Data Structures – Week #1

Introduction

Goals

- We will learn methods of how to
  - (explicit goal) organize or **structure large amounts of data in the main memory (MM)** considering efficiency; i.e,
    - ***memory space*** **and**
    - ***execution time***
  - (implicit goal) gain additional experience on
    - what data structures to use for solving what kind of problems
    - programming

Goals continued…1

- **Explicit Goal**
  - We look for answers to the following question:

"*How do we store data in MM* such that

- ***execution time*** grows as *slow* as possible with the growing size of input data, and
- data uses up *minimum* ***memory space***?"

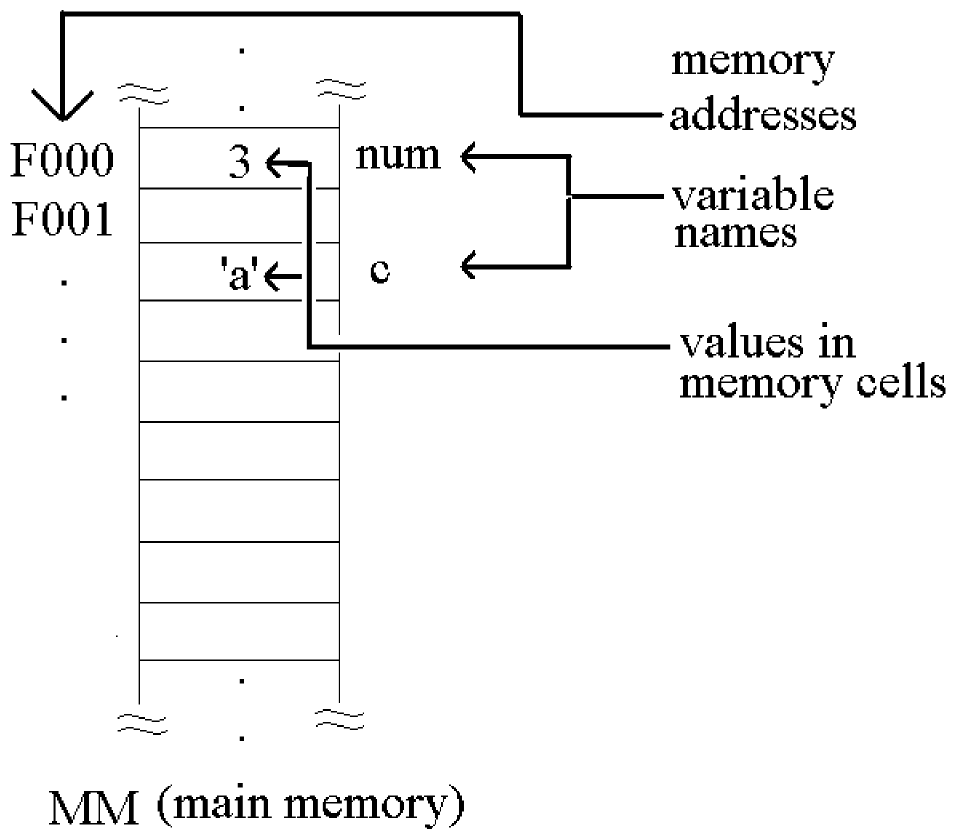
Goals continued…2

- As a tool to calculate the execution time of algorithms, we will learn the basic principles of **algorithm analysis**.
- To efficiently structure data in MM, we will thoroughly discuss the
  - ***static***, (arrays)
  - ***dynamic*** (pointers)

  ways of *memory allocations*, two fundemantal

  Implementation tools for data structures.

Representation of Main Memory



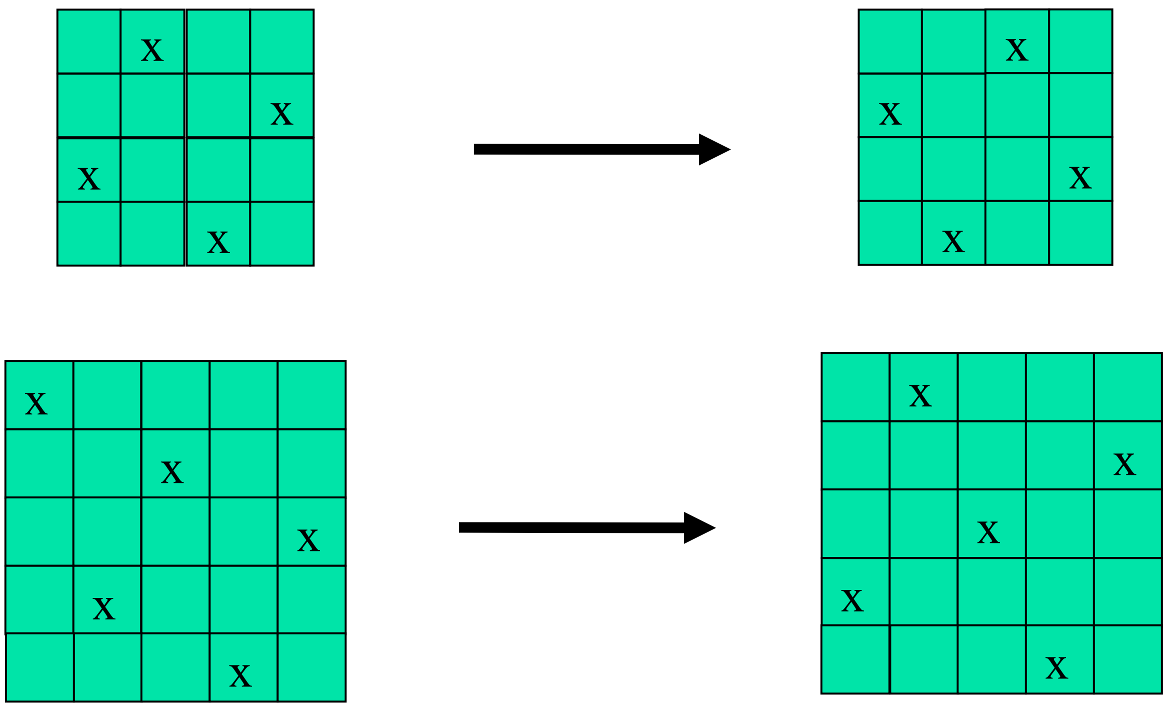
Examples for efficient vs. inefficient data structures

- 8-Queen problem
  - 1D array vs. 2D array representation results in saving memory space
  - Search for proper spot (square) using horse moves save time over square-by-square search

- Fibonacci series: A lookup table avoids redundant recursive calls and saves time

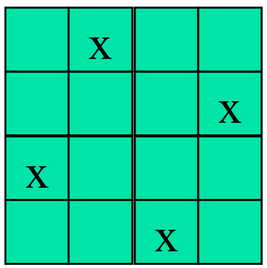
Examples for efficient vs. inefficient data structures

8-Queen problem



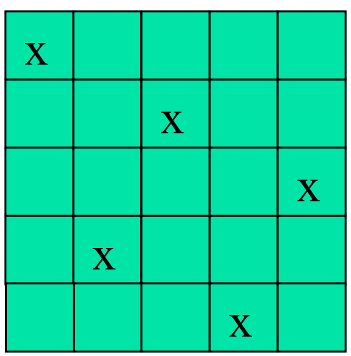
Examples for efficient vs. inefficient data structures

8-Queen problem



```
int a[4][4];
....
a[0][1]=1;
a[1][3]=1;
a[2][0]=1;
a[3][2]=1;
```

inefficient:
more memory
space required



```
int a[5];
....
a[0]=0;
a[1]=2;
a[2]=4;
a[3]=1;
a[4]=3;
```

efficient:
less memory
space required

Math Review

- Exponents

$$x^a x^b = x^{a+b}; \qquad \frac{x^a}{x^b} = x^{a-b}; \qquad (x^a)^b = x^{ab};$$

- Logarithms

$$y = x^a \Leftrightarrow \log_x y = a, \quad y > 0; \qquad \log_x y = \frac{\log_z y}{\log_z x}, \quad z > 0;$$

$$\log xy = \log x + \log y; \quad \log \frac{1}{x} = -\log x; \quad \log x^a = a \log x$$

Math Review

- Arithmetic Series: Series where the variable of summation is the base.

$$\sum_{i=1}^{k+1} i = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2};$$

$$\frac{k(k+1)}{2} + \frac{2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$$

Math Review

- Geometric Series: Series at which the variable of summation is the exponent.

$$\sum_{i=0}^{n} a^i = \frac{1-a^{n+1}}{1-a}, \quad 0 < a < 1; \quad \sum_{i=0}^{n} a^i = \frac{a^{n+1}-1}{a-1}, \quad a \in N^+ - \{1\};$$

$$\lim_{n\to\infty} \sum_{i=0}^{n} a^i = \frac{1}{1-a}, \quad 0 < a < 1;$$

$$s = \lim_{n\to\infty} \sum_{i=0}^{n} a^i = 1 + a + a^2 + a^3 + a^4 + \ldots = \frac{1}{1-a};$$

$$as = \lim_{n\to\infty} a \sum_{i=0}^{n} a^i = a + a^2 + a^3 + a^4 + \ldots = \frac{a}{1-a};$$

$$\Rightarrow \quad s - as = s(1-a) = 1$$

Math Review

- Geometric Series…cont'd
- An example to using above formulas to calculate another geometric series

$$s = \sum_{i=1}^{\infty} \frac{i}{2^i};$$

$$s = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \cdots + \frac{i}{2^i} + \cdots$$

$$2s = 1 + \frac{2}{2} + \frac{3}{2^2} + \frac{4}{2^3} + \cdots + \frac{i}{2^{i-1}} + \cdots$$

$$s = 2s - s = 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^i} + \cdots$$

$$s = \sum_{i=0}^{\infty} \frac{1}{2^i} = 2;$$

Math Review

- Proofs
  - Proof by Induction
    - Steps
      - Prove the base case ($k=1$)
      - Assume hypothesis holds for $k=n$
      - Prove hypothesis for $k=n+1$
  - Proof by counterexample
    - Prove the hypothesis wrong by an example
  - Proof by contradiction ($A \Rightarrow B \Leftrightarrow \sim B \Rightarrow \sim A$)
    - Assume hypothesis is wrong,
    - Try to prove this
    - See the contradictory result

Math Review

- Proof examples (Proofs… cont'd)
- Proof by Induction
  - Hypothesis

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

  - Steps
    1. Prove true for n=1: $\sum_{i=1}^{1} i = 1$
    2. Assume true for n=k: $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$
    3. Prove true for n=k+1:

$$\sum_{i=1}^{k+1} i = \frac{k(k+1)}{2} + k + 1 = \frac{(k+1)(k+2)}{2};$$

$$\frac{k(k+1)}{2} + \frac{2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$$

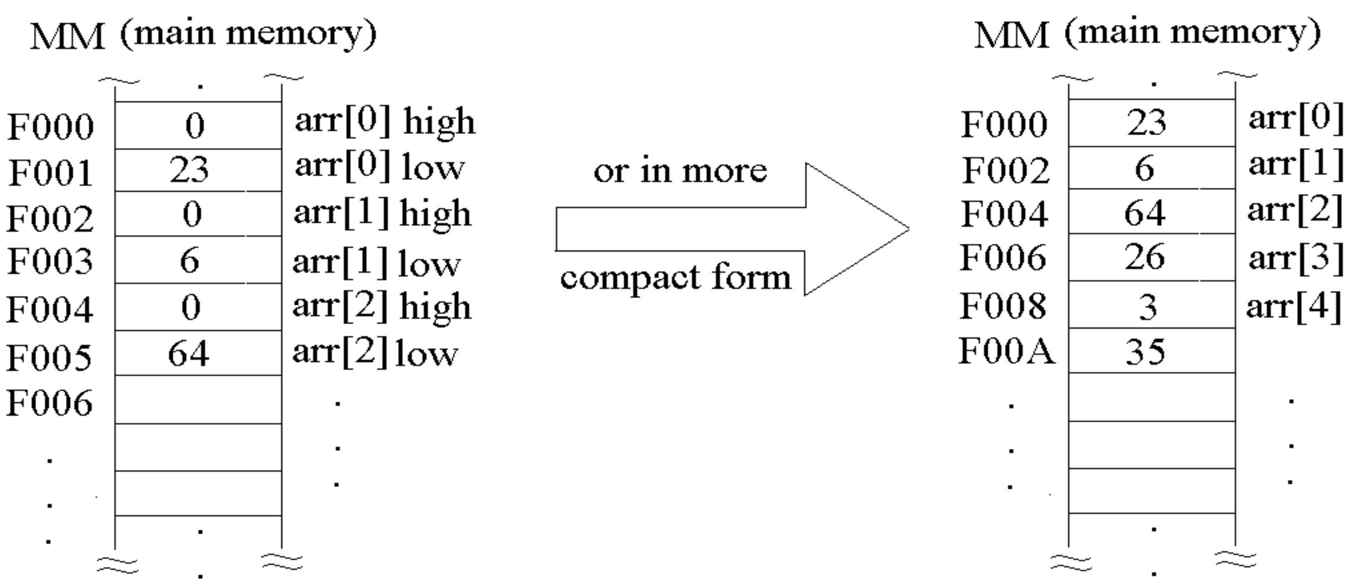
Arrays

- Static data structures that
  - represent **contiguous** memory locations holding **data of same type**
  - provide ***direct access*** to data they hold
  - have a ***constant size*** determined up front (at the beginning of) the run time

Arrays… cont'd

- An integer array example in C
- int arr[12]; //12 integers

Index →	0	1	2	3	4	5	6	7	8	9	10	11	
Value	23	6	64	26	3	35	8	56	39	48	41	12	arr



Multidimensional Arrays

- To represent data with multiple dimensions, multidimensional array may be employed.
- Multidimensional arrays are structures specified with
  - the data value, and
  - as many indices as the dimensions of array
- Example:
  - `int arr2D[r][c];`

Multidimensional Arrays

$$\begin{bmatrix} m[0][0] & m[0][1] & m[0][2] & \cdots & m[0][c-1] \\ m[1][0] & m[1][1] & m[1][2] & \cdots & m[1][c-1] \\ m[2][0] & m[2][1] & m[2][2] & \cdots & m[2][c-1] \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ m[r-1][0] & m[r-1][1] & m[r-1][2] & & m[r-1][c-1] \end{bmatrix}$$

- ***m***: **a two dimensional (2D) array with *r* rows and *c* columns**
- **Row-major representation: 2D array is implemented row-by-row.**
- **Column-major representation: 2D array is implemented column-first.**
- **In row-major rep., *m[i][j]* is the entry of the above matrix *m* at i+1th row and j+1th column. "i" and "j" are row and column indices, respectively.**
- **How many elements?** $n = r*c$ **elements**

Row-major Implementation

- Question: How can we store the matrix in a 1D array in a row-major fashion or how can we map the 2D array *m* to a 1D array *a*?

l elements

a	…	***m[0][0]***	…	***m[0][c-1]***	…	***m[r-1][0]***	…	***m[r-1][c-1]***	…

index: k $\longrightarrow$ ***k=l*** ***k=l+c-1*** ***k=l+(r-1)c+0*** ***k=l+(r-1)c+c-1***

In general, ***m[i][j]*** is placed at ***a[k]*** where ***k=l+ic+j*.**

Implementation Details of Arrays

- *Array names are pointers* that point to the first byte of the first element of the array.
    a) double vect[row_limit];// vect is a pointer!!!
- Arrays may be efficiently passed to functions using their ***name*** and their ***size*** where
    - the name specifies the beginning address of the array
    - the size states the bounds of the index values.

3. Arrays can only be copied element by element.

Implementation Details… cont'd

```
#define maxrow …;
#define maxcol …;
…
int main() {
      int minirow;
      double min;
      double probability_matrix[maxrow][maxcol];
      … ;   //probability matrix initialized!!!
      min=minrow(probability_matrix,maxrow,maxcol,&minirow);
      …;
      return 0;
}
```

Implementation Details… cont'd

```
double minrow(double arr[][maxcol], int xpos, int ypos, int *ind){
// finds minimum of sum of rows of the matrix and returns the sum
// and the row index with minimum sum.
    double min;
    …;
    mn = <a large number>;
    for (i=0; i<=xpos; i++) {
        sum=0;
        for (j=0; j<=ypos; j++)
            sum += darr[i][j];
        if (mn > sum) { mn=sum; *ind=i; }   // call by reference!!!
    }
    return mn;
}
```

Records

- As opposed to **arrays** in which we keep data of the same type, we keep related data of various types in a **record**.
- Records are used to encapsulate (keep together) related data.
- Records are composite, and hence, user-defined data types.
- In C, records are formed using the reserved word "struct."

Struct

- We declare as an example a student record called “stdType”.

- We declare first the data types required for individual fields of the record stdType, and then the record stdType itself.

Struct… Example

```
enum genderType = {female, male};       // enumerated type declared…
typedef enum genderType genderType;  // name of enumerated type shortened…
struct instrType {
…                                                 // left for you as exercise!!!
}

typedef struct instrType instrType;
struct classType {     // fields (attributes in OOP) of a course declared…
      char classCode[8];
      char className[60];
      instrType instructor;
      struct classType *clsptr;
}
typedef struct classType classType; // name of structure shortened…
```

Struct… Example continues

```
struct stdType {
      char id[8];                               //key
                  //personal info
      char name[15];
      char surname[25];
      genderType gender;                              //enumerated type
      …
                  //student info
      classType current_classes[10]; //...or      classType *cur_clsptr
      classType classes_taken[50];   //...or      classType *taken_clsptr
      float grade;
      unsigned int credits_earned;
      …
                  //next record's first byte's address
      struct stdType *sptr;            //address of next student record
}
```

Memory Issues

- Arrays can be used within records.
  - Ex: classType current_classes[10]; // from previous slide
- Each element of an array can be a record.
  - stdType students[1000];
- Using an array of classType for keeping taken classes wastes memory space (Why?)
  - Any alternatives?
- How will we keep student records in MM?
  - In an array?
  - Advantages?
  - Disadvantages?

Array Representation

Advantages

- Direct access (i.e., faster execution)

Disadvantages

- Not suitable for changing number of student records
  - The higher the extent of memory waste the smaller the number of student records required to store than that at the initial case.
  - The (constant) size of array requires extension which is impossible for static arrays in case the number exceeds the bounds of the array.

The other alternative is **pointers** that provide **dynamic memory allocation**

indices	0	1	2	…	…	n-3	n-2	n-1
students	std 1	std 2	std 3	…	…	std n-2	std n-1	std n

Array Representation

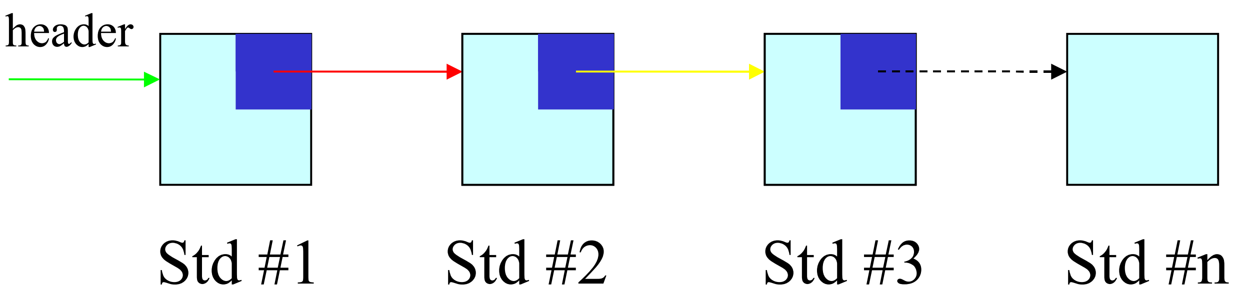
Pointers

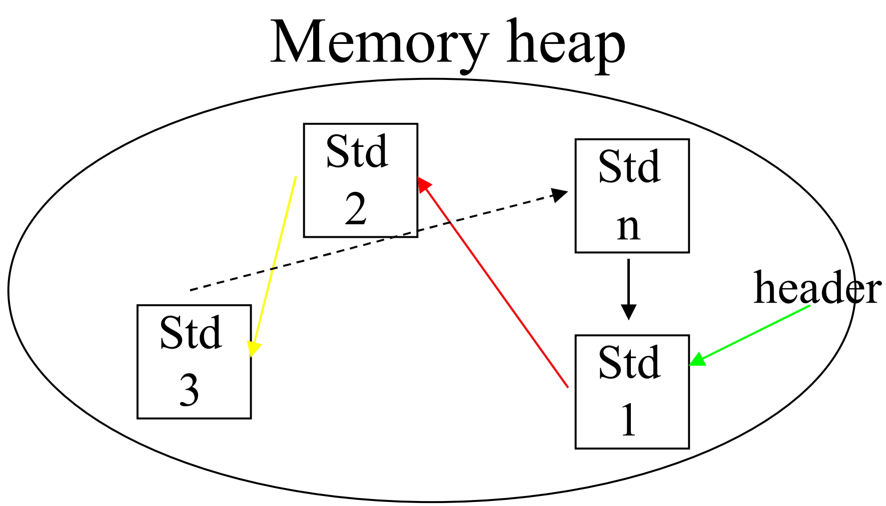
- Pointers are variables that hold memory addresses.

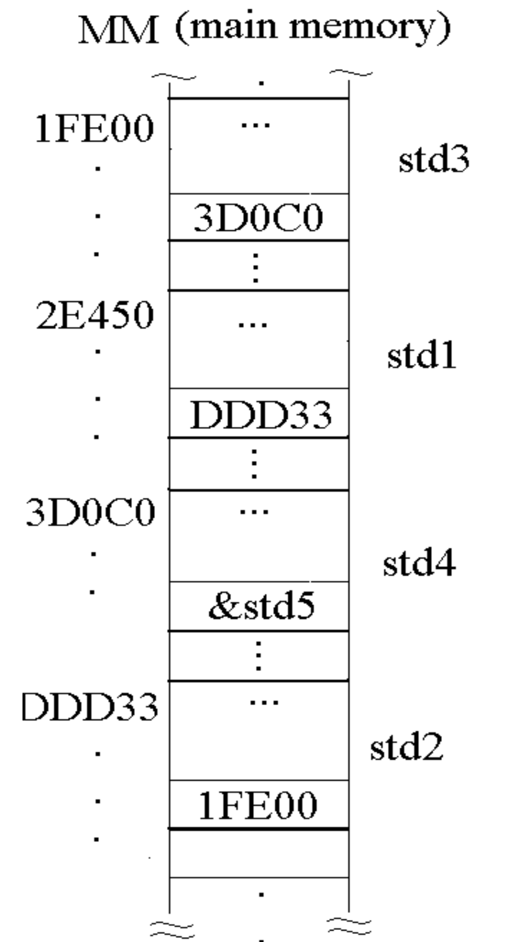
- Declaration of a pointer is based on the type of data of which the pointer holds the memory address.

  - Ex: `stdtype *stdptr;`

Linked List Representation







Value of header=**2E450**

Dynamic Memory Allocation

```
header=(*stdtype) malloc(sizeof(stdtype));
//Copy the info of first student to node pointed to by header
s =(*stdtype) malloc(sizeof(stdtype));
//Copy info of second student to node pointed to by header
Header->sptr=s;
…
```

Arrays vs. Pointers

- Static data structures
- Represented by an index and associated value
- Consecutive memory cells
- Direct access (+)
- Constant size (-)
- Memory not released during runtime (-)

- Dynamic data structures
- Represented by a record of information and address of next node
- Randomly located in heap (cause for need to keep address of next node)
- Sequential access (-)
- Flexible size (+)
- Memory space allocatable and releasable during runtime (+)