Micro Theory IIIa - Concave, Convex, Quasi

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Econ 500 Micro Theory Index

Convexity Definitions

- A convex combination of x, y is $\lambda x + (1-\lambda)y$.
- A set A is convex if for any $x, y \in A$, the convex combo of x, y is also in A.
- A set is strictly convex if for any $x, y \in A$, the convex combo is in the interior of A
- Preferences $\succsim$ are convex if the upper contour sets are convex, i.e. if $\forall x \in X$ the set $y \in X : y \succsim x$ is convex.
- Preferences are strictly convex if $y \succsim x$ and $z \succsim x$ with $y \neq z$ means that all convex combinations of y, z are strictly preferred to x.

Quasiconcavity Definitions

Smiley face

- u is quasiconcave (QCV) if the upper contour sets are convex: $\forall x,\, y : u(y) \geq x$ is convex.
- u is strictly QCV (SQCV) if $u(y) \geq u(x)$ and $u(z) \geq u(x)$ implies that all convex combinations of y, z have strictly higher utility than x. This is the inverse of Jensen's inequality.

Simpler conditions:

- u is QCV if $\forall x, y,\, u(\lambda x + (1-\lambda)y) \geq \min\{u(x), u(y)\}$
- u is SQCV if $\forall x, y,\, u(\lambda x + (1-\lambda)y) > \min\{u(x), u(y)\}$

This all tells us that SOC hold for utility maximization.

Quasiconvexity

Frowny Face

Quasiconvexity (QCX) is quite similar to quasiconcavity.

- Intuitively: we now want lower contour sets to be convex.
- u is QCX when $f(\lambda x + (1-\lambda)y) \leq \max\{f(x), f(y)\}$. In other words, convex combinations lie below their endpoints.
- Strictly quasiconvex - SQCX - if $f(\lambda x + (1-\lambda)y) < \max\{f(x), f(y)\}$