SDSU COMP521
Fall 2023

Final Project - Topic 2

December 14, 2023

Problem Description

Numerically estimating differential functions of arbitrary degree can be difficult to implement. We have done first and second derivatives with finite difference schemes, but think about how a third, fourth, or tenth order derivative would look. An alternate path to these techniques is to use the inverse of a Vandermonde matrix to calculate the derivatives of polynomials. The inverse of a Vandermonde matrix can be directly computed, making the technique computationally cheap.

Using the paper from Hassan et. al.[1], implement the scheme for calculating the closed form of the Vandermonde matrix inverse. Test the scheme on the following functions with orders of accuracy $(O) \in [1, 2, 3, 4, 5]$;.

Equation 1

Using the function

$$f(x) = e^x$$

estimate the derivatives using the inverse Vandermonde matrix for step sizes $h \in [0.1, 0.050.0250.0125]$ on the interval $x \in [0, 1]$. Do this for the first and second derivatives. Show and discuss the accuracy of the scheme, and the time needed to solve. Compare this to a standard finite difference scheme of your choice.

Equation 2

Using the function

$$f(x) = e^{-5x} \sin(75x)$$

estimate the first derivative using the inverse Vandermonde matrix with step size $h = 0.01$ on the domain $x \in [0, 1]$. Many times the derivative must be estimated from observational data which may be noisy, and this noise can significantly effect the accuracy of a numerical scheme. Add some noise to the initial function data, and discuss how well the method works for higher order derivatives with noisy data. Compare the speed of the solution with MATLABs technique for finding the derivative of data. How do they compare? How about when there are more data points $(h = 0.001, 0.0001)$?

Deliverable

You have to present a **REPORT** and submit it as a file in .PDF format. This report must describe the solution of each problem. It must describe and explain the results. Do not forget to identify the plots and tables. The report must present any script used.

The report must cover all areas from the project guidelines handout. Make sure to include:

- The report presents a plot with the error metrics for different first and second derivatives of equation 1. Plots should show expected and actual order of convergence.
- Plot the results for the finest grid for each equation.
- A plot with the times needed for the Vandermonde matrix and equivalent MATLAB estimates.

- Of course, as always, discuss your results. Be sure to include specifics about errors, error rate, etc.

References

[1] Hassan Zohair Hassan, AA Mohamad, and GE Atteia. An algorithm for the finite difference approximation of derivatives with arbitrary degree and order of accuracy. *Journal of Computational and Applied Mathematics*, 236(10):2622–2631, 2012.