# CSE 102 - Midterm Study Guide

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## Contents

# 1 Introductory Material Review

## 1.1 Asymptotic Bounds

**Definition 1** (Big-O)**.** $f(n) = O(g(n))$ if there exists a positive constant $c$ and an integer $n_0$ such that $f(n) \leq c \cdot g(n)$ for all $n \geq n_0$.

**Definition 2** (Big-$\Omega$)**.** $f(n) = \Omega(g(n))$ if there exists a positive constant $c$ and an integer $n_0$ such that $c \cdot g(n) \leq f(n)$ for all $n \geq n_0$.

**Definition 3** (Big-$\Theta$)**.** $f(n) = \Theta(g(n))$ if there exists positive constants $c_1$, $c_2$, and an integer $n_0$ such that $c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$ for all $n \geq n_0$.

## 1.2 Inductive Proofs

**Definition 4** (Inductive Proof)**.** Simples rules of induction taken from CSE 16 with Prof. Tracy Larrabee.

1. Write down the Left Hand Side of $P(k+1)$.
2. Rewrite $P(k+1)$ to include Left Hand Side $P(k)$.
3. Replace Left Hand Side of $P(k)$ with Right Hand Side of $P(k)$.
4. Rewrite so Right Hand Side of $P(k)$ becomes Right Hand Side of $P(k+1)$.

**Examples:**

*Proof.* For all $n \in \mathbb{Z}^+$ the number $n^2 + n$ is even
**Base Case (n = 1):**

$$n^2 + n = 1 + 1 = 2, \text{ which is even}$$

**Inductive Hypothesis:**

$$\text{Assume } n^2 + n \text{ is even, prove } (n+1)^2 + (n+1) \text{ is even}$$

$$\begin{aligned}
(n+1)^2 + (n+1) &= n^2 + 2n + 1 + n + 1 && (1)\\
&= n^2 + n + 2n + 2 && (2)\\
2p = n^2 + n&\text{, 2p is the definition of even} && (3)\\
&= 2p + 2n + 2 && (4)\\
&= 2(p + n + 1)\text{, which is even} && (5)
\end{aligned}$$

$\square$

# 2 Solving Recurrence Relations

## 2.1 Master Theorem

**Definition 5** (Master Theorem)**.** Let $a \geq 1$ and $b > 1$ be constants, let $f(n)$ be a function, and let $T(n)$ be defined on the nonnegative integers by the recurrence

$$T(n) = aT(n/b) + f(n)$$

where we interpret $n/b$ to mean either $\lfloor n/b \rfloor$ or $\lceil n/b \rceil$. Then $T(n)$ has the following asymptotic bounds:

1. If $f(n) = O(n^{\log_b a - \epsilon})$ for some constant $\epsilon > 0$, then $T(n) = \Theta(n^{\log_b a})$.
2. If $f(n) = \Theta(n^{\log_b a})$, then $T(n) = \Theta(n^{\log_b a} \log n)$.
3. If $f(n) = \Omega(n^{\log_b a + \epsilon})$ for some constant $\epsilon > 0$, and if $af(n/b) \leq kf(n)$ for some constant $k < 1$ and all sufficiently large $n$, then $T(n) = \Theta(f(n))$.

## 2.2 Unpacking Tree / Algebraic Pattern

## 2.3 Substitution

## 2.4 Guess and Verify

# 3 Algorithms

## 3.1 Binary Search

## 3.2 Sorting

### 3.2.1 Lower Bounds

## 3.3 Merge Sort

## 3.4 Number of leaves / depth as proof for lower asymptotic bounds

## 3.5 Quick Select

## 3.6 Dynamic Programming

### 3.6.1 Fibonacci

### 3.6.2 Binomial Coefficients

### 3.6.3 Maximize independent set

# 4 Practice Problems

## 4.1 Introductory Material Review

**Problem 4.1.** Let $f(n) = 100n^2 + 10n + 1000$. Use the definition of Big-O to prove $f(n) = O(n^2)$.

**Answer:** HW1 - Ex.2

**Problem 4.2.** Let $f(n) = 100n + 0.001n \log n$. Use the definition of Big-O to prove $f(n) = O(n \log n)$.

**Answer:** HW1 - Ex.2

**Problem 4.3.** Let $f(n) = 50n \log n + 30n$. Use the definition of Big-O to prove $f(n) = O(n^3)$.

**Answer:** HW1 - Ex.2

**Problem 4.4.** Prove that $T(n) = 2T(n-1) + 1$ is $T(n) = 2^n - 1$.

**Answer:** HW1 - Ex.1 Part 2

## 4.2 Solving Recurrence Relations

**Problem 4.5.** Find tight asymptotic bounds (big-Theta) for $T(n) = 2T(n/4) + n^2\sqrt{n}$

**Answer:** HW3 - Ex.4

**Problem 4.6.** Find tight asymptotic bounds (big-Theta) for $T(n) = T(n-1) + \frac{1}{n}$

**Answer:** HW3 - Ex.4

**Problem 4.7.** Find tight asymptotic bounds (big-Theta) for $T(n) = 1600T(n/4) + n!$ (hint: answering this shouldn't require too many, if any, difficult calculations)

**Answer:** HW3 - Ex.4

**Problem 4.8.** Find tight asymptotic bounds (big-Theta) for $T(n) = 6T(n/3) + n^4/\log^{25} n$ (hint: answering this shouldn't require too many, if any, difficult calculations)

**Answer:** HW3 - Ex.4

**Problem 4.9.** Find tight asymptotic bounds (big-Theta) for $T(n) = \sqrt{n}T(\sqrt{n}) + n$ (hint: when everything fails, you guess and check)

**Answer:** HW3 - Ex.4

**Problem 4.10.** Find tight asymptotic bounds (big-Theta) for $T(n) = T(n/2) + n(5 - \cos^2 n \sin^{20} n)$ (hint: answering this shouldn't require too many, if any, difficult calculations, just think the most basic trigonometric inequality)

**Answer:** HW3 - Ex.4

**Problem 4.11.** Find tight asymptotic bounds (big-Theta) for $T(n) = \alpha T(n/4) + n^2$ (hint: your answer should depend on the $\alpha$ parameter)

**Answer:** HW3 - Ex.4

**Problem 4.12.** Find tight asymptotic bounds (big-Theta) for $T(n) = 5T(n/5) + \frac{n}{\log_5 n}$ (hint: think of $n = 5^m$. Also the recursion $T(n) = T(n-1) + \frac{1}{n}$ above may come in handy.)

**Answer:** HW3 - Ex.4

## 4.3 Algorithms