Micro Theory IIIb - Hemicontinuity

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We are concerned now with correspondences, which can map x to some set of ys. Intuitively that corresponds to flat regions on indifference curves.

Upper Hemicontinuous

Suppose a correspondence $f : X \to Y$. Suppose $f(x) \subseteq$ some compact subset of Y. As a heuristic, I say that f is hemicontinuous if the graph of f is a closed set. That means:

- For any convergent sequence $x_n \in X : x_n \to x$, and
- if there is $y_n \in f(x_n)$ that converges to y; then
- $y \in f(x)$.

Intuitively, imagine the correspondence given by $f(x) = [x, x+1)$. Suppose $x_n = 1 - 1/n$. This converges to 1. Then $f(x_4)$ will be $[3/4, 7/4)$. Choose y_n as $2 - 2/n$. Then $y_4 = 2 - 2/4 = 3/2 \in f(x_4)$. It's apparent that y_n converges to 2 , but 2 is not in $f(1)$.

Lower Hemicontinous

More chill condition. No compact codomain needed.

If $\forall x_n \to x$, $y \in f(x)$ there exists some $y_n \in f(x_n)$ such that $n \geq N \implies y_n \to y$.