EndGameSolutions Final Report

Zafran A. Arif

Spring 2022

Contents

1 Introduction

I would like to say thank you to *EndGameSolutions* for trusting me to assist your company in working on a team-building project. The given problem and supplied data have been reviewed by me and I will provide several mathematical models and the solution for the problem in this report. This project is very interesting and challenging at the same time since there are many conditions to satisfy. I will be using Linear Optimization to solve the problems and MATLAB to assist me with the numbers.

2 Project Description and Guide

The title of this project is **Multi-period Team Assignment Planning**. From the given data, I noticed that we have five different projects for the next nine weeks. The timelines and team size for each project are given in this table:

Project Name	Au	Br	Cs	Dy	Eu
Time Window (first week - last week)	1-3	4-5	6-9	1-4	5-9
Team Size (min - max)	5-8	5-9	6-12	6-11	8-15

I have received the Excel file (*EndGameSolutions.xlsx*) containing all scores for each of the 50 available staff members. All these 50 available staff members has been recently rated and given scores, ranging from 0 to 100, for productivity, creativity and teamwork. A sample is shown in this table:

Staff Member	Productivity	Creativity	Teamwork
1	96	85	61
2	42	59	10
...	...	...	...
50	63	85	32

From the given data above, I understand that *EndGameSolutions* would like to maximize the total productivity by assigning 50 available staff members to the projects subject to the following conditions:

1. The average productivity score of staff assigned to each project should be at least 70.
2. The average creativity score of staff assigned to each project should be at least 50.
3. The average teamwork score of staff assigned to each project should be at least 50.
4. A given staff member can be assigned to more than one project as long as the projects do not overlap in time. (No one can work more than one project at a time.)
5. At least one senior staff member must be assigned to each project, and no senior staff member can be assigned to more than one project.
6. Not all staff need to be assigned to a project.

3 Mathematical Models for Each Conditions

First of all, let's define the decision variable x_{ij} as the number of staff members i in a certain project j where $i = 1, ..., 50$ and $j = 1, 2, 3, 4, 5$. Moreover, we need to assign the staff members to each project, thus

$$x_{ij} = \begin{cases} 1 & \text{if staff member } i \text{ is assigned to project } j \\ 0 & \text{otherwise.} \end{cases}$$

Let p_i be the productivity scores, c_i be the creativity scores, and t_i be the teamwork scores of the staff members. We are required to maximize the total productivity of all assigned staff members to every projects. Let z be our objective variable, then we have the objective function

$$z = \sum_{i=1}^{50} \sum_{j=1}^{5} p_i x_{ij}$$

In the formula above, we are summing all of the productivity to each staff members.

3.1 First Condition

We are asked to set the standard for *EndGameSolutions* by setting the minimum average productivity score to of staff assigned to each project to 70.

$$\frac{\sum_{i=1}^{50} p_i x_{ij}}{\sum_{i=1}^{50} x_{ij}} \geq 70 \quad \text{for } j = 1, ..., 5$$

or

$$\sum_{i=1}^{50} (p_i - 70) x_{ij} \geq 0 \quad \text{for } j = 1, ..., 5$$

3.2 Second Condition

We are asked to set the standard for *EndGameSolutions* by setting the minimum average creativity score to of staff assigned to each project to 50.

$$\frac{\sum_{i=1}^{50} c_i x_{ij}}{\sum_{i=1}^{50} x_{ij}} \geq 50 \quad \text{for } j = 1, ..., 5$$

or

$$\sum_{i=1}^{50} (c_i - 50) x_{ij} \geq 0 \quad \text{for } j = 1, ..., 5$$

3.3 Third Condition

We are asked to set the standard for *EndGameSolutions* by setting the minimum average teamwork score to of staff assigned to each project to 50.

$$\frac{\sum_{i=1}^{50} t_i x_{ij}}{\sum_{i=1}^{50} x_{ij}} \geq 50 \quad \text{for } j = 1, ..., 5$$

or

$$\sum_{i=1}^{50} (t_i - 50) x_{ij} \geq 0 \quad \text{for } j = 1, ..., 5$$

3.4 Fourth Condition

In this condition, we are required to make sure that a staff member can be assigned to more than one project as long as the projects do not overlap in time.

Notice that four projects are overlapping in time: Project Au & Dy, project Br & Dy, project Br & Eu, and project Cs & Eu. Recall that Au is the first project, Dy is the fourth, Cs is the third, and Eu is the fifth, so we're going to call them j for the respective numbers.

Thus, for $i = 1, ..., 50$, we have the equations for the overlapping projects

$$\begin{aligned}
\textbf{Au \& Dy:} \quad & x_{i1} + x_{i4} \leq 1 \\
\textbf{Br \& Dy:} \quad & x_{i2} + x_{i4} \leq 1 \\
\textbf{Br \& Eu:} \quad & x_{i2} + x_{i5} \leq 1 \\
\textbf{Cs \& Eu:} \quad & x_{i3} + x_{i5} \leq 1
\end{aligned}$$

3.5 Fifth Condition

Now, we're asked to make sure that at least one senior staff member be assigned to each project, but they can't be in more than one project.

The model for at least one senior staff member be assigned to each project is

$$\sum_{i=1}^{8} x_{ij} \geq 1 \quad \text{for } j = 1, 2, 3, 4, 5$$

The model for the senior staff can't be in more than one project is

$$\sum_{j=1}^{5} x_{ij} \leq 1 \quad \text{for } i = 1, ..., 8$$

3.6 Sixth Condition

Although that *EndGameSolutions* has 50 available staff members, not all staff need to be assigned to a project. No mathematical model needed for this condition.

3.7 Seventh Condition

Remember that each project has different team size, thus

$$\text{The team size of Project Au:} \quad 5 \leq \sum_{i=1}^{50} x_{i1} \leq 8$$

$$\text{The team size of Project Br:} \quad 5 \leq \sum_{i=1}^{50} x_{i2} \leq 9$$

$$\text{The team size of Project Cs:} \quad 6 \leq \sum_{i=1}^{50} x_{i3} \leq 12$$

$$\text{The team size of Project Dy:} \quad 6 \leq \sum_{i=1}^{50} x_{i4} \leq 11$$

$$\text{The team size of Project Eu:} \quad 5 \leq \sum_{i=1}^{50} x_{i5} \leq 15$$

4 Final Mathematical Model

$$
\begin{aligned}
\max_{x} \quad & z = \sum_{i=1}^{50} \sum_{j=1}^{5} p_i x_{ij} \\
\text{s.t.} \quad & \sum_{i=1}^{50} (p_i - 70) x_{ij} \geq 0 \quad \text{for } j = 1,2,3,4,5 \\
& \sum_{i=1}^{50} (c_i - 50) x_{ij} \geq 0 \quad \text{for } j = 1,2,3,4,5 \\
& \sum_{i=1}^{50} (t_i - 50) x_{ij} \geq 0 \quad \text{for } j = 1,2,3,4,5 \\
& x_{i1} + x_{i4} \leq 1 \quad \text{for } i = 1,...,50 \\
& x_{i2} + x_{i4} \leq 1 \quad \text{for } i = 1,...,50 \\
& x_{i2} + x_{i5} \leq 1 \quad \text{for } i = 1,...,50 \\
& x_{i3} + x_{i5} \leq 1 \quad \text{for } i = 1,...,50 \\
& \sum_{i=1}^{8} x_{ij} \geq 1 \quad \text{for } j = 1,2,3,4,5 \\
& \sum_{j=1}^{5} x_{ij} \leq 1 \quad \text{for } i = 1,...,8 \\
& 5 \leq \sum_{i=1}^{50} x_{i1} \leq 8 \\
& 5 \leq \sum_{i=1}^{50} x_{i2} \leq 9 \\
& 6 \leq \sum_{i=1}^{50} x_{i3} \leq 12 \\
& 6 \leq \sum_{i=1}^{50} x_{i4} \leq 11 \\
& 5 \leq \sum_{i=1}^{50} x_{i5} \leq 15 \\
& x_{ij} \in \mathbb{Z}
\end{aligned}
$$

5 Result

After doing several calculations to the model that we have created, we can see that *EndGameSolutions* need to spread their staff members into the projects to get an optimal solution. I introduce a strategy where we get the best total productivity and here is a table to help us visualize the optimal solution.

Project Name	Productivity (total)	Productivity (average)	Creativity (average)	Teamwork (average)	Staff Members
Au	422	70.33	50.33	50.17	**6**,14,30,37,41,48
Br	431	71.83	50.00	50.00	**1**,2,7,16,40,48
Cs	420	70.00	50.50	50.00	**3**,18,35,41,47,48
Dy	422	70.33	50.00	50.00	**4**,31,32,42,43,44
Eu	420	70.00	50.33	50.00	**8**,10,12,24,26,37
Total	2115	70.49	50.23	50.03	

6 Executive Summary

The project **Multi-period Team Assignment Planning** that was trusted to me to handle is done. Remember that every staff members have a productivity, creativity, and teamwork score. Thus, I had to do very careful mathematical modeling and calculation to ensure the best output.

From the table in **Section 5. Result**, we can conclude that it is possible to create such teams that need to satisfy several conditions (refer to **Section 2. Project Description and Guide** on page 2) to maximize the total productivity. Hence, I introduced a strategy to get the best output so that *EndGameSolutions* can assign the available staff members and maximize their productivity.

Recall that the goal is to maximize the total productivity of the available staff members. The table that I provided is the best strategy to maximize productivity with the best average productivity, creativity, and teamwork scores. The scores on the table met the required minimum where at least the average score for productivity is 70. Also, the average creativity and teamwork scores are both above 50. We also have a nice even spread of staff members in each project. With six staff members in a project, there is at least one senior staff member to help and lead the team. Therefore, all of the constraints are met and we have a nice optimal solution to the problems.

Although it is clear that the scores in the table are similar, and there is no significant difference among the teams, notice that Project Br has the best team compared to the other projects. This team that is led by senior staff member **1** has the highest total productivity score (431) among other teams. The average productivity score is also the highest with a score of 71.83. However, the creativity and teamwork scores are both 50 which is not too prominent.

In conclusion, I suggest *EndGameSolutions* use the provided strategy to maximize the total productivity of the available staff members in each project. That is, assign staff members **6**,14,30,37,41,48 to **Project Au**, **1**,2,7,16,40,48 to **Project Br**, **3**,18,35,41,47,48 to **Project Cs**, **4**,31,32,42,43,44 to **Project Dy**, and **8**,10,12,24,26,37 to **Project Eu**.