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1 Solution summary

Q No	$O(n)$
1	$\log n$
2	$\log n$
3	$\log n$
4	$n^2 \cdot \log n$
5	$n \cdot \log n$
6	$n^{(1/2)}$
7	n^2

2 Detailed Solution

2.1 Find $T(n)$ and Big-Oh

```
A()  
{  
    while(n>1)  
    {  
        n=n/2;  
    }  
}
```

Statement	$O(n)$
$n > 1$	$(\log_2 n) + 1$
$n = n/2$	$\log_2 n$

$T(n) = \sum O(n)$
 $T(n) = \log_2 n + 1 + \log_2 n$
 $T(n) = 2 * \log_2 n + 1$
 let $f(n) = T(n)$
 $f(n) = O(g(n))$ if $f(n) \leq c(g(n))$
 $2 * \log_2 n + 1 \leq c * \log_2 n$
 dividing by $\log_2 n$ both sides
 $c = 4$ and $N \geq 2$
 $f(n) \leq 4 * \log_2 n$ for $N \geq 2$
 Time complexity is $= O(\log n)$

2.2 Find T(n) and Big-Oh

```

A()
{
    for (i=1; i<=n; i=i*2)
        print("welcome");
}

```

Statement	$O(n)$
$i = 1$	1
$i \leq n$	$\log_2 n + 1$
$i = i * 2$	$\log_2 n$
print	$\log_2 n$

$T(n) = \sum O(n)$
 $T(n) = 1 + \log_2 n + 1 + \log_2 n + \log_2 n$
 $T(n) = 3 * \log_2 n + 2$
 Time complexity $= O(\log n)$

2.3 Find T(n) and Big-Oh

```

A()
{
    for (i=1; i<=n; i=i*3)
        print("welcome");
}

```

Statement	$O(n)$
$i = 1$	1
$i \leq n$	$\log_3 n + 1$
$i = i*3$	$\log_3 n$
print	$\log_3 n$

$T(n) = \sum O(n)$
 $T(n) = 1 + \log_3 n + 1 + \log_3 n + \log_3 n$
 $T(n) = 3*\log_3 n + 2$
 Time complexity = $O(\log n)$

2.4 Find $T(n)$ and Big-Oh

```

A()
{
    int i, j, k;
    for (i=n/2; i<=n; i++)
        for (j=1; j<=n/2; j++)
            for (k=1; k<=n; k=k*2)
{
    print("welcome");
}
}

```

Statement	$O(n)$
$i = n/2$	1
$i \leq n$	$n/2 + 1$
$i++$	$n/2$
$j = 1$	$n/2(1)$
$j \leq n/2$	$n/2(n/2 + 1)$
$j++$	$n/2(n/2)$
$k = 1$	$n^2/4(1)$
$k \leq n$	$n^2/4(\log_2 n + 1)$
$k = k*2$	$n^2/4(\log_2 n)$
print	$n^2/4(1)$

$T(n) = \sum O(n)$
 $T(n) = n^2/4(\log_2 n) + \dots$
 Time complexity $O(n^2 * \log n)$

2.5 Find T(n) and Big-Oh

```

A()
{
    int i,j,k;
    for (i=n/2; i<=n; i++)
        {
            for (j=1; j<=n; j=2*j)
        {
            for (k=1; k<=n; k=k*2)
                print("welcome");
        }
    }
}

```

Statement	O(n)
i,j,k	3
i=n/2	1
i <= n	n/2+1
i++	n/2
j = 1	n/2(1)
j <= n	n/2(log ₂ n + 1)
j = j*2	n/2(log ₂ n)
k = 1	n/2(log ₂ n)(1)
k <= n	n/2(log ₂ n)(log ₂ n)+1
k = k*2	n/2(log ₂ n)(log ₂ n)

$$T(n) = \sum O(n)$$

$$T(n) = 3 + 1 + n/2+1 + n/2 + n/2 + n/2(\log_2 n + 1) \dots$$

$$T(n) = n^*(\log_2 n)^2 + \log_2 n + n + c$$

$$T(n) = 2*n(\log_2 n) + \log_2 n + n + c$$

$$f(n) = T(n)$$

$$2*n(\log_2 n) + \log_2 n + n \leq c*n(\log_2 n)$$

$$c = 3.5 \text{ when } n > 2$$

$$\text{Time complexity} = O(n*\log n)$$

2.6 Find T(n) and Big-Oh

```

AC()
{
    i=1,s=1;

```

```

while (s<=n)
{
    i++;
    s=s+i;
    print("welcome");
}
}

```

Statement	O(n)
i =1, s=1	2
s <= 2	$k(k+1)/2 + 1 > n$
i++	$k(k+1)/2 > n$
s = s+i	$k(k+1)/2 > n$

$T(n) = \sum O(n)$
 $T(n) = 2 + 3*k(k+1)/2 > n$
 $k*(k+1) > c*n$
 $k^2 = c*n$
 $k = n^{(1/2)}$
 Time complexity = $O(n^{(1/2)})$

2.7 Find T(n) and Big-Oh

```

A()
{
    int i, j, k, n;
    for (i=1; i<=n; i++)
    {
        for (j=1; j<=i; j++)
    {
        for (k=1; k<=100; k++)
        {
            print("welcome");
        }
    }
}
}
}

```

Statement	$O(n)$
i,j,k,n	4
i=1	1
i <= n	n+1
i++	n
j=1	n(1)
j <= 1	n(n+1)/2+1
j++	n(n+1)/2
k = 1	n(n+1)/2
k <= 100	n(n+1)/2*(100)+1
k++	n(n+1)/2*(100)
print	n(n+1)/2*(100)

$$T(n) = \sum O(n)$$

$$T(n) = c*n(n+1)$$

$$\text{Time complexity} = O(n^2)$$