Micro Theory IIa - Elasticity

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Elasticity Basics

In utility maximization and other calculus problems we'll have quantities like $\frac{\partial x_i(p,w)}{\partial p_k}$. This tells you: if the price of good k increases a tiny amount, how much more good x_i will you demand? For normalization reasons, we might want this in percentage terms: If the price of good k increases by $q\%$, by what percentage will demand for x_i increase?

We can get this by turning our 'tiny amounts' into 'tiny percentages':

$$\epsilon_{ik} = \frac{\partial x_i(p,w)}{x_i(p,w)} \Big/ \frac{\partial p_k}{p_k} = \frac{\partial x_i(p,w)}{\partial p_k} \cdot \frac{p_k}{x_i(p.w)}$$

Observing that $\frac{\partial}{\partial x} \log a(x) = a'(x)/a(x)$, we can transform this:

$$\epsilon_{ik} = \frac{\partial \log x_i(p,w)}{\partial \log p_k}$$

ϵ_{ik} is the price elasticity of good i with respect to price k. If $i \neq k$, we can call this a cross-price elasticity of demand.

With the same tool we can construct ϵ_{iw}, the income elasticity of any given good:

$$\epsilon_{iw} = \frac{\partial x_i(p,w)}{\partial w} \cdot \frac{w}{x_i(p,w)} = \frac{\partial \log x_i(p,w)}{\partial \log w}$$

Marginal Rate of Substitution

Define the marginal rate of substitution as the number of units of i required to compensate for a unit loss in good j. This is simply:

$$MRS_{ij} = \frac{MU_i}{MU_j}$$

the ratio of the marginal utilities of i and j. We can think of this as the (negative) slope of the indifference curve. Thus, a concave indifference curve arises if MU_i falls as i rises, and/or MU_J rises as j falls (diminishing marginal utility); then as $j \to 0$, more and more of i is required to substitute for j.

Elasticity of Substitution

Construct an elasticity of substitution by asking: 'how much does the ratio of goods demanded change when the relative prices change'? In other words,

$$E_{ik} = \frac{\partial(x_i(p,w)/x_j(p,w))}{\partial(p_i/p_j)} \cdot \frac{p_i/p_j}{x_i(p,w)/x_j(p,w)} = \frac{\partial \log[x_j(p,w)/x_i(p,w)]}{\partial \log(p_1/p_2)}$$

The first-order conditions for utility maximization tell us that $p_1/p_2 = MU_i/MU_j = MRS_{ij}$. So we can also write:

$$E_{ik} = \frac{\partial \log[x_j(p,w)/x_i(p,w)]}{\partial \log MRS_{ij}}$$

The income elasticity can, I think, be expressed as a special case of the elasticity of substitution between good i and the numeraire good/'money' with price 1.