# Midterm Study Guide

### Zack Traczyk CSE 102 - Vaggos, Winter 2024

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### 1 Introductory Material Review

#### 1.1 Asymptotic Bounds

**Definition 1** (Big-O). f(n) = O(g(n)) if there exists a positive constant c and an integer  $n_0$  such that  $f(n) \le c \cdot g(n)$  for all  $n \ge n_0$ .

**Definition 2** (Big- $\Omega$ ).  $f(n) = \Omega(g(n))$  if there exists a positive constant c and an integer  $n_0$  such that  $c \cdot g(n) \leq f(n)$  for all  $n \geq n_0$ .

**Definition 3** (Big- $\Theta$ ).  $f(n) = \Theta(g(n))$  if there exists positive constants  $c_1$ ,  $c_2$ , and an integer  $n_0$  such that  $c_1 \cdot g(n) \le f(n) \le c_2 \cdot g(n)$  for all  $n \ge n_0$ .

#### 1.2 Inductive Proofs

#### **2** Solving Recurrence Relations

- 2.1 Master Theorem
- 2.2 Unpacking Tree / Algebraic Pattern
- 2.3 Substitution
- 2.4 Guess and Verify
- 2.5 Practice Problems
- 2.5.1 HW3 Ex.4

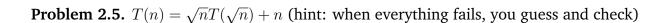
Like in many previous exercises and homeworks, find tight asymptotic bounds (big-Theta) for T(n) in each of the cases.

**Problem 2.1.**  $T(n) = 2T(n/4) + n^2\sqrt{n}$ 

**Problem 2.2.**  $T(n) = T(n-1) + \frac{1}{n}$ 

**Problem 2.3.** T(n) = 1600T(n/4) + n! (hint: answering this shouldn't require too many, if any, difficult calculations)

**Problem 2.4.**  $T(n) = 6T(n/3) + n^4/\log^{25} n$  (hint: answering this shouldn't require too many, if any, difficult calculations)



**Problem 2.6.**  $T(n) = T(n/2) + n(5 - \cos^2 n \sin^{20} n)$  (hint: answering this shouldn't require too many, if any, difficult calculations, just think the most basic trigonometric inequality)

**Problem 2.7.**  $T(n) = \alpha T(n/4) + n^2$  (hint: your answer should depend on the  $\alpha$  parameter)

**Problem 2.8.**  $T(n)=5T(n/5)+\frac{n}{\log_5 n}$  (hint: think of  $n=5^m$ . Also the recursion  $T(n)=T(n-1)+\frac{1}{n}$  above may come in handy.)

## 3 Algorithms

- 3.1 Binary Search
- 3.2 Sorting
- 3.2.1 Lower Bounds
- 3.3 Merge Sort
- 3.4 Number of leaves / depth as proof for lower asymptotic bounds
- 3.5 Quick Select
- 3.6 Dynamic Programming
- 3.6.1 Fibonacci
- 3.6.2 Binomial Coefficients
- 3.6.3 Maximize independent set