$(x+h)^2$

$= x^2 + 2xh + h^2$

Derivatives

الاشتقاق

we denote it by:

$$D f(x) = f'(x)$$

$$\frac{d}{dx} f(x) = f'(x)$$

$$\frac{d}{dy} f(x) \; ?? \; x$$

$$\frac{d}{dx}(x^2) = 2x \; ??$$

$$\frac{d}{dx} f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx} x^2 = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2x+h)}{h} = 2x$$

$$\frac{d}{dx} x^2 = 2x$$

$$\frac{d}{dx} x^3 = 3x^2$$

$$\frac{d}{dx} 3x^4 = 12x^3$$

$$\frac{d}{dx} \sqrt{x} = \frac{d}{dx} (x)^{\frac{1}{2}}$$

$$= \frac{1}{2} x^{-\frac{1}{2}}$$

$$= \frac{1}{2\sqrt{x}}$$

$\frac{1}{2} - 1$

$\frac{2}{3} - 1$

$\frac{2}{3} - \frac{3}{3} = \frac{-1}{3}$

$\frac{17}{18} - \frac{18}{18} = \frac{-1}{18}$

$$\frac{d}{dx} \sqrt[3]{x^2} = \frac{d}{dx} (x)^{\frac{2}{3}}$$

$$= \frac{2}{3} x^{-\frac{1}{3}}$$

$$\frac{d}{dx} x^{-\frac{1}{3}} = \frac{1}{9} x^{-\frac{1}{3}-\frac{3}{3}}$$

$$= \frac{1}{9} x^{-\frac{4}{3}}$$

Ex: $\frac{d}{dx}\frac{5}{2x^4} = \frac{d}{dx}\left(\frac{5}{2}x^{-4}\right)$

$= (-4)\left(\frac{5}{2}\right)x^{-5}$

$= -10x^{-5}$

$\frac{d}{dx}(x+2)(x-3)$

$= \frac{d}{dx}\left[x^2 - 3x + 2x - 6\right]$

$= \frac{d}{dx}\left[x^2 - x - 6\right] = 2x - 1$

Higher order derivatives

المشتقات من الرتب العليا

$f'(x) = \frac{d}{dx} f(x)$, first derivatives

$f''(x) = \frac{d^2 f(x)}{dx^2}$, second ..

$f'''(x) = \frac{d^3 f(x)}{dx^3}$, third ..

⋮

$f^{(10)}(x) = \frac{d^{10} f(x)}{dx^{10}}$, tenth ..

⋮

$\frac{d^{100} f(x)}{dx^{100}} = f^{(100)}$

ابتداءً من المشتقة الرابعة

$\frac{d^4 f(x)}{dx^4} = f^{(4)}(x)$

$\frac{d^{117} f(x)}{dx^{117}} = f^{(117)}(x)$

f^4 , $f^{(4)}$

Ex: Find fourth derivatives of

$$f(x) = 3x^5 - 12x^3 + 24x + 10$$

$$f'(x) = 15x^4 - 36x^2 + 24$$

$$f''(x) = 60x^3 - 72x$$

$$f'''(x) = 180x^2 - 72$$

$$f^{(4)}(x) = 360x$$

$$f^{(5)}(x) = 360,$$

$$f^{(6)}(x) = 0$$

Find $\dfrac{d^{114}}{dx^{114}}(5 + x^{112})$

$$= 0$$

Product rule

قاعدة الضرب (الدوال)

$$\frac{d}{dx}[f(x)\cdot g(x)] = f'(x)\,g(x) + g'(x)\,f(x)$$

Ex: Find $\frac{d}{dx}(x^2+2)(x^3-3x+5)$

$$= 2x(x^3-3x+5) + (x^2+2)(3x^2-3)$$

$$= 2x\cdot x^3 - 2x + 5 + x^2 + 2\cdot 3x^2 - 3$$

Ex: $\frac{d}{dx}\left[x\sqrt{x}\right]$ (الأول: x، الثاني: $\sqrt{x}$)

$$= \sqrt{x} + x\left(\frac{1}{2\sqrt{x}}\right)$$

$$x\sqrt{x} = x^1 x^{1/2} = x^{3/2}$$

$$\frac{d}{dx}x\sqrt{x} = \frac{d}{dx}\left(x^{3/2}\right) = \frac{3}{2}x^{1/2}$$

$$\frac{d}{dx}\left[(25x^4 - 13x^2)\left(\frac{2}{x} + x^3\right)\right]$$

$$= \left(100x^3 - 26x\right)\left(\frac{2}{x} + x^3\right) +$$

$$+ \left(-2x^2 + 3x^2\right)\left(25x^4 - 13x^2\right)$$

$$\frac{d}{dx}\left(5x^3 - \frac{1}{x}\right)\left(\sqrt{x} + 2\right)$$

$$= \left(15x^2 + \frac{1}{x^2}\right)\left(\sqrt{x} + 2\right) +$$

$$+ \left(5x^3 - \frac{1}{x}\right)\left(\frac{1}{2\sqrt{x}}\right)$$

$$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}x^{-1} = -x^{-2} = \frac{-1}{x^2}$$

$$\frac{d}{dx}\left(\frac{5}{x^4}\right) = \frac{d}{dx}(5x^{-4}) = -20x^{-5} = \frac{-20}{x^5}$$

$$\frac{-20}{x^5} \qquad \frac{d}{dx}\frac{-2}{x^{-2}} = \frac{4}{x^{-1}}$$

$$\frac{d}{dx}\left(\frac{n}{x^m}\right) = \frac{-nm}{x^{m+1}}$$

$$\frac{d}{dx}\sqrt{x} = \frac{d}{dx}x^{\frac{1}{2}} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2}\frac{1}{x^{\frac{1}{2}}} = \frac{1}{2}\frac{1}{\sqrt{x}}$$