# CSE 102 - Midterm Study Guide

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### 1 Introductory Material Review

#### 1.1 Asymptotic Bounds

**Definition 1** (Big-O). f(n) = O(g(n)) if there exists a positive constant c and an integer  $n_0$  such that  $f(n) \le c \cdot g(n)$  for all  $n \ge n_0$ .

**Definition 2** (Big- $\Omega$ ).  $f(n) = \Omega(g(n))$  if there exists a positive constant c and an integer  $n_0$  such that  $c \cdot g(n) \leq f(n)$  for all  $n \geq n_0$ .

**Definition 3** (Big- $\Theta$ ).  $f(n) = \Theta(g(n))$  if there exists positive constants  $c_1$ ,  $c_2$ , and an integer  $n_0$  such that  $c_1 \cdot g(n) \le f(n) \le c_2 \cdot g(n)$  for all  $n \ge n_0$ .

#### 1.2 Inductive Proofs

**Definition 4** (Inductive Proof). Simples rules of induction taken from CSE 16 with Prof. Tracy Larrabee.

- 1. Write down the Left Hand Side of P(k+1).
- 2. Rewrite P(k+1) to include Left Hand Side P(k).
- 3. Replace Left Hand Side of P(k) with Right Hand Side of P(k).
- 4. Rewrite so Right Hand Side of P(k) becomes Right Hand Side of P(k+1).

#### **Examples:**

*Proof.* For all  $n \in \mathbb{Z}^+$  the number  $n^2 + n$  is even

Base Case (n = 1):

$$n^2 + n = 1 + 1 = 2$$
, which is even

### **Inductive Hypothesis:**

Assume  $n^2 + n$  is even, prove  $(n + 1)^2 + (n + 1)$  is even

$$(n+1)^2 + (n+1) = n^2 + 2n + 1 + n + 1$$
(1)

$$= n^2 + n + 2n + 2 \tag{2}$$

$$2p = n^2 + n$$
, 2p is the definition of even (3)

$$=2p+2n+2\tag{4}$$

$$= 2(p+n+1), \text{ which is even}$$
 (5)

### 1.3 Practice Problems

#### 1.3.1 HW1 - Ex.1 Part 2

**Problem 1.1.** Prove that T(n) = 2T(n-1) + 1 is  $T(n) = 2^n - 1$ .

## 2 Solving Recurrence Relations

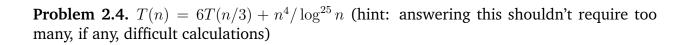
- 2.1 Master Theorem
- 2.2 Unpacking Tree / Algebraic Pattern
- 2.3 Substitution
- 2.4 Guess and Verify
- 2.5 Practice Problems
- 2.5.1 HW3 Ex.4

Like in many previous exercises and homeworks, find tight asymptotic bounds (big-Theta) for T(n) in each of the cases.

**Problem 2.1.**  $T(n) = 2T(n/4) + n^2\sqrt{n}$ 

**Problem 2.2.**  $T(n) = T(n-1) + \frac{1}{n}$ 

**Problem 2.3.** T(n) = 1600T(n/4) + n! (hint: answering this shouldn't require too many, if any, difficult calculations)



**Problem 2.5.**  $T(n) = \sqrt{n}T(\sqrt{n}) + n$  (hint: when everything fails, you guess and check)

**Problem 2.6.**  $T(n) = T(n/2) + n(5 - \cos^2 n \sin^{20} n)$  (hint: answering this shouldn't require too many, if any, difficult calculations, just think the most basic trigonometric inequality)

**Problem 2.7.**  $T(n) = \alpha T(n/4) + n^2$  (hint: your answer should depend on the  $\alpha$  parameter)

**Problem 2.8.**  $T(n) = 5T(n/5) + \frac{n}{\log_5 n}$  (hint: think of  $n = 5^m$ . Also the recursion  $T(n) = T(n-1) + \frac{1}{n}$  above may come in handy.)

# 3 Algorithms

- 3.1 Binary Search
- 3.2 Sorting
- 3.2.1 Lower Bounds
- 3.3 Merge Sort
- 3.4 Number of leaves / depth as proof for lower asymptotic bounds
- 3.5 Quick Select
- 3.6 Dynamic Programming
- 3.6.1 Fibonacci
- 3.6.2 Binomial Coefficients
- 3.6.3 Maximize independent set