MAT274 Lecture Notes 2/8/2006 3.1 2nd order linear homogenous equation with constant coeff.A second order ODE has form

$$y'' + p(t)y' + q(t)y = g(t)$$

If g(t) = 0 for all t the equation homogenous (NODE) In this section we will look at homogenous equation with constant coefficient

$$ay'' + by' + cy = 0 \text{ — a, b, c = numbers}$$

Initial conditions: typically have the form

$$y(t_0) = y_0 \text{ — } y'(t_0) = y_0'$$

Thus the solution passes through (see ya) and the slope of tangent at t_0 is y_0' example:

$$\begin{aligned} &y'' - y = 0 \quad \text{2nd order linear} \\ &\text{solutions} \quad y = e^t \; y_2 = e^{-t} \end{aligned}$$

In general y(t) = c1 e$\hat{\text{t}}$ + c2 e$\hat{\text{-}}$t (called general solution) is a solution with c1 and c2 any constant

$$\begin{aligned} y'(t) &= c_1 e^t - c_2 e^{-t} \\ y''(t) &= c_1 e^t - c_2 e^{-t} \\ y''(t) - y(t) &= c_1 e^t - c_2 e^{-t} - (c_1 e^t + c_2 e^{-t} = 0 \end{aligned}$$

Now let's add initial conditions y(0) = 3 and y'(0) = 1 find c1 and c2

$$\begin{aligned} y(0) \to t = 0 \text{ and } y = 3 &\quad 3 = c_1 + c_2 \\ y' = c_1 e^t - c_2 e^{-t} \to y'(0) = 1 &\quad 1 = c_1 - c_2 \end{aligned}$$

Solve

$$
\begin{aligned}
3 &= c_1 + c_2 \\
1 &= c_1 - c_2 \\
\hline
4 &= 2c_1 \to c_1 = 1, \quad c_2 = 1
\end{aligned}
$$

$y = 2e^t + e^{-t}$

In general: To solve a 2nd order NODE a y" + b y' + c y = 0 (*) we begin by assuming a solution of the form

$$y = e^{rt}$$

plug in

$y = e^{rt} \quad y' = re^{rt} \quad y'' = r^2e^{rt}$

$ay'' + by' + cy = ar^2e^{rt} + bre^{rt} + ce^{rt} = 0$

$e^{rt}(ar^2 + br + c = 0)$

exponential is never 0

$$ar^2 + br + c = 0 \to \text{characteristic equation of NODE}$$

Solve the equation for r

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \to b$$

we have 3 possibilities

$$b^2 - 4ac > 0 \to \text{ 2 real solutions}(r_1 \text{ and } r_2)$$

$$\to y = e^{r_1 t} \quad y = e^{r_2 t}$$

$$b^2 - 4ac = 0 \to \text{one real solution } r_1 \text{ we will see}$$

$$b^2 - 4ac < 0 \to \text{2 complex solutions (sec 3.4)}$$

In this section we will assume we have 2 real solutions r1 and r2

$$\to \text{ general solution } y = c_1e^{r_1 t} + c_2e^{r_2 t}$$

example: find general solution of

$$y'' + y' + 2y = 0$$

characteristic equation

$$r_2 + r - 12 = 0$$
$$(r-3)(r+4) = 0$$
$$r_1 = 3 \quad r_2 = -4$$

$$\text{general solution: } \to y = c_1 e^{3t} + c_2 e^{-4t}$$

example:

$$4y'' - y = 0 \quad y(-2) = 1 \text{quad} y'(-2) = 1$$

$$\text{characteristic equation}$$
$$4r^2 - 1 = 0 \to \text{ be careful}$$
$$r = \pm \frac{1}{2}$$
$$y = c_1 e^{\frac{r}{2}} + c_2^{\frac{-r}{2}}$$

Now find c1 and c2 using IC

$$y(-2) = 1 \to 1 = c_1 e^{-1} + c_2 e$$
$$y' = \frac{c_1}{2} e_{\frac{t}{2}} - \frac{c_2}{2} e^{\frac{-t}{2}}$$
$$y'(-2) = -1 \to -1 = \frac{c_1}{2} e^{-1} - \frac{c_2}{2} e$$

$$1 = c_1 e^{-1} + c_2 e \to 1 = c e^{-1} + c_2 e$$
$$-1 = \frac{c_1}{2} e^{-1} - \frac{c_2}{2} e^{1} \to -2 = c_1 e^{-1} - c_2 e$$
$$-1 = 2 c_1 e^{-1}$$
$$c_1 = \frac{-e}{2} \to c_2 = \frac{3}{2} e^{-1}$$

plug in C1 and C2 into general solutoin.

$$y = \frac{e^{1+\frac{t}{2}}}{2} + \frac{3}{2} e^{-1-\frac{t}{2}} \to \text{ note it goes to } \infty$$

example

$$2y'' - 3y' = 0 \quad y(0) = 1 \quad y'(0) = 3$$
$$2r^2 - 3r = 0 \quad r = 0, r = \frac{3}{2}$$

$$y = c_1 + c_2 e^{\frac{3}{2}t}$$
$$y(0) = 1 \to 1 = c_1 + c_2$$

$$y' = \frac{3}{2} c_2 e^{\frac{3}{2}t}, y'(0) = 3 \to 3 = \frac{3}{2} c_2$$

$$c_2 = 2 \to c_1 = -1$$
$$y = -1 + 2e^{\frac{3}{2}t}$$

MATH 272 Lecture Notes 19/13/2005 Iterated Integrals over Rectangles:suppose f(x,y) is continuous over a rectangle (a ¡= x ¡= b, c ¡= y ¡= d)

$$\text{want to evaluate} \quad \int\int_R f(x,y)\mathrm{d}A$$
$$\int\int_R f(x,y)\mathrm{d}A = \int_a^b \int_c^d f(x,y)\mathrm{d}y\mathrm{d}x$$

assume x is constant, integrate yexample

$$\int\int_R x + t\mathrm{d}A, R = 0 \le x \le 1, 2 \le y \le 3$$
$$\int_0^1 \int_2^3 x + y\mathrm{d}x\mathrm{d}y = \int_0^1 xy + \frac{y^2}{2}]_2^3 \,\mathrm{d}x = \int_0^1 3x + \frac{9}{2} - 2x - 2\mathrm{d}x$$
$$= \int_0^1 x + \frac{5}{2}\,\mathrm{d}x = \frac{x^2}{2} + \frac{5}{2}x]_0^1 = 3$$

we can reverse the orderFubini theorem:

$$\iint_R f(x,y)\,\mathrm{d}A = \int_a^b \int_c^d f(x,y)\,\mathrm{d}y\mathrm{d}x = \int_c^d \int_a^b f(x,y)\mathrm{d}x\mathrm{d}y$$

example:

$$\iint_R \frac{xy^2}{x^2+1}\,\mathrm{d}A\,\mathrm{R} : 0 \le x \le 1 \ -3 \le y \le 3$$
$$\int_0^1 \int_{-3} 3\frac{xy^2}{x^2+1}\,\mathrm{d}x\mathrm{yx} = \int_0^1 10\left(\frac{x}{x^2+1}\right)\,\mathrm{d}x$$

$$18\int_0^1 \frac{x}{x^2+1} = \frac{u}{2}\int \frac{1}{u} = 9\ln(x^2+1)]_\infty^1 = 9\ln 2$$

Now change order:

$$\begin{aligned}
\int_{-3}^{3}\int_0^1 \frac{xy^2}{x^2+1}\,\mathrm{d}x\mathrm{d}y &= \int_{-3}^{3} y^2 \int_0^1 \frac{x}{x^2+1}\,\mathrm{d}x\mathrm{d}y \\
&= \int_{-3}^{3} y^2 \frac{1}{2}\ln(x^2+1)]_0^1 \mathrm{d}y \\
&= \frac{\ln 2}{2}\int_{-3} 3y^2 \mathrm{d}y \\
&= \frac{18}{2}\ln 2 = 9\ln(2)
\end{aligned}$$

example

$$\iint_R y\sin(xy)\mathrm{d}A \quad 1 \le x \le 2 \quad 0 \le y \le \frac{\pi}{2}$$

$$\int_0^{\frac{\pi}{2}}\int_1^2 y\sin(xy)\mathrm{d}x\mathrm{d}y = \int_0^{\frac{\pi}{2}} y\frac{-\cos(xy)}{y}]_1^2 \mathrm{d}y$$

$$\int_0^{\frac{\pi}{2}} -\cos(2y)+\cos(y)\mathrm{d}y = \frac{-\sin(2y)}{2}+\sin(y)]_0^{\frac{\pi}{2}}$$

$$\frac{-\sin(\frac{\pi}{2})}{2}+\sin(\frac{\pi}{2})\ldots = \sin\frac{\pi}{2} = 1$$