

SDSU COMP521

Fall 2023

Homework 07 - Due Date: Nov. 21 2023

Problem 1

Solve the one-way wave equation (hyperbolic PDE):

$$u_t + u_x = 0$$

where

$$u(x, 0) = u_0(x) = \begin{cases} 1 - |x| & \text{if } |x| < 1 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

Use **Lax-Friedrichs** with space and time domains of $x \in [-2, 2]$ and $t \in [0, 8]$ respectively. Use left boundary condition $u(-2, t) = u_0(-2 - t)$ and right boundary condition $u(2, t) = u_0(2 - t)$. **Write your own code.**

1. Show the plot of the numerical and exact solutions $u(x, t)$ at the first and last time points and three points inside the interval (your choice). Analyze and discuss the behavior.
2. Plot the error metric on a log-log plot for different values of Δx you choose, at the final time step. Based on this plot, determine the order of spatial accuracy.

Problem 2

Find the numerical solution for the following heat equation:

$$u_t - u_{xx} = 0 \quad \text{for } 0 < x < 1 \quad \text{and} \quad 0 \leq t \leq 0.1$$

, with the initial condition $u(x, 0) = f(x) = \sin(\pi x) + \sin(3\pi x) \quad \forall x \in [0, 1]$ and boundary conditions:

$$\begin{aligned} u(0, t) = c_1 = 0 & \quad \text{for } x = 0 \quad \text{and} \quad 0 \leq t \leq 0.1 \\ u(1, t) = c_2 = 0 & \quad \text{for } x = 1 \quad \text{and} \quad 0 \leq t \leq 0.1 \end{aligned}$$

Solve the problem using the explicit scheme learned in class. Start with $\Delta x = 0.2$ and $\Delta t = 0.02$.

1. Show the plot of the numerical and exact solutions U and $u(x, t)$. You could use contour plots. Analyze and discuss the behavior. The exact solution is: $u(x, t) = \sin(\pi x)e^{-\pi^2 t} + \sin(3\pi x)e^{-9\pi^2 t}$
2. Plot the error metric on a log-log plot for different values of Δx you choose, at the final time step. Based on this plot, determine the order of spatial accuracy.

Deliverable

You have to present a **REPORT** and submit it as a file in .PDF format. This report must describe the solution of each problem. It must describe and explain the results. Do not forget to identify the plots and tables. The report must submit the script used to call the functions.