Midterm Study Guide

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Contents

1 Introductory Material Review

1.1 Asymptotic Bounds

Definition 1 (Big-O). $f(n) = O(g(n))$ if there exists a positive constant c and an integer n_0 such that $f(n) \leq c \cdot g(n)$ for all $n \geq n_0$.

Definition 2 (Big-Ω). $f(n) = \Omega(g(n))$ if there exists a positive constant c and an integer n_0 such that $c \cdot g(n) \leq f(n)$ for all $n \geq n_0$.

Definition 3 (Big-Θ). $f(n) = \Theta(g(n))$ if there exists positive constants c_1, c_2, and an integer n_0 such that $c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$ for all $n \geq n_0$.

1.2 Inductive Proofs

2 Solving Recurrence Relations

2.1 Master Theorem

2.2 Unpacking Tree / Algebraic Pattern

2.3 Substitution

2.4 Guess and Verify

2.5 Practice Problems

2.5.1 HW3 - Ex.4

Like in many previous exercises and homeworks, find tight asymptotic bounds (big-Theta) for $T(n)$ in each of the cases.

Problem 2.1. $T(n) = 2T(n/4) + n^2\sqrt{n}$

Problem 2.2. $T(n) = T(n-1) + \frac{1}{n}$

Problem 2.3. $T(n) = 1600T(n/4) + n!$ (hint: answering this shouldn't require too many, if any, difficult calculations)

Problem 2.4. $T(n) = 6T(n/3) + n^4/\log^{25} n$ (hint: answering this shouldn't require too many, if any, difficult calculations)

Problem 2.5. $T(n) = \sqrt{n}T(\sqrt{n}) + n$ (hint: when everything fails, you guess and check)

Problem 2.6. $T(n) = T(n/2) + n(5 - \cos^2 n \sin^{20} n)$ (hint: answering this shouldn't require too many, if any, difficult calculations, just think the most basic trigonometric inequality)

Problem 2.7. $T(n) = \alpha T(n/4) + n^2$ (hint: your answer should depend on the α parameter)

Problem 2.8. $T(n) = 5T(n/5) + \frac{n}{\log_5 n}$ (hint: think of $n = 5^m$. Also the recursion $T(n) = T(n-1) + \frac{1}{n}$ above may come in handy.)

3 Algorithms

3.1 Binary Search

3.2 Sorting

3.2.1 Lower Bounds

3.3 Merge Sort

3.4 Number of leaves / depth as proof for lower asymptotic bounds

3.5 Quick Select

3.6 Dynamic Programming

3.6.1 Fibonacci

3.6.2 Binomial Coefficients

3.6.3 Maximize independent set