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Digital Signal Processing
Lecture 2

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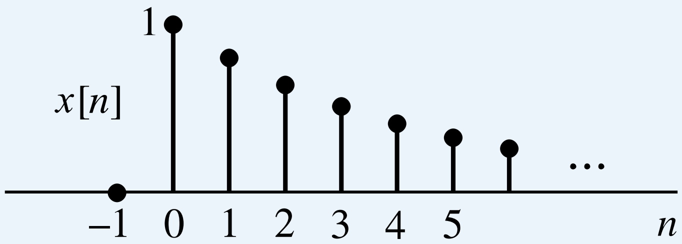
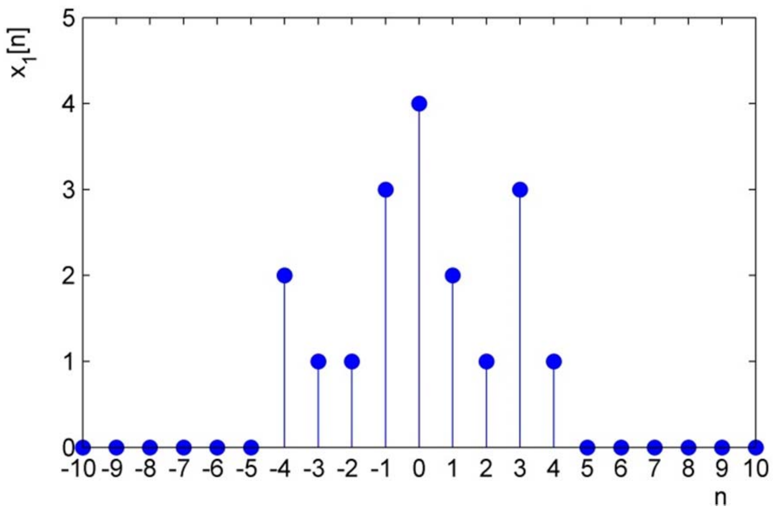
Discrete Time Signals

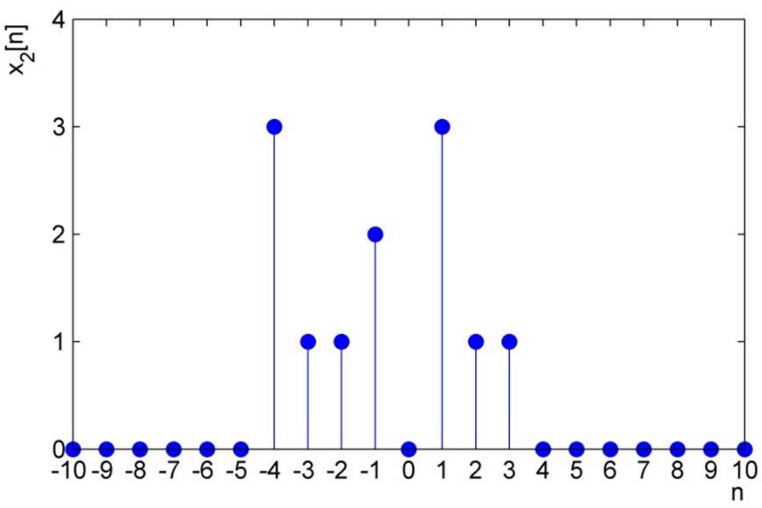
Representation	Example
Functional	$x[n] = \begin{cases} \left(\frac{1}{2}\right)^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$
Tabular	$\begin{array}{c\|cccccccc} n & \ldots & -2 & -1 & 0 & 1 & 2 & 3 & \ldots \\ \hline x[n] & \ldots & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{4} & \frac{1}{8} & \ldots \end{array}$
Sequence	$x[n] = \{ \ldots \; 0 \; \underset{\uparrow}{1} \; \frac{1}{2} \; \frac{1}{4} \; \frac{1}{8} \; \ldots \}$
Pictorial	

[1] The symbol $\uparrow$ denotes the index $n = 0$; it is omitted when the table starts at $n = 0$.

Discrete Time Signals

✓ A discrete time signal is a sequence of numbers





Operations on Sequences

- Transformations of the dependent variable

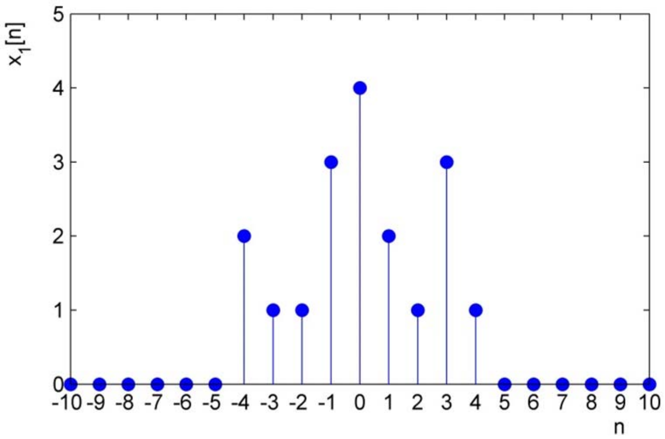
$$
\begin{aligned}
y[n] &= x_1[n] + x_2[n], && \text{(signal addition)} \\
y[n] &= x_1[n] - x_2[n], && \text{(signal subtraction)} \\
y[n] &= x_1[n] \cdot x_2[n], && \text{(signal multiplication)} \\
y[n] &= x_1[n] / x_2[n], && \text{(signal division)} \\
y[n] &= a \cdot x_2[n]. && \text{(signal scaling)}
\end{aligned}
$$

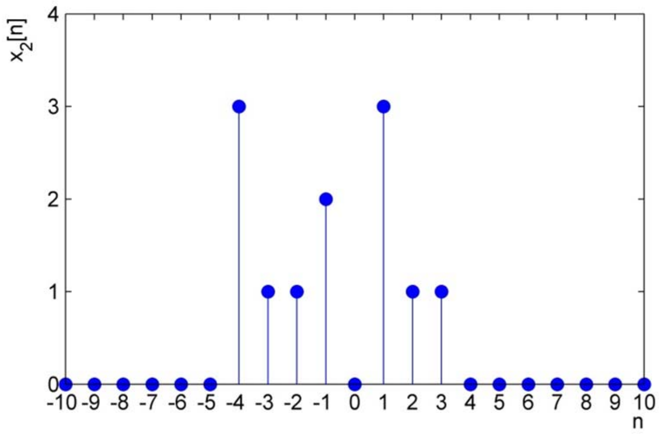
- Transformations of the independent variable
  - Time-reversal or folding: $y[n] = x[-n]$
  - Time-shifting: $y[n] = x[n - n_0]$
  - Shifting and folding are not commutative operations

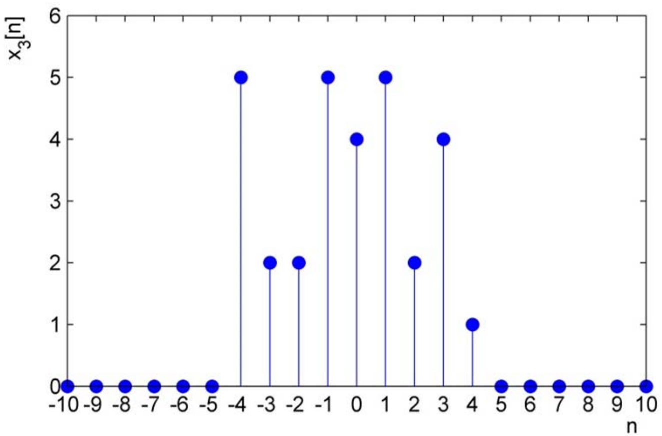
$$x[n] \stackrel{\text{shift}}{\rightarrow} x[n - n_0] \stackrel{\text{fold}}{\rightarrow} x[-n - n_0] \neq x[n] \stackrel{\text{fold}}{\rightarrow} x[-n] \stackrel{\text{shift}}{\rightarrow} x[-n + n_0]$$

Operations on Sequences

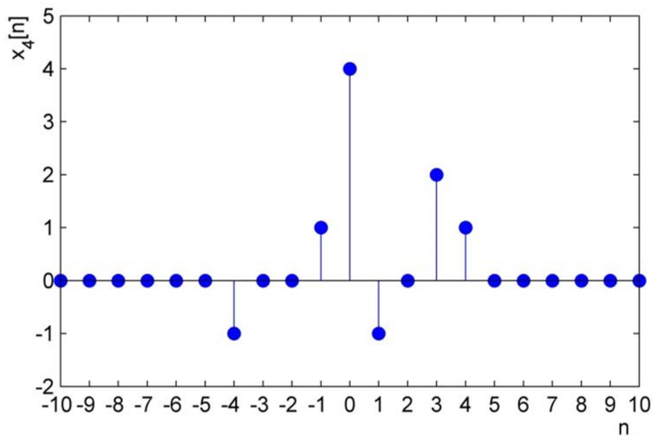
✓ Addition/Subtraction







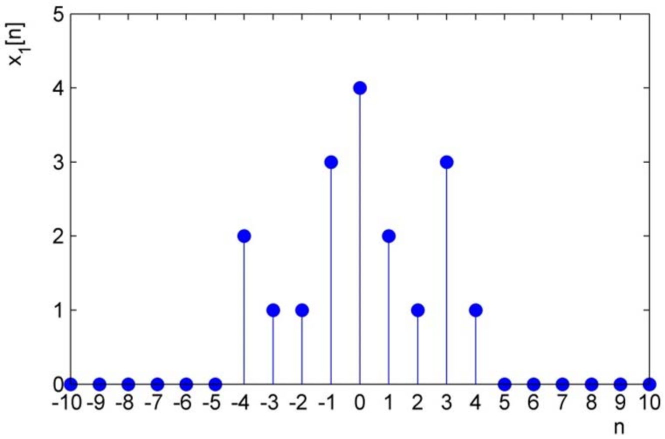
$x_3[n]=x_1[n]+x_2[n]$

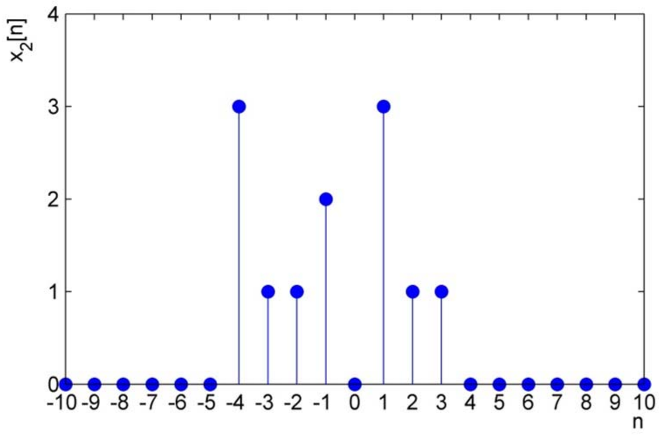


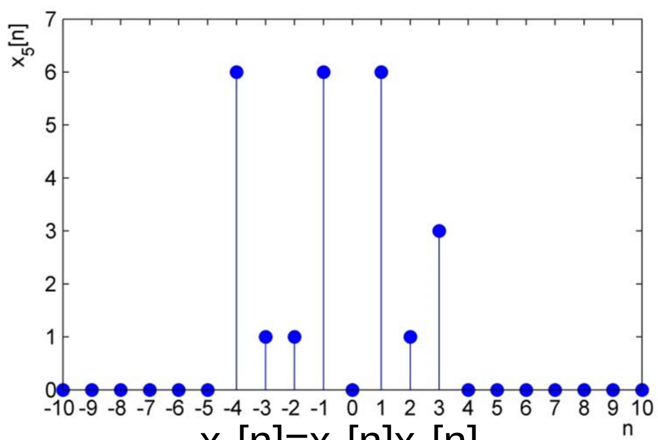
$x_4[n]=x_1[n]-x_2[n]$

Operations on Sequences

✓ **Multiplication**





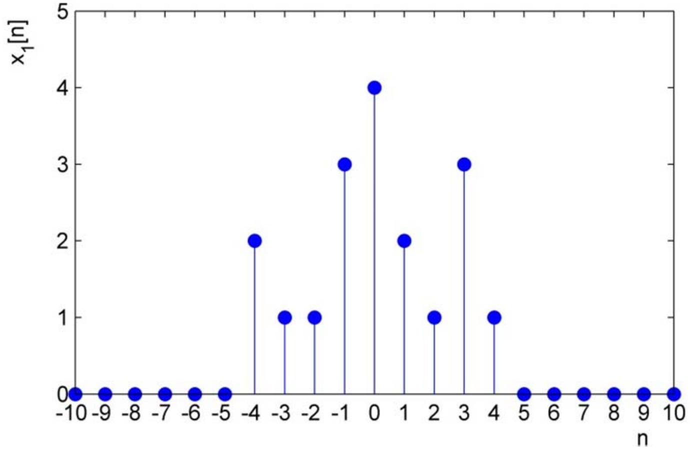


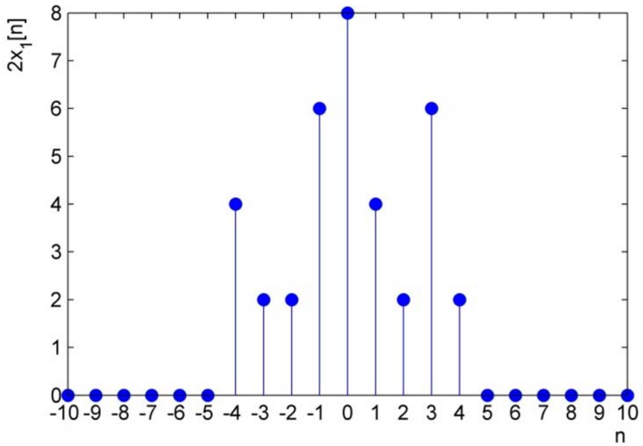
$x_5[n]=x_1[n]x_2[n]$

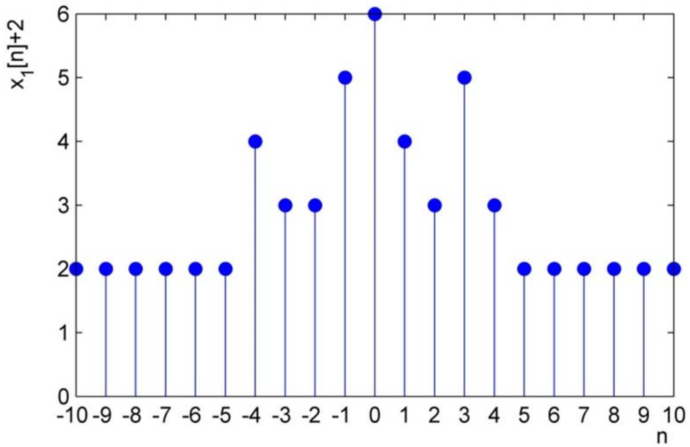
✓ How will you apply multiplication if the sizes are not the same?

Operations on Sequences

✓ **Multiplication & Addition with constants**

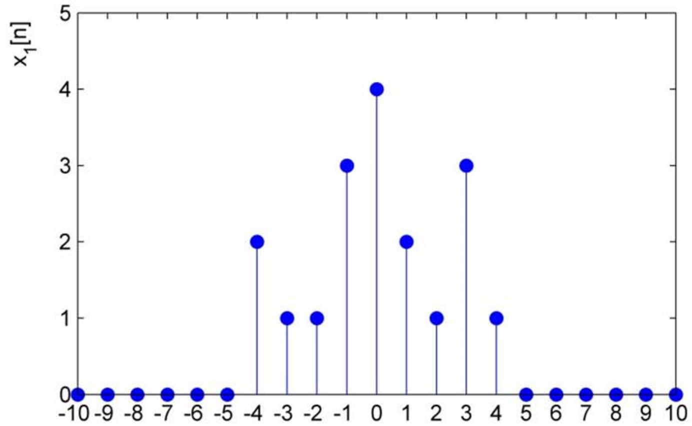


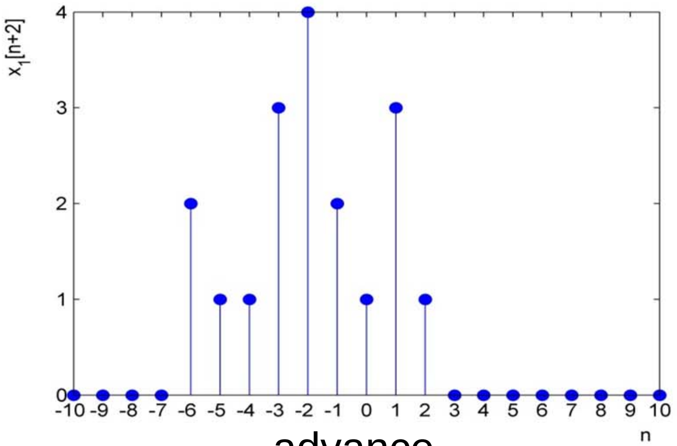




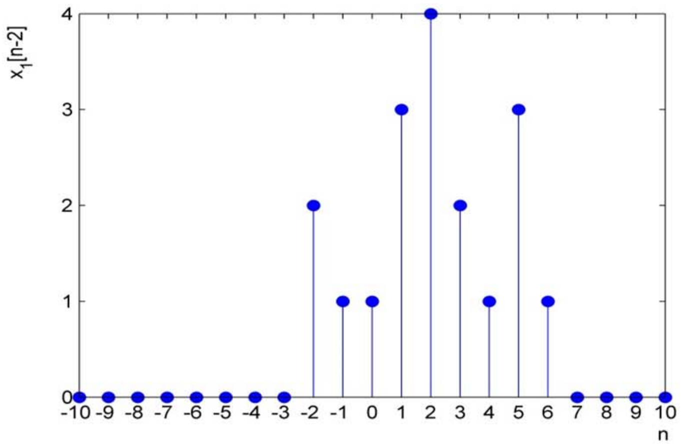
Operations on Sequences

- ✓ **Shifting** $y[n]=x[n-n_0]$ → If $n_0>0$ → time delay If $n_0<0$ → time advance



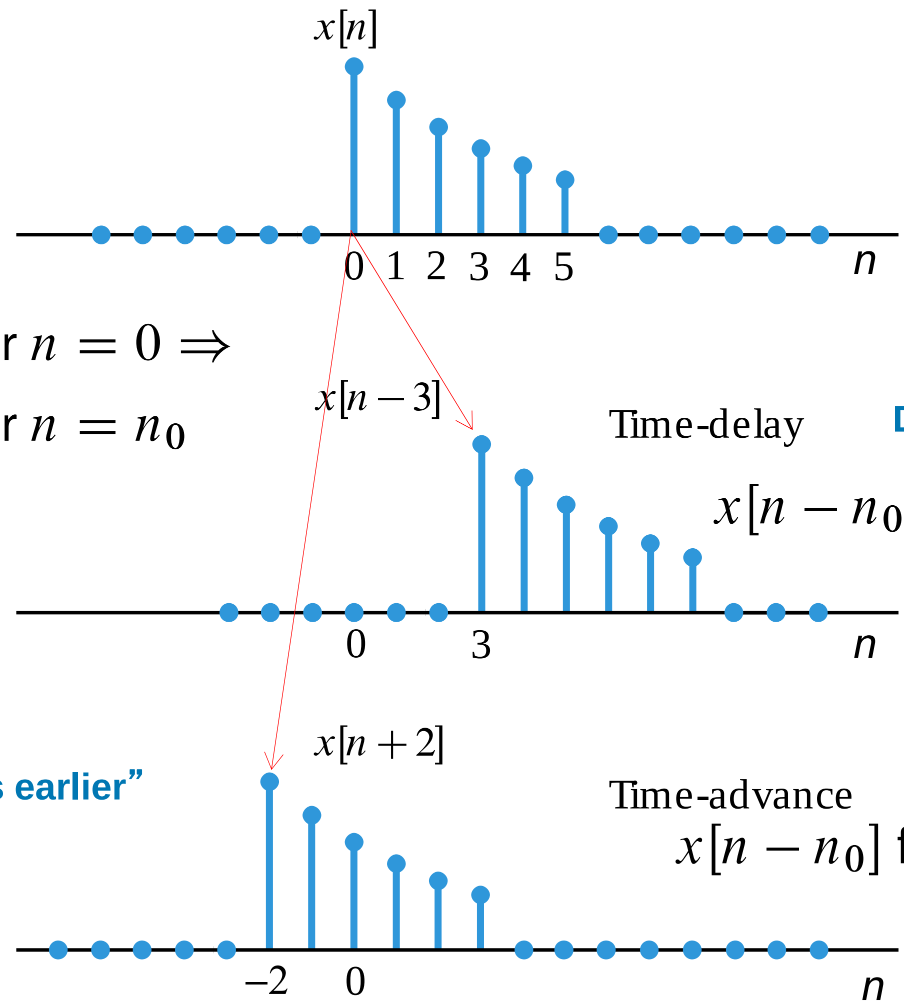


advance



delay

Operations on Sequences



$x[n] = x[0]$ for $n = 0 \Rightarrow$

$x[n - n_0] = x[0]$ for $n = n_0$

Time-delay

Delay ➔ "Happens later"

$x[n - n_0]$ for $n_0 > 0$ $(n_0 = 3)$

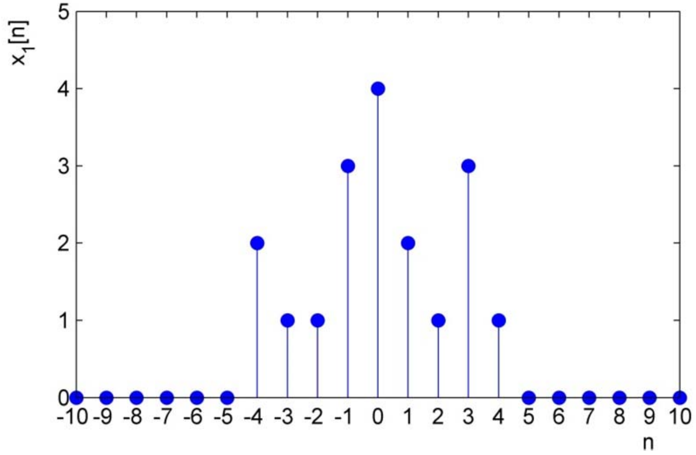
Advance ➔ "Happens earlier"

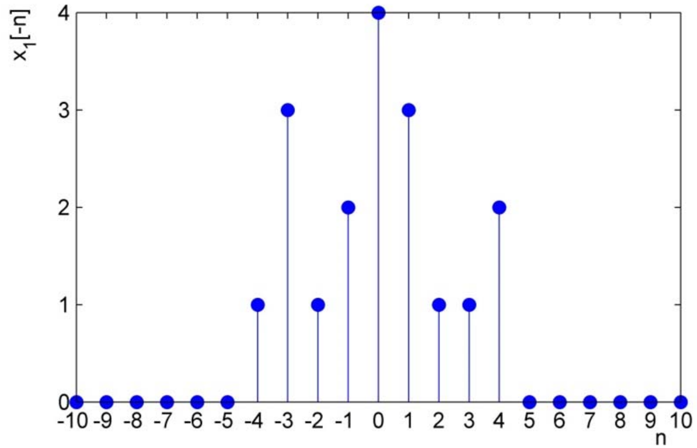
Time-advance

$x[n - n_0]$ for $n_0 < 0$ $(n_0 = -2)$

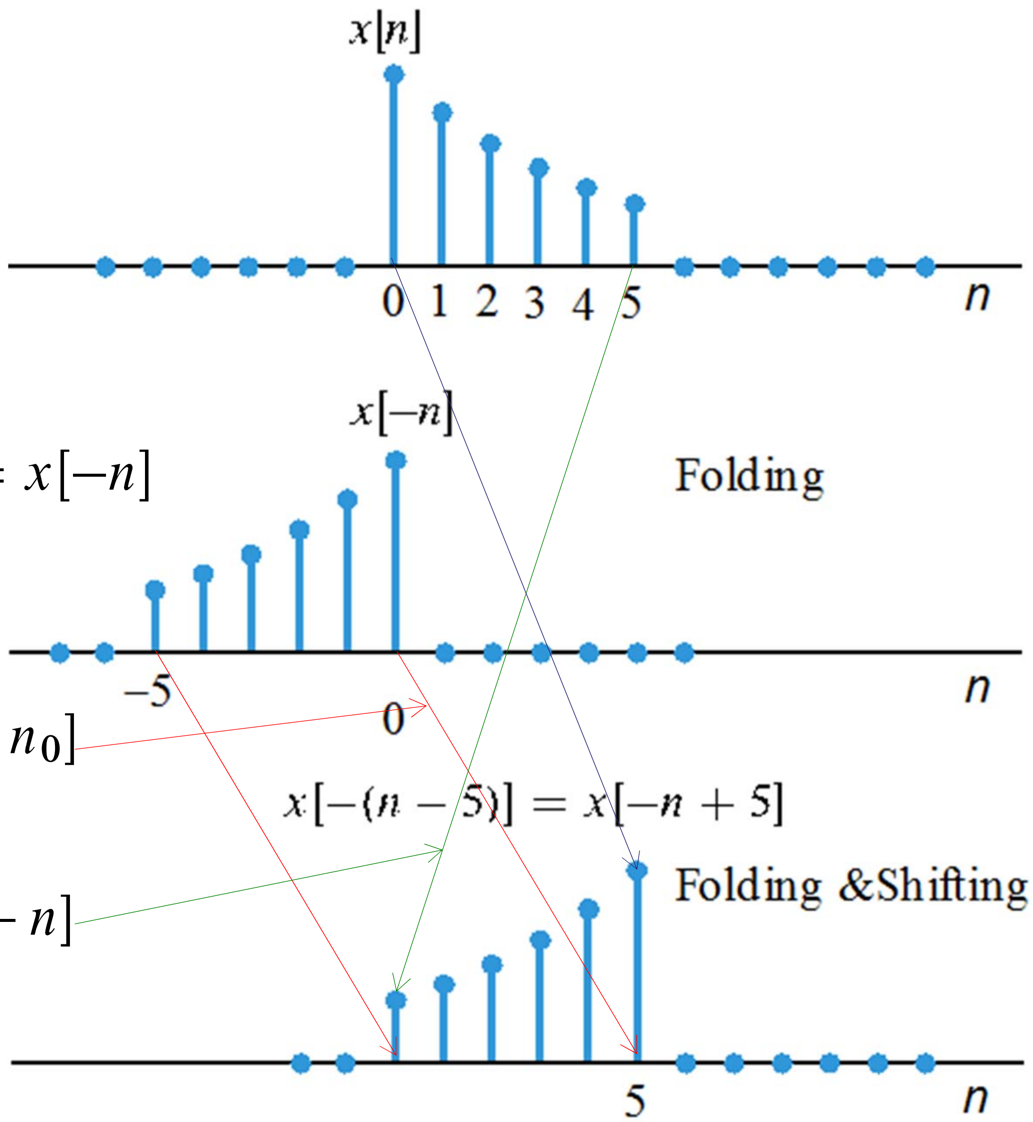
Operations on Sequences

✓ Time Reverse: y[n]= x[-n]





Operations on Sequences



$$y_1[n] = x[-n]$$

Time Reversal

$$y_2[n] = y_1[n - n_0]$$

$$y_2[n] = x[n_0 - n]$$

Shifting

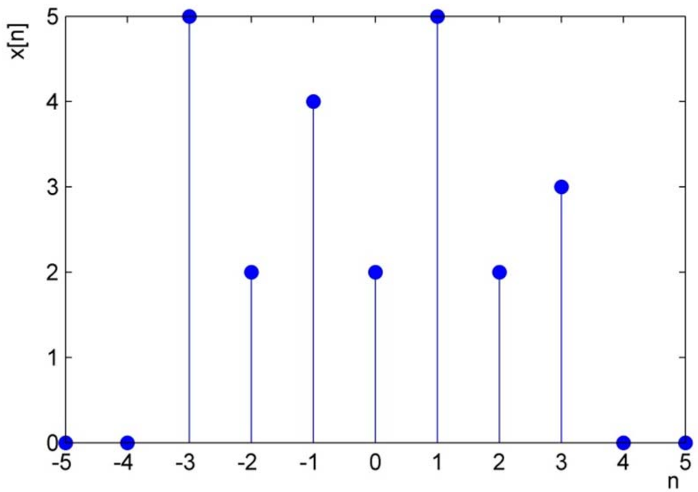
$$n_0 = 5$$

Operations on Sequences

$$y[n] = x[\alpha n]$$ → Signal compression

$$y[n] = x\left[\frac{n}{\alpha}\right]$$ → Signal stretching

$x[n]$ is given below. Estimate $x[2n]$ and $x[n/2]$



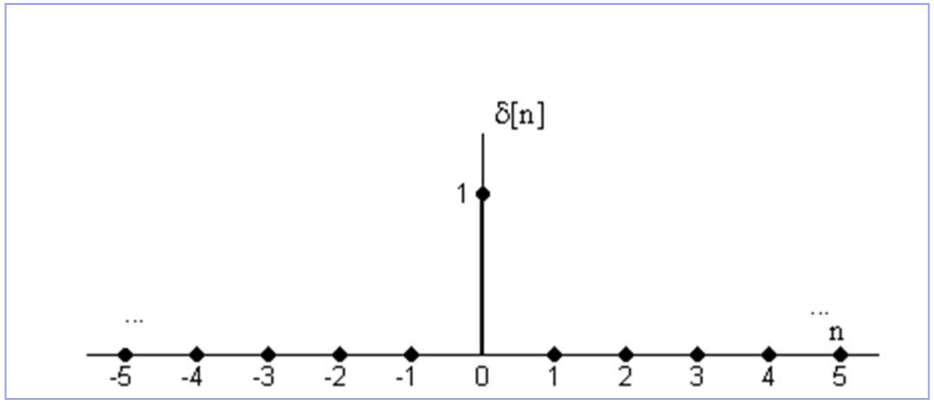
Linear Interpolation

$$y[n]=\begin{cases} x[n/2] & ,\ n\text{:even} \\ \frac{1}{2}\{x[(n-1)/2]+x[(n+1)/2]\} & ,\ n\text{:odd} \end{cases}$$

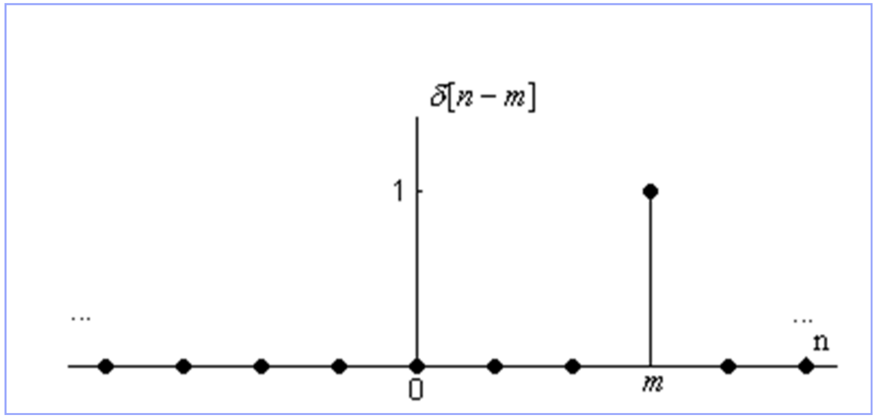
Elementary Digital Signals

✓ Unit Impulse (Digital Impulse, Discrete Time Impulse):

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

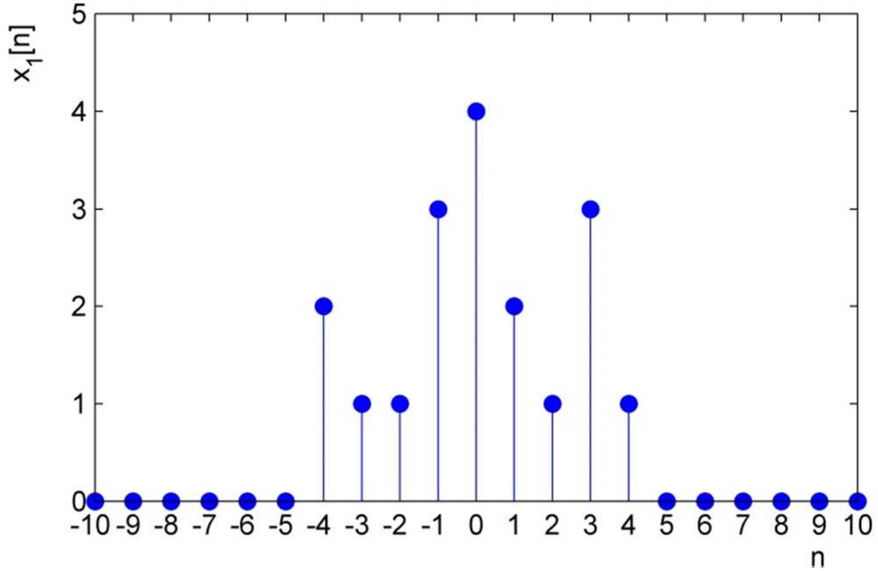


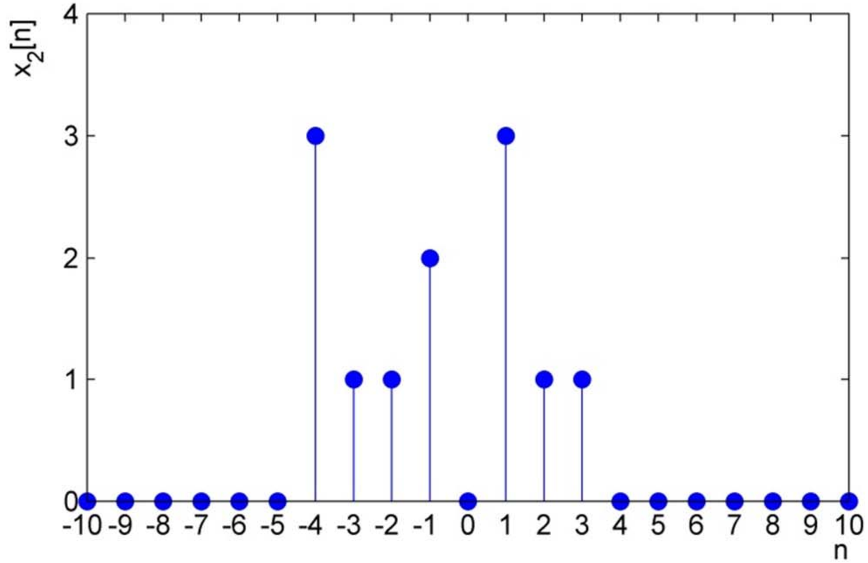
$$\delta[n-m] = \begin{cases} 1 & n = m \\ 0 & n \neq m \end{cases}$$



Exercises

- ✓ Write the signals given below in terms of $\delta[n]$





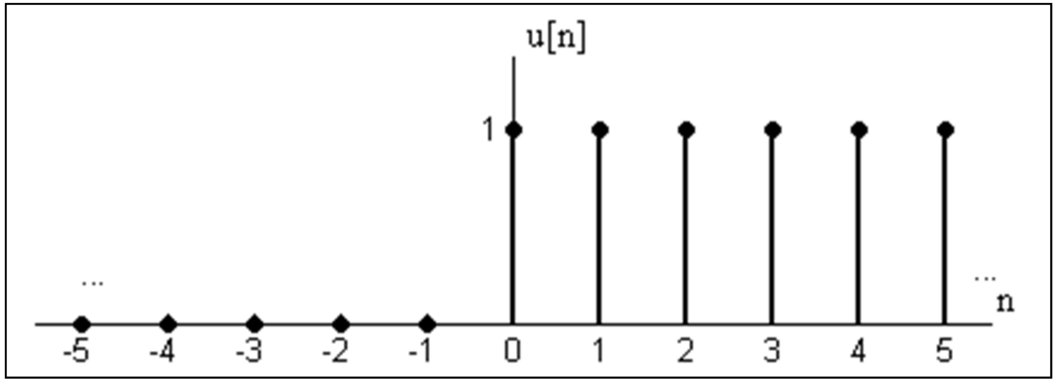
- ✓ if $x[n]=\delta[n]+\delta[n-1]+\delta[n-2]+4\delta[n-3]+5\delta[n-4]$

$$x[-n+2]=?$$

Elementary Digital Signals

✓ Unit Step Sequence:

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

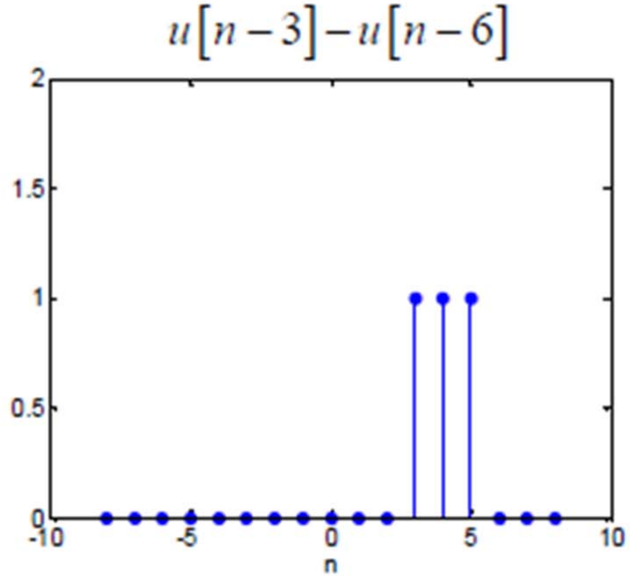


✓ Unit Impulse and Unit Step Sequences are related as follows:

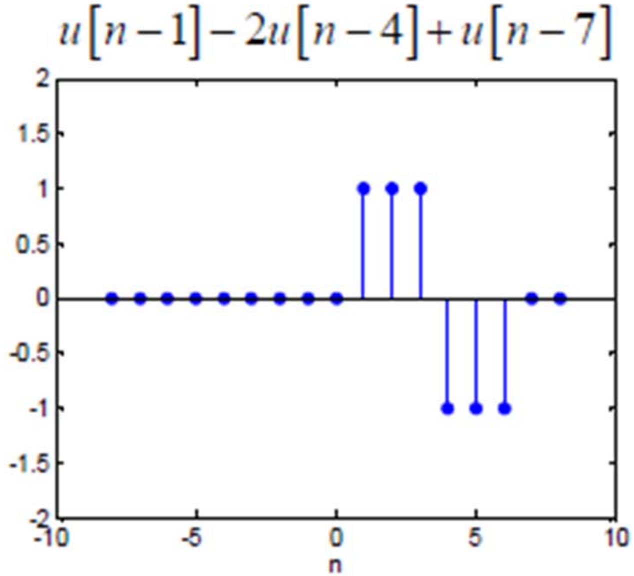
$$u[n] = \sum_{k=0}^{\infty} \delta(n-k)$$

$$\delta[n] = u[n] - u[n-1]$$

Exercises



$u[n-3]-u[n-6]$
2
1.5
1
0.5
0
-10
-5
0
5
10
n

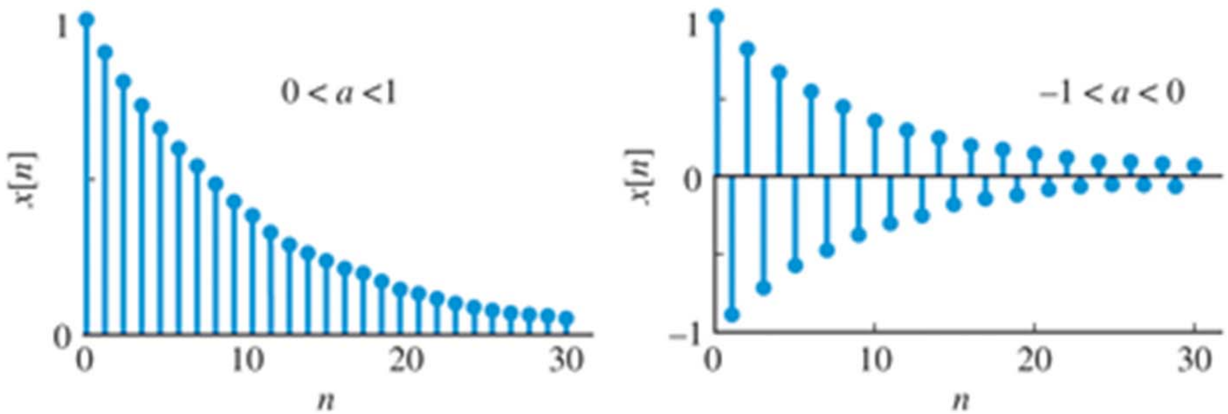


$u[n-1]-2u[n-4]+u[n-7]$
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-10
-5
0
5
10
n

Elementary Digital Signals

- ✓ Real exponential sequence:

$$\boxed{x[n] = a^n}$$



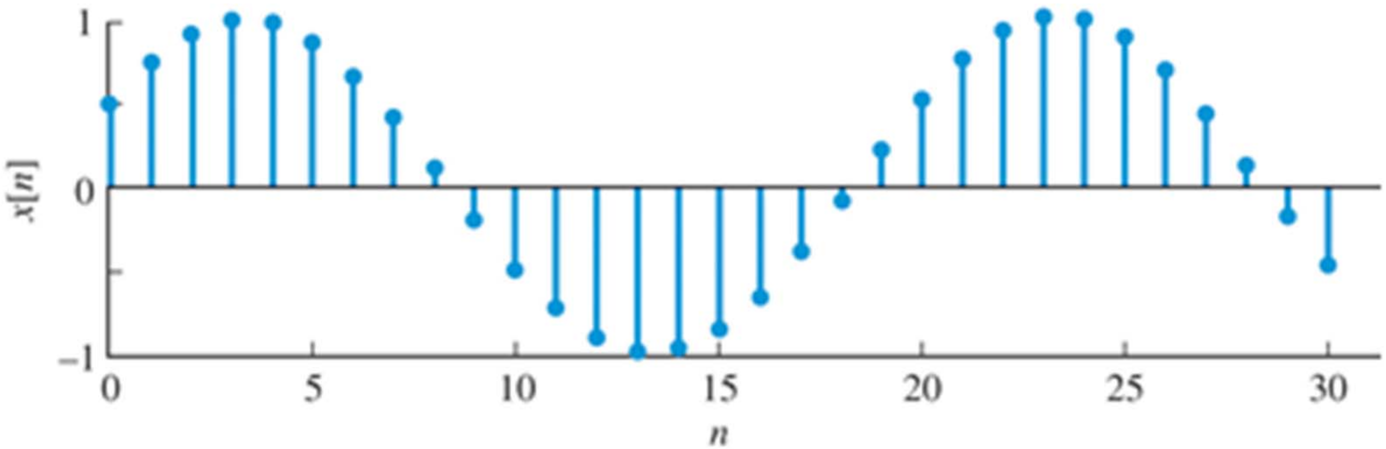
Elementary Digital Signals

✓ Sinusoidal sequence:

$$x[n] = A\cos(\omega n + \theta)$$

$$\cos(\omega n) = \frac{e^{j\omega n} + e^{-j\omega n}}{2}$$

$$\sin(\omega n) = \frac{e^{j\omega n} - e^{-j\omega n}}{2j}$$

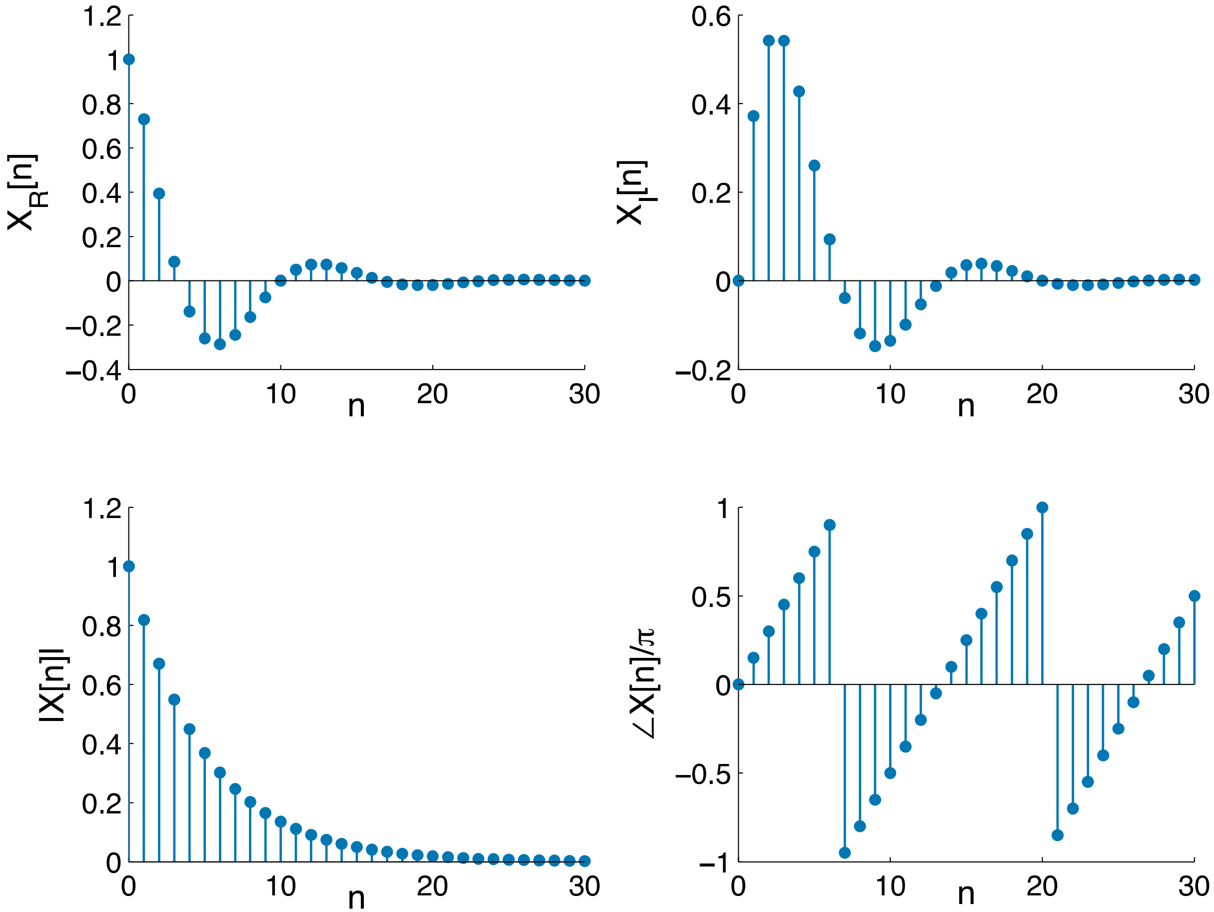


ω → Digital frequency, unit: [radians] or [rad/sample]

Elementary Digital Signals

- ✓ Complex exponential sequence:

$$x[n] = a^n = (r\,e^{j\omega_0})^n = r^n e^{j\omega_0 n} = r^n \cos\omega_0 n + j r^n \sin\omega_0 n$$



Properties of Sinusoidal/Complex Signals

$$x[n] = A\cos(\omega n + \phi)$$

$$\cos[(\omega + 2\pi r)n] = \cos(\omega n)$$

$$\text{x[n]=}Ae^{j(\omega n+\phi)}$$

$$e^{j(\omega+2\pi r)n} = e^{j\omega n}e^{2\pi rn} = e^{j\omega n}$$

We need only consider frequencies in an interval of length 2π for all discrete time real sinusoidal and exponential signals!

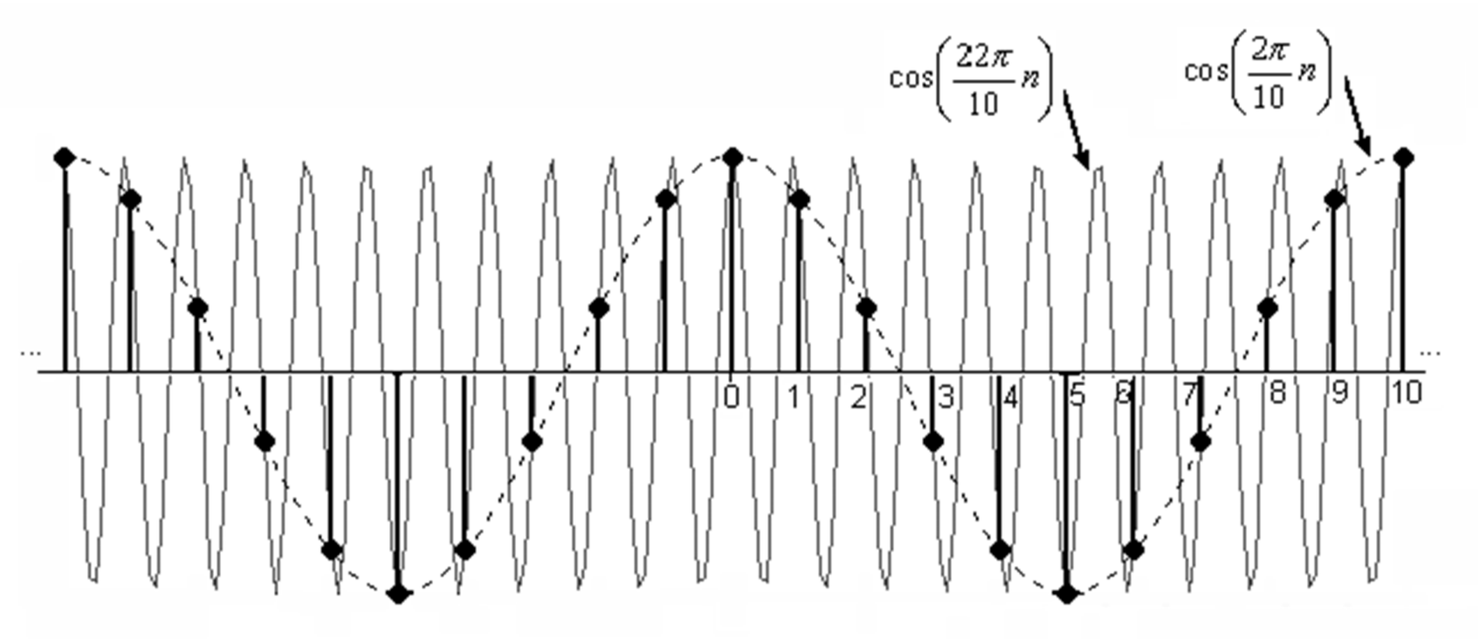
Typically, we will choose either $-\pi < \omega \leq \pi$

or

$$0 \leq \omega < 2\pi$$

Properties of Sinusoidal Signals

$$\cos[(\omega + 2\pi r)n] = \cos(\omega n)$$



Periodicity of Discrete Time Signals

$$x[n] = x[n+N] \text{ for all } n$$

$$A\cos(\omega n + \phi) = A\cos(\omega n + \omega N + \phi)$$

Periodicity requires that:

$$\omega N = 2\pi k, \; k \text{ is integer}$$

For the complex exponential signals:

$$e^{j\omega(n+N)} = e^{j\omega n}$$

Examples

Is $x[n] = \cos\left(\frac{\pi}{11}n\right)$ periodic?

Is $x[n] = 5e^{j4\pi n/7}$ periodic?