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Digital Signal Processing
Lecture 5-Supplementary

Quote of the Day

The longer mathematics lives the more abstract – and therefore, possibly also the more practical – it becomes.

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Correlation

- Correlation ~ identifies similarity between sequences:

Cross correlation of x against y

$$r_{xy}[\ell] = \sum_{n=-\infty}^{\infty} x[n]y[n-\ell]$$

"lag"

- Note: $r_{yx}[\ell] = \sum_{n=-\infty}^{\infty} y[n]x[n-\ell]$

call $m = n - \ell$

$$= \sum_{m=-\infty}^{\infty} y[m+\ell]x[m] = r_{xy}[-\ell]$$

Correlation

- Correlation: $r_{xy}[n] = \sum_{k=-\infty}^{\infty} x[k]y[k-n]$
- Convolution: $x[n] \circledast y[n] = \sum_{k=-\infty}^{\infty} x[k]y[n-k]$
- Hence: $r_{xy}[n] = x[n] \circledast y[-n]$

Correlation may be calculated by convolving with time-reversed sequence

Autocorrelation

- Autocorrelation (AC) is correlation of signal with itself:

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- Note: $r_{xx}[0] = \sum_{n=-\infty}^{\infty} x^2[n] = \varepsilon_x$ **Energy** of sequence $x[n]$

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Correlation maxima

- Note: $r_{xx}[\ell] \le r_{xx}[0] \Rightarrow \left| \frac{r_{xx}[\ell]}{r_{xx}[0]} \right| \le 1$
- Similarly: $r_{xy}[\ell] \le \sqrt{\varepsilon_x \varepsilon_y} \Rightarrow \frac{r_{xy}[\ell]}{\sqrt{r_{xx}[0] r_{yy}[0]}} \le 1$