Time Series
First Exam

Question	Points	Max Points
1		10
2		10
3		12
4		5
5		5
6		8
Total		50

Instructions:

The examination lasts 90 minutes. You are not allowed to use a material. No interaction with anyone except the instructor is allowed. There will be another file for the simulation questions called"Exam1_Simulation.RMD".

Rhodes College Honor Code:
"As a member of the Rhodes community, I pledge I will not lie, cheat, or steal, and that I will report any such violation that I may witness."

Signature:

(printed name)

(signature)

1. (a) (3 pts) Carefully and briefly describe a **white noise.**

(b) (3 pts) In the simulation section; simulate and plot a white noise with mean zero and standard deviation equal to 5.

(c) (4 pts) Is your simulated noise w_t stationary; how you can tell both graphically and mathematically? give a detailed answer.

2. (a) (3 pts) Using the simulated noise w_t. Let $v_t = \frac{1}{3}(w_{t-1} + w_t + w_{t+1})$. In the simulation section; simulate and plot v_t.

(b) (3 pts) Explain **in detail** why would someone condiser such a process, v_t. Support your points with matheatical calcualtions.

(c) (4 pts) Show that v_t is stationary and find its autocorrelation function (ACF).

3. Consider the random walk with drift model

$$x_t = \delta + x_{t-1} + w_t$$

for $t = 1, 2, \ldots$, $X_0 = 0$. where w_t is white noise with variance σ^2.

(a) (3 pts) Show that (**step by step**) the model can be written as $x_t = \delta t + \sum_{k=1}^{t} w_k$

(b) (4 pts) Find the mean function and the **autocovariance** function of x_t. Explain the details of the calculations.

(c) (3 pts) In the simulation section; simulate and plot a **any path** of the process x_t.

(d) (2 pts) Argue: is process x_t stationary? Use both the **plot and the mathemtical calcuations** in your arguments.

4. (5 pts) Find the cross-correlation function (CCF) between the processes; $x_t = w_t + w_{t-1}$ and $y_t = w_t - w_{t-1}$. Are the series jointly stationary?

5. (5 pts) Considering the simultaneous monthly readings of the SOI and an index for the number of new fish (Recruitment). Imagine you are now in a job interview, and you are asked to analyze the following plots, (write clearly using only well-defined terms). Make at least three clear points from each plot and the more points the better

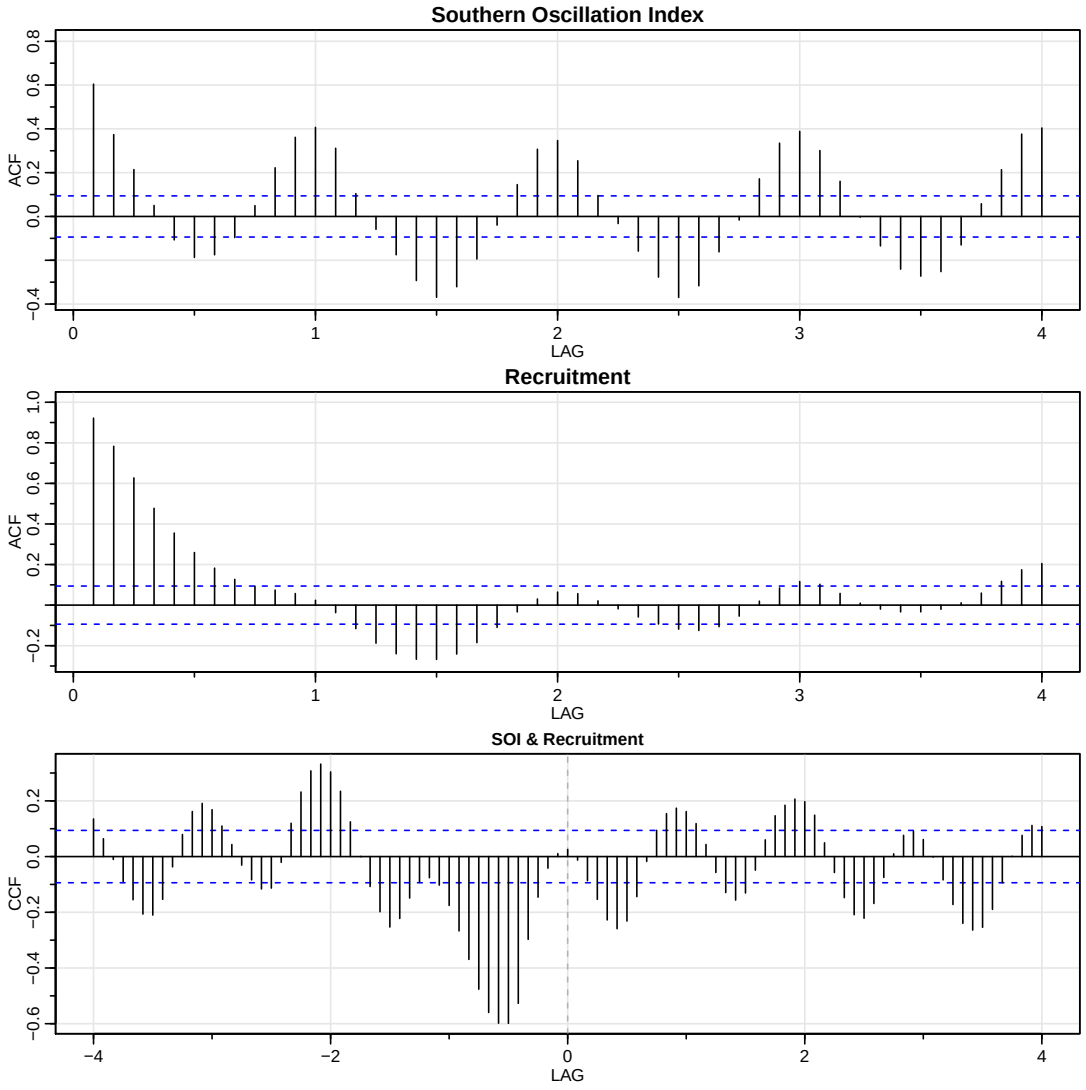
```
par(mfrow=c(3,1))
acf1(soi, 48, main="Southern Oscillation Index")
```

```
##  [1]  0.60  0.37  0.21  0.05 -0.11 -0.19 -0.18 -0.10  0.05  0.22  0.36
## [12]  0.41  0.31  0.10 -0.06 -0.17 -0.29 -0.37 -0.32 -0.19 -0.04  0.15
## [23]  0.31  0.35  0.25  0.10 -0.03 -0.16 -0.28 -0.37 -0.32 -0.16 -0.02
## [34]  0.17  0.33  0.39  0.30  0.16  0.00 -0.13 -0.24 -0.27 -0.25 -0.13
## [45]  0.06  0.21  0.38  0.40
```

```
acf1(rec, 48, main="Recruitment")
```

```
##  [1]  0.92  0.78  0.63  0.48  0.36  0.26  0.18  0.13  0.09  0.07  0.06
## [12]  0.02 -0.04 -0.12 -0.19 -0.24 -0.27 -0.27 -0.24 -0.19 -0.11 -0.03
## [23]  0.03  0.06  0.06  0.02 -0.02 -0.06 -0.09 -0.12 -0.13 -0.11 -0.05
## [34]  0.02  0.08  0.12  0.10  0.06  0.01 -0.02 -0.03 -0.03 -0.02  0.01
## [45]  0.06  0.12  0.17  0.20
```

```
ccf2(soi, rec, 48, main="SOI & Recruitment")
```


Southern Oscillation Index
ACF
LAG
Recruitment
ACF
LAG
SOI & Recruitment
CCF
LAG

6. As promissed: Let x_t be AR(1) model. $x_t = \phi x_{t-1} + w_t$ where w_t is white noise with mean 0 and standard deviation σ.

   (a) (3 pts) In the simulatioin section. Simulate and plot a path of an AR(1) process and plot the autocorrelation function for the simulated process.

   (b) (5 pts) Show that $\rho_x(h) = \phi^h$.