
                or (whoseMove == 'Min' and newVal < provisional):
                        provisional = newVal
        return provisional
```



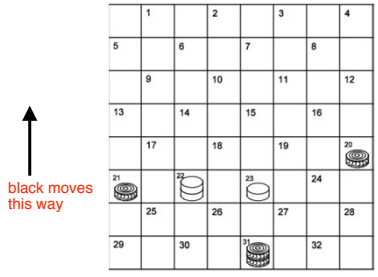
# Checkers Static Evaluation

Example in Checkers:
$h(s) = 5x_1 + x_2$
Where $x_1$ = Max's king advantage;
$x_2$ = Max's single man advantage.

photo courtesy of thegamesupply.com.



# Checkers Example




Black to move,
White = "Min",
Black = "Max"

Ss
State-space Search

# Minimax Search Example