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Digital Signal Processing
Lecture 2-Supplementary

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Discrete-time Sinusoids

- General discrete-time (DT) sinusoid sequence:

$$x[n] = A\cos(\omega_o n + \emptyset), \qquad -\infty < n < \infty$$

A - amplitude

$\emptyset$ - phase (radians)

n - sample number (an integer)

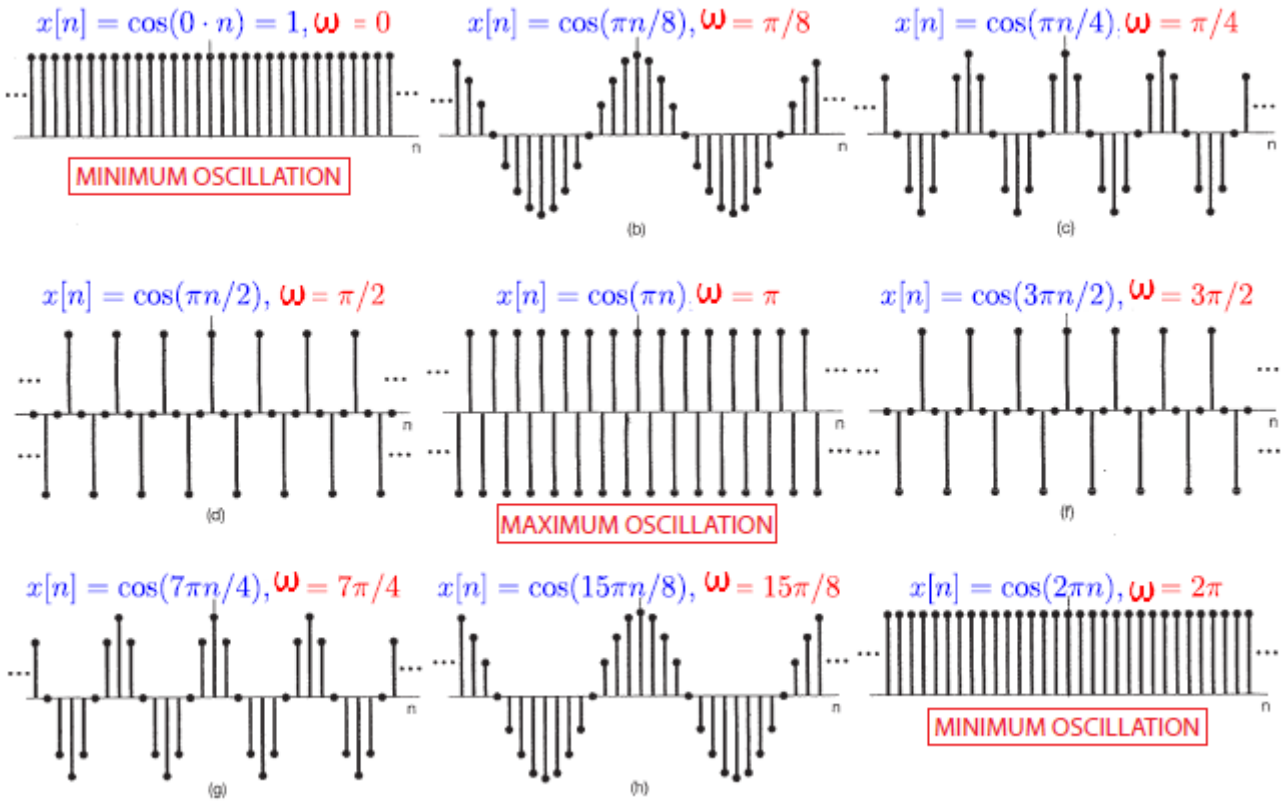
ω_o - frequency (radians/sample)

$\omega_o = 2\pi f_o$, where f_o is the frequency (cycles/sample)

Discrete-time Sinusoids

Discrete-Time Sinusoids: Frequency and Rate of Oscillation

Rate of oscillation increases as ω increases UP TO A POINT then decreases again and then increases again and then decreases again



Discrete-time Sinusoids

The rate of oscillation of : 1) Decreases as goes from $-\pi$ to 0; 2) Increases as goes from 0 to π

Frequencies around $\omega=2\pi k$ are "low" frequency; Frequencies around $\omega=\pi(2k+1)$ are "high" frequency.

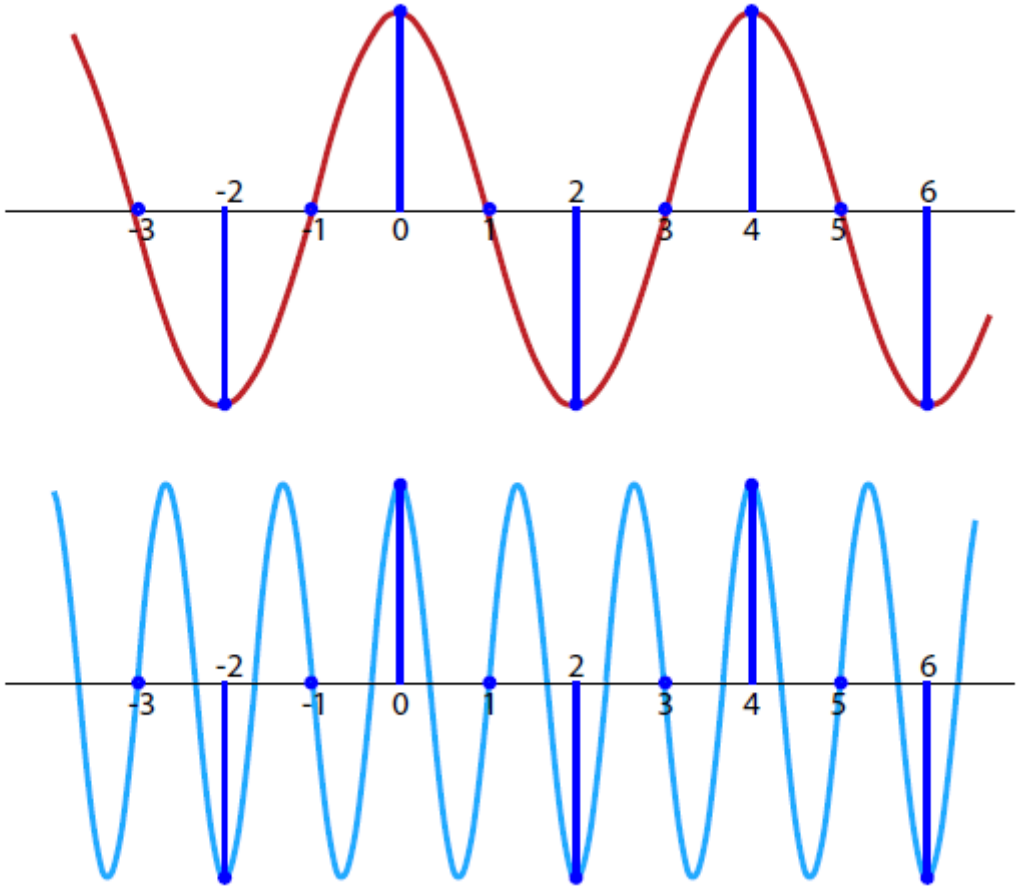
Let

$$x_1[n] = A\cos(\omega_1 n + \phi) \quad \text{and} \quad x_2[n] = A\cos(\omega_2 n + \phi)$$

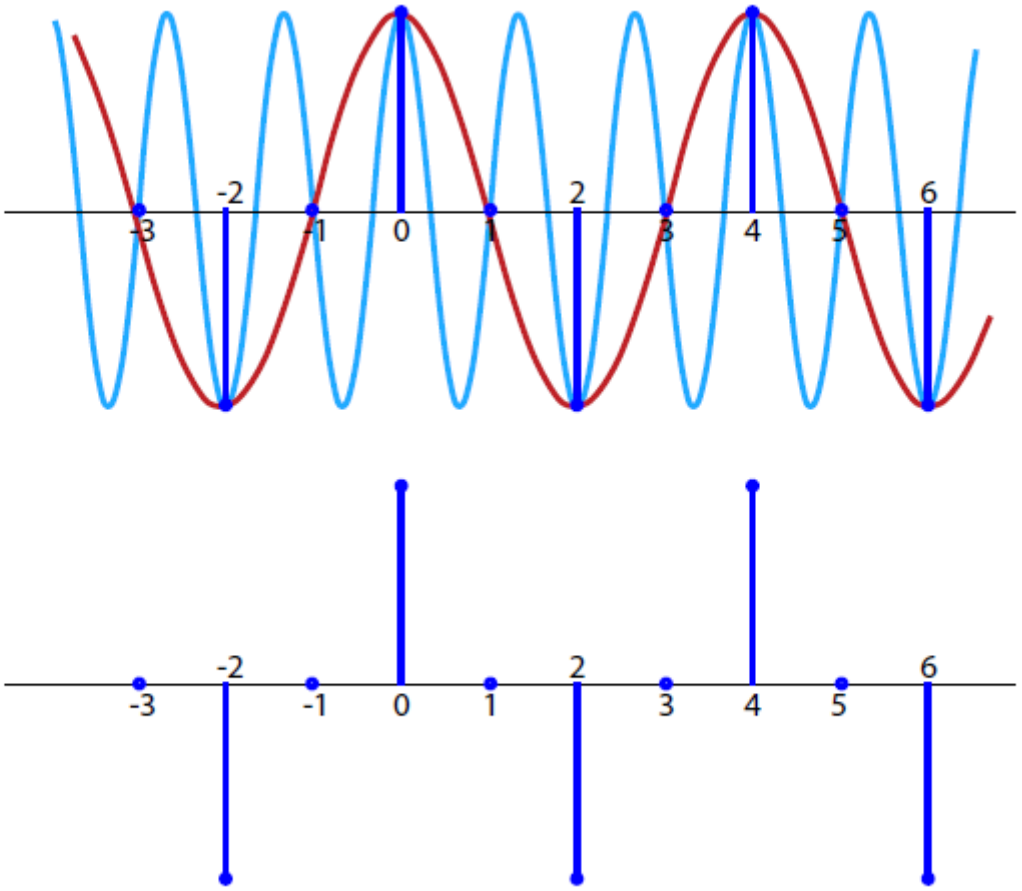
and $\omega_2 = \omega_1 + 2\pi k$ where $k \in \mathbb{Z}$:

$$\begin{aligned} x_2[n] &= A\cos(\omega_2 n + \phi) \\ &= A\cos(\omega_1 + 2\pi k)n + \phi) \\ &= A\cos(\omega_1 n + 2\pi kn + \phi) \\ &= A\cos(\omega_1 n + \phi) = x_1[n] \end{aligned}$$

Discrete-time Sinusoids

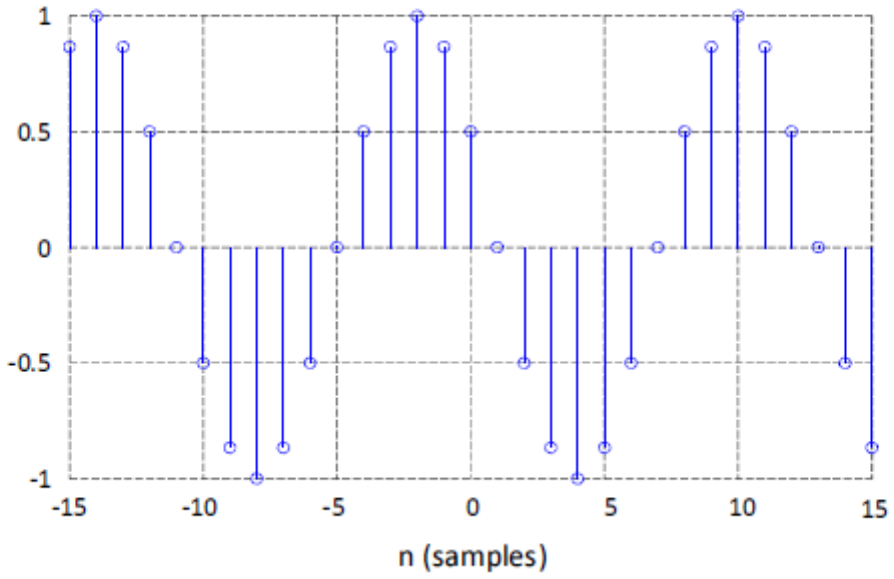


Discrete-time Sinusoids



Discrete-time Sinusoids

- For the sinusoid plot below:
  - What is the period of the sinusoid (call it *N*)?
  - What is the discrete-time frequency of the sinusoid?
  - What is the phase shift in samples? In radians?



Discrete-time Sinusoids

- The period can be determined by counting the number of samples per repeating cycle, N = 12
- There are 12 samples per 1 cycle:
  - f_o is 1 cycle/12 samples, $f_o = \frac{1}{12}$ cycles/sample
  - $\omega_o = 2\pi\frac{1}{12} = \frac{\pi}{6}$ radians/sample
- The phase shift is two samples to the left:
  - Phase shift = 2 samples * $\frac{\pi}{6}$ radians/sample = $\frac{\pi}{3}$ radians
  - Phase shift is positive as it is shifted to the left
- $x[n] = \cos\left(\frac{\pi}{6}n + \frac{\pi}{3}\right)$