

Assignment 4:

The results of Trapezoidal and Simpson method:

N	Trapezoidal_Error	Trapezoidal_Order	Simpson_Error	Simpson_Order
20	0.016275	NaN	5.6311e-05	NaN
40	0.0040265	2.0151	3.5187e-06	4.0003
80	0.001004	2.0038	2.1983e-07	4.0006
160	0.00025083	2.0009	1.3713e-08	4.0027

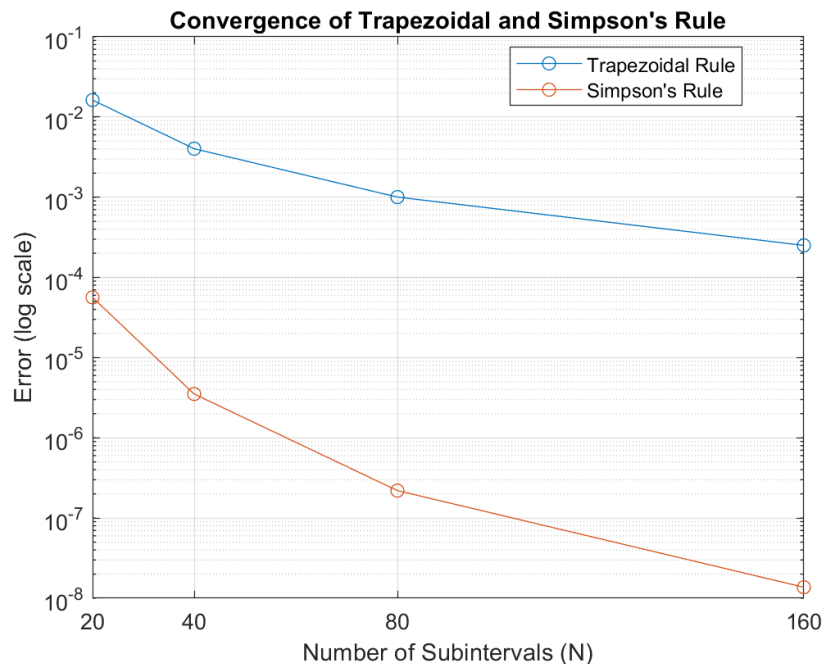
trapezoidal_error is the error associated with using the Trapezoidal Rule for numerical integration. The error decreases as N increases because with more subintervals, the Trapezoidal Rule provides a better approximation to the exact integral. Essentially, as N grows, the approximation becomes more accurate.

trapezoidal_order is order of accuracy for the Trapezoidal Rule which is around 2 as we expected. This means that the error decreases by a factor of N^2 as we double the number of subintervals.

simpson_error is the error associated with Simpson's Rule. Similar to the Trapezoidal Rule, the error decreases as N increases because Simpson's Rule becomes more accurate with finer subintervals. This reduction in error is due to a more accurate representation of the function under the curve.

simpson_order is order of accuracy of approximately 4. This is higher than the Trapezoidal Rule. The 4 signifies that the error decreases by a factor of N^4 as we double the number of subintervals. Therefore, increasing N has a more pronounced effect on reducing the error with Simpson's Rule compared to the Trapezoidal Rule.

The order of accuracy for each rule provides insight into how quickly the error decreases with increasing N, with Simpson's Rule showing a faster convergence rate due to its higher order.



For the adaptive quadratic, the process began by selecting the errors of the Simpson and Trapezoidal methods at $N = 160$. Subsequently, the error of the adaptive quadratic method was set to these selected errors. The number of iterations required for the adaptive method to attain the desired errors, based on the Simpson and Trapezoidal methods, was then computed.

The number of iterations for the adaptive method was 128, which was close to the value of 160, corresponding to the Simpson error, indicating a sensible result and this method works better than Simpson. However, for the Trapezoidal method, the number of iterations was calculated to be 12, which appeared to be an unexpected outcome. I went over the code over and over, but I could not understand what is the problem. I appreciate any feedback and I try to implement them in my code.

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Number of iterations (Adaptive Simpson's Rule) for error 0.0000000137133307 (Simpson's Rule N=160): 128  
Number of iterations (Adaptive Simpson's Rule) for error 0.0002508336528969 (Trapezoidal Rule N=160): 12
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